GAUGE BOSON PAIRS at the LHC:
ELECTROWEAK CORRECTIONS
and MONTE CARLO IMPLEMENTATION

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(with A. Bierweiler, S. Gieseke, T. Kasprzik, A. Penin, S. Uccirati)

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I. Introduction

"Typical" size of electroweak corrections: \( \frac{\alpha_{\text{weak}}}{\pi} \approx 10^{-2} \)

new aspects at LHC: \( \sqrt{s} \approx 1\text{-}2\text{TeV} \gg M_{W,Z} \)

strong enhancement of negative corrections

one-loop example: massive U(1)

\[ \Rightarrow \text{Born} \ast \left[ 1 + \frac{\alpha}{4\pi} \left( - \ln^2 \frac{s}{M^2} + 3 \ln \frac{s}{M^2} - \frac{7}{2} + \frac{\pi^2}{3} \right) \right] \]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\frac{s}{M^2} & - \ln^2 \frac{s}{M^2} & + 3 \ln \frac{s}{M^2} & - \frac{7}{2} + \frac{\pi^2}{3} & \Sigma & \ast 4 \frac{\alpha_{\text{weak}}}{4\pi} \\
\hline
\left( \frac{1000}{80} \right)^2 & -25.52 & +15.15 & -0.21 & -10.6 & -13\% \\
\left( \frac{2000}{80} \right)^2 & -41.44 & +19.31 & -0.21 & -22.3 & -27\% \\
\hline
\end{array}
\]

(five-fermion cross section \( \Rightarrow \) factor 4)
• leading log$^2$ multiplied by $(\text{charge})^2 = I(I + 1) = \begin{cases} \frac{3}{4} & I = 1/2 \\ 2 & I = 1 \end{cases}$
  ⇒ further enhancement for $W$-pairs by nearly factor 2.

• important subleading logarithms (NLL+...)

• One-loop up to $\mathcal{O}(40\%)$ → two-loop terms may be relevant

• interplay between electroweak and QCD corrections

• important differences between fermions and electroweak gauge bosons

• important differences between long. and transverse gauge bosons ($I = 1/2$ vs. $I = 1$)

• Small transverse momenta, close to $WW$ threshold!
  → complete calculation with masses required
Gauge Boson Pair Production: $W^+W^-, W^\pm Z, ZZ, \gamma\gamma$

**Two Approaches:**

- **dominant, logarithmically enhanced terms via evolution equation & separation of QED**
  
  ⇒ one- and two-loop terms in NNLL

  J.H.K., Metzler, Penin, Uccirati: JHEP 1106 (2011) 143

  related work based on SCET: Manohar,…

- **one-loop calculation, including $M^2_W/\hat{s}$ terms and real radiation: full NLO**

  Bierweiler, Kasprzik, J.H.K., Uccirati

  earlier work: logarithmically enhanced terms only, including W decays and off-shell effects

  Accomando, Denner, Kaiser

- **Aim: Combine the two approaches**

  ⇒ Implementation in Monte Carlo
II. One-Loop Results
II.1. W-Pairs: Leading Order

\[ q q Z/\gamma W W Z/\gamma W q q^\prime W W q q^\prime W W q q^\prime W W q q^\prime W W q q^\prime W W \]

\[ u\bar{u} \to W^+W^-, \ \sqrt{s} = 1 \ \text{TeV} \]

- Strong enhancement for \( \Theta \to 180^\circ \)
- Dominance of transverse W
Also included: $\gamma\gamma \to WW$

![Diagram of $\gamma\gamma \to WW$](image)

also included: $gg \to WW$

![Diagram of $gg \to WW$](image)
One Loop Corrections

- On-shell, $G_\mu$-scheme ($M_W, M_Z, G_\mu$)

- Virtual and real photon radiation included
  remaining collinear singularities "absorbed" in PDFs
  ($q\bar{q}$: MSTW2008LO; $\gamma\gamma$: MRST2004QED/NNPDF2.3QED)

- Two independent calculations

- NLO QCD recalculated
  → giant corrections from events with soft W and hard jet: jet or $p_T$-veto
  to select "balanced" pairs

(related results: Baglio, Ninh, Weber)
• surprisingly large contribution from $\gamma\gamma \rightarrow WW$ at large $\hat{s}$ and large rapidity (small $\hat{t}$)

• large negative corrections at large $\hat{s}$ and small rapidity

• pronounced modification of angular distribution ($\rightarrow$ mimics anomalous couplings) (compare Accomando, Denner, Kaiser)

• sizable QCD corrections

• combination of QCD and EW corrections? additive vs. multiplicative!
Total cross sections LHC8, $p_T > p_{T,\text{cut}}$ ($\int L \, dt = 25 \, \text{fb}^{-1}$ per experiment)
Total cross sections LHC14, $p_T > p_T,\text{cut}$

(Hope for $\int L dt = 300$ fb$^{-1}$ per experiment)

assume 10% efficiency

→ up to $p_T^\text{cut} \sim 800$ GeV
Differential LO cross sections for the W-boson rapidity gap with a minimal invariant mass of 1000 GeV at the LHC14. On the right-hand-side, the corresponding relative rates due to photon- and gluon-induced channels w.r.t. the $q\bar{q}$-contributions are shown, as well as the EW corrections.
Real $W, Z$ Radiation: Compensation?

- soft and/or collinear radiation may (partly) compensate or overcompensate virtual corrections:

- model study (Bell, J.K., Rittinger arXiv:1004.4117; EPJC) 
  strong dependence on cuts! 
  asymptotic energies (multi-TeV)

- semi-realistic evaluation (on-shell $W, Z$) (Bierweiler, Kasprzik, J.K.) 
  $q\bar{q} \rightarrow W^+W^-(\gamma)$ (Born + one-loop) 
  vs. $q\bar{q} \rightarrow W^+W^-Z$ 
  $q\bar{q} \rightarrow W^+W^-W^+ + \text{c.c.}$

- request large invariant mass of V-pair 
  or large $p_T$ of both Vs
Aim: real radiation taken care of by MC ⇒ different final states:

→ different angular distributions

\( W^+W^- : q\bar{q} \rightarrow W^+W^-Z, \ W^+W^-W^\pm \)

(\( \delta_{VV\gamma} \) strongly depends on cuts; here: \( p_{T,\gamma} > 15 \text{ GeV}, \ |y_\gamma| < 2.5 \))
II.2. ZZ, WZ, $\gamma\gamma$

Similar setup, qualitative similar results

$$pp \rightarrow V_1 V_2 (+\gamma) + X$$

$\sqrt{s} = 14$ TeV

- Particularly large corrections for ZZ at large $p_T$: $\sim -40\%$
- Small $p_T$: $\sim -4\%$
particularly large corrections for ZZ at large $M_{ZZ}$: $\sim -40\%$
small $M_{ZZ}$: $\sim -4\%$
large effects on $\Delta y \equiv y_{V1} - y_{V2}$ distribution for large $M_{V1V2}$

$$\frac{d\sigma}{d\Delta y} \bigg|_{M_{V1V2}} \equiv \text{angular distribution in } V_1V_2 \text{ restframe}$$

$\rightarrow$ impact on search for anomalous couplings
# ZZ production: polarization and decays

## Impact of NLO corrections?

<table>
<thead>
<tr>
<th>ZZ polarizations</th>
<th>summed</th>
<th>LL</th>
<th>L+</th>
<th>++</th>
<th>+–</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>default cuts</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_{\text{LO}}/\text{pb} )</td>
<td>7.067</td>
<td>0.402</td>
<td>0.734</td>
<td>0.100</td>
<td>4.997</td>
</tr>
<tr>
<td>( \delta \sigma_{\text{weak}}/\text{pb} )</td>
<td>-0.338(-0.292)</td>
<td>-0.015(-0.014)</td>
<td>-0.029(-0.025)</td>
<td>-0.004(-0.003)</td>
<td>-0.257(-0.223)</td>
</tr>
<tr>
<td><strong>( p_{T,Z} &gt; 500 \text{ GeV} )</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_{\text{LO}}/\text{pb} )</td>
<td>( 10^{-2} \times [0.499] )</td>
<td>( 10^{-7} \times [0.921] )</td>
<td>( 10^{-4} \times [0.334] )</td>
<td>( 10^{-7} \times [0.230] )</td>
<td>( 10^{-2} \times [0.492] )</td>
</tr>
<tr>
<td>( \delta \sigma_{\text{weak}}/\text{pb} )</td>
<td>( -0.195(-0.148) )</td>
<td>( -4.70(+5.577) )</td>
<td>( -0.087(-0.067) )</td>
<td>( -0.426(-0.185) )</td>
<td>( -0.192(-0.147) )</td>
</tr>
<tr>
<td><strong>( p_{T,Z} &gt; 1000 \text{ GeV} )</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_{\text{LO}}/\text{pb} )</td>
<td>( 10^{-3} \times [0.146] )</td>
<td>( 10^{-9} \times [0.189] )</td>
<td>( 10^{-6} \times [0.306] )</td>
<td>( 10^{-10} \times [0.475] )</td>
<td>( 10^{-3} \times [0.146] )</td>
</tr>
<tr>
<td>( \delta \sigma_{\text{weak}}/\text{pb} )</td>
<td>( -0.088(-0.062) )</td>
<td>( -4.319(+30.04) )</td>
<td>( -0.126(-0.090) )</td>
<td>( -2.953(+2.295) )</td>
<td>( -0.088(-0.062) )</td>
</tr>
</tbody>
</table>

**low** \( p_T \): universal corrections \( \sim -4\% \)

\[ \implies \text{angular distribution unchanged} \]

**high** \( p_T \): large corrections, only one combination \( (+–) \) survives

\[ \implies \text{angular distribution unchanged} \]

\[ \implies \text{universal correction factor} \]
**Assumption:** factorization of EW and QCD corrections

\[
d\sigma_{QCD \times EW} = K_{\text{weak}}(\hat{s}, \hat{t}) \times d\sigma_{QCD}
\]

\[
d\sigma_{QCD} \overset{\text{\textbf{best}}}{=} \text{best prediction for QCD-corrected cross section}
\]
• \( \hat{s}, \hat{t} \) from matching of kinematics:
\[
\hat{s} = M_{V_1 V_2}^2; \quad \hat{t} = ?
\]

– small \( p_T \) of \( V_1 V_2 \) \( \implies \hat{\Theta} = \) scattering angle in \( V_1 V_2 \) restframe (collinear or soft QCD corrections)

– large \( p_T \) of \( V_1 V_2 \) (associated with hard jet!)
  define directions of incoming protons in \( V_1 V_2 \) restframe
  \[
  e_{1,2}^* \equiv \frac{p_{1,2}^*}{|p_{1,2}^*|}
  \]
  scattering axis: \( \hat{e}^* \equiv \frac{e_1^* - e_2^*}{|e_1^* - e_2^*|} \)
  scattering angle: \( \cos \hat{\Theta} = \frac{p_{V_1}^*}{|p_{V_1}^*|} \cdot \hat{e}^* \)

• or (HERWIG)
  \( \hat{s}, \hat{t} \) provided by the generator
NLO corrections (electroweak) with full QED replaced by virtual + endpoint subtraction. (Dittmaier) (remainder = real radiation - endpoint subtraction < 1%)

$pp \rightarrow WW$ at 14 TeV:
Simulation for $pp \rightarrow ZZ \rightarrow e^+e^-\mu^+\mu^- + X$ at 13 TeV, $M_{ZZ}$ and $p_{T,l}$ (average lepton $p_T$) distributions

- **Standard HERWIG++ setup used**
  (v2.6.3, with simple add-on for EW corrections, 2M events), ZZ at NLO QCD matched with parton showers, hadronization included, underlying event switched off

- **huge QCD corrections at large $p_{T,Z}$**, factorized ansatz not justified
  $\rightarrow$ jet veto, cut on $p_{T,ZZ}$; veto: $|\sum_i p_{T,l}^i| < 0.3 \sum_i |p_{T,l}^i|$
Simulation for $pp \rightarrow (W^+ \rightarrow e^+\nu_e (W^- \rightarrow \mu^-\bar{\nu}_\mu) + X$ at 13 TeV, $M_{WW}$ and leading-$W$ $p_{T,W}$ distributions

- **Standard Herwig++ setup used**
  (v2.6.3, with simple add-on for EW corrections, 2M events), WW at NLO QCD $\oplus$ parton shower, hadronization included, underlying event switched off

- **V+E approximate results consistent with arXiv:1208.3147**
IV. Two Loop Results (Sudakov Logarithms)

one-loop: $\sim -40\%$

two-loop: $\sim ?$

(Vast amount of literature since $\sim 2000$)

Karlsruhe (Jantzen, J.K., Metzler, Penin, Smirnov, Uccirati)

Fadin, Lipatov, Martin, Melles

PSI (Denner, Melles, Pozzorini, …)

Ciafaloni, …

Manohar, …
A) Form Factor and Evolution Equations

\[ p_2 \]

Born:
\[ \mathcal{F}_{\text{Born}} = \overline{\psi}(p_2)\gamma_{\mu}\psi(p_1) \]

\[ Q^2 = -q^2 \]

\[ \frac{\partial}{\partial \ln Q^2} \mathcal{F} = \left[ \int_{M^2}^{Q^2} \frac{dx}{x} \gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2)) \right] \mathcal{F} \]

Collins, Sen

\[ \Rightarrow \mathcal{F} = \mathcal{F}_{\text{Born}} F_0(\alpha(M^2)) \exp \left\{ \int_{M^2}^{Q^2} \frac{dx}{x} \left[ \int_{M^2}^{x} \frac{dx'}{x'} \gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2)) \right] \right\} \]
aim: $N^4LL \Rightarrow$ corresponds to all terms of the form:

$$\alpha^n \left[ \mathcal{L}^{2n} + \mathcal{L}^{2n-1} + \mathcal{L}^{2n-2} + \mathcal{L}^{2n-3} + \mathcal{L}^{2n-4} \right]_{\text{LL NLL NNLL N}^3\text{LL N}^4\text{LL}}$$

$$\mathcal{L} \equiv \ln(Q^2/M^2)$$

NNLL requires running of $\alpha$ (i.e. $\beta_0$ and $\beta_1$) and:

- $\zeta(\alpha), \xi(\alpha), F_0(\alpha)$ up to $O(\alpha)$ (one-loop)
- $\gamma(\alpha)$ up to $O(\alpha^2)$ (massless two loop)

$N^3LL$ requires two-loop calculation in high-energy limit including linear logarithms (available for non-abelian theory)

$N^4LL$ requires complete two-loop calculation in high-energy limit (available for abelian theory)
B) Two-Loop Results: Massive U(1) Model

\[
\mathcal{F}_\alpha(M, Q) = \mathcal{F}_{\text{Born}} \left[ 1 + \frac{\alpha}{4\pi} f^{(1)} + \left( \frac{\alpha}{4\pi} \right)^2 f^{(2)} + \ldots \right]
\]

\[
f^{(2)} = \frac{1}{2} \mathcal{L}^4 - 3 \mathcal{L}^3 + \left( 8 + \frac{2}{3}\pi^2 \right) \mathcal{L}^2 - \left( 9 + 4\pi^2 - 24\zeta_3 \right) \mathcal{L} \\
+ \frac{25}{2} + \frac{52}{3}\pi^2 + 80\zeta_3 - \frac{52}{15}\pi^4 - \frac{32}{3}\pi^2 \ln^2 2 + \frac{32}{3} \ln^4 2 + 256 \text{Li}_4 \left( \frac{1}{2} \right)
\]

\[
\mathcal{L} \equiv \ln(Q^2/M^2)
\]
C) Massive SU(2) form factor in 2-loop approximation

2-loop vertex diagrams (massless fermions, massive bosons):

Abelian ($C_F^2$):

non-Abelian ($C_FC_A$): last 2 +

Higgs: + 1-loop×1-loop corrections + renormalization

fermion ($C_FT_Fn_f$):
\[ f_2 = \frac{9}{32} \mathcal{L}^4 - \frac{19}{48} \mathcal{L}^3 - \left( -\frac{7}{8} \pi^2 + \frac{463}{48} \right) \mathcal{L}^2 \]

\[ + \left( \frac{39 \text{Cl}_2 \left( \frac{\pi}{3} \right)}{2 \sqrt{3}} + \frac{45}{4} \frac{\pi}{\sqrt{3}} - \frac{61}{2} \zeta_3 - \frac{11}{24} \pi^2 + 29 \right) \mathcal{L} \]
Extensions to four-fermion scattering (Penin et al.) and W-pair production (J.K., Metzler, Penin, Uccirati)

General form (for transverse $W$):

$$\mathcal{A} = \alpha(\mu_T) \mathcal{Z}_\psi \mathcal{Z}_W \tilde{A}$$

$\mathcal{Z}_{W,\psi} = \text{form factors of quark or } W\text{-field}$

$\tilde{A} = \text{reduced amplitude (matrix in isospin space)}$

Solution of

$$\frac{\partial}{\partial \ln Q^2} \tilde{A} = \chi(\alpha(Q^2)) \tilde{A}$$

($\chi \equiv \text{anomalous dimension matrix}$)

up to quadratic logarithms only one-loop terms ($\ln^2, \ln^1, \ln^0$) of $q\bar{q} \rightarrow WW$ and

the two-loop term $\gamma^{(2)}$ of the anomalous dimension (form factor!) required.
**Complication:** massless photon! ⇒ QED subtracted

\[ p p \rightarrow W_T^+W_T^- \]

\[ p p \rightarrow W_L^+W_L^- \]

\[ \sqrt{s} = 14 \text{ TeV} \]

\[ \frac{d\sigma_{\text{NLO}}}{d\sigma_{\text{LO}}} \]

\[ \frac{d\sigma_{\text{NNLO}}}{d\sigma_{\text{LO}}} \]

\[ (W_{T,L} : \text{Isospin } I = 1, 1/2; \Rightarrow \text{relative factor } 6/11) \]
V. Summary

- Weak corrections at LHC14 may reach $-40\%$

- Complicated impact on differential distributions → fake anomalous couplings; sensitive to cuts

- First steps toward combination of QCD and EW corrections in Monte Carlo generator

- Dominant two-loop corrections (partially) available