Implications of an R-scan for Charm Physics

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SFB TR9

I. **CHARM and BOTTOM MASSES**

- m_Q from Sum Rules
- Experimental Analysis: m_c

II. **Resonant Production of χ_{c1} and χ_{c2}**

- Concept
- Phenomenology
- Luminosity Requirements

III. **SUMMARY**

I. CHARM and BOTTOM MASSES

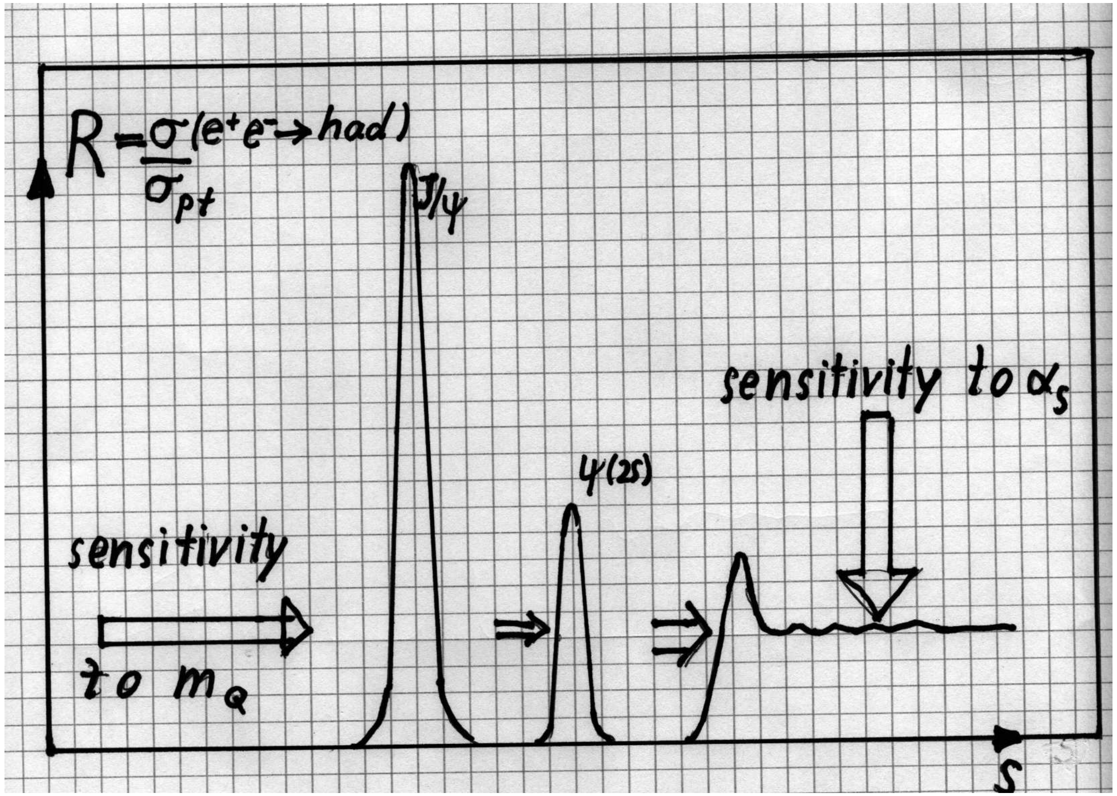
in collaboration with

K. Chetyrkin, Y. Kiyo, A. Maier, P. Maierhöfer, P. Marquard,

A. Smirnov, M. Steinhauser, C. Sturm

I. 1. m_Q from SVZ Sum Rules, Moments and Tadpoles

Main Idea (SVZ)



Sensitivity to m_Q from location of $Q\overline{Q}$ threshold.

Some definitions:

$$\left(-q^2 g_{\mu\nu} + q_\mu q_\nu\right) \Pi(q^2) \equiv i \int \mathrm{d}x\, e^{iqx} \langle T j_\mu(x) j_\nu(0) \rangle$$

with the electromagnetic current j_μ.

$$R(s) = 12\pi \,\mathrm{Im}\left[\Pi(q^2 = s + i\varepsilon)\right]$$

Taylor expansion: $\Pi_Q(q^2) = Q_Q^2 \frac{3}{16\pi^2} \sum_{n \geq 0} \bar{C}_n z^n$

with $z = q^2/(4m_Q^2)$ and $m_Q = m_Q(\mu)$ the $\overline{\mathrm{MS}}$ mass.

$$\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s}{\pi} \bar{C}_n^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \bar{C}_n^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 \bar{C}_n^{(3)} + \ldots$$

generic form

$$
\begin{aligned}
\bar{C}_n = {} & \boxed{\bar{C}_n^{(0)}} \\
& + \frac{\alpha_s}{\pi}\left(\boxed{\bar{C}_n^{(10)}} + \bar{C}_n^{(11)} l_{m_c} \right) \\
& + \left(\frac{\alpha_s}{\pi}\right)^2 \left(\boxed{\bar{C}_n^{(20)}} + \bar{C}_n^{(21)} l_{m_c} + \bar{C}_n^{(22)} l_{m_c}^2 \right) \\
& + \left(\frac{\alpha_s}{\pi}\right)^3 \left(\boxed{\bar{C}_n^{(30)}} + \bar{C}_n^{(31)} l_{m_c} + \bar{C}_n^{(32)} l_{m_c}^2 + \bar{C}_n^{(33)} l_{m_c}^3 \right) \\
& + \ldots
\end{aligned}
$$

with $\alpha_s = \alpha_s(\mu)$, $l_{m_c} = \ln\left(\frac{m_c^2}{\mu^2}\right)$

$\bar{C}_n^{(ij)}$ = pure numbers

for $j \geq 1$ from RG

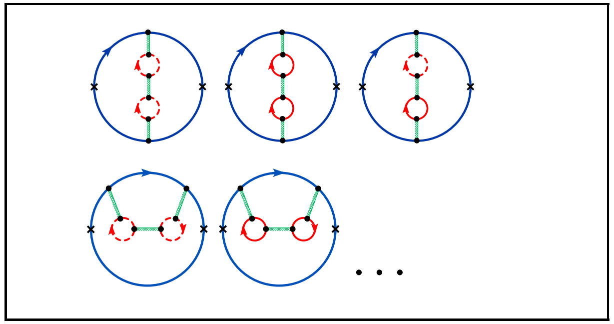
for $j = 0$: calculation

Analysis in N^{3}LO

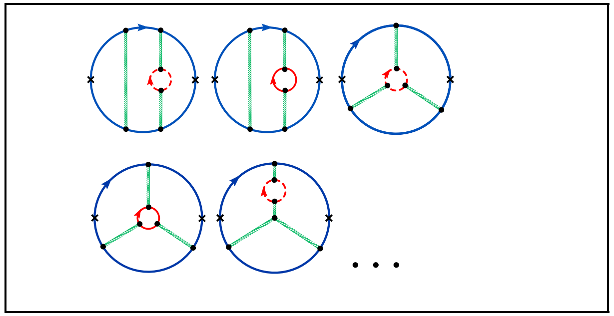
Algebraic reduction to 13 master integrals (Laporta algorithm);

numerical and analytical evaluation of master integrals

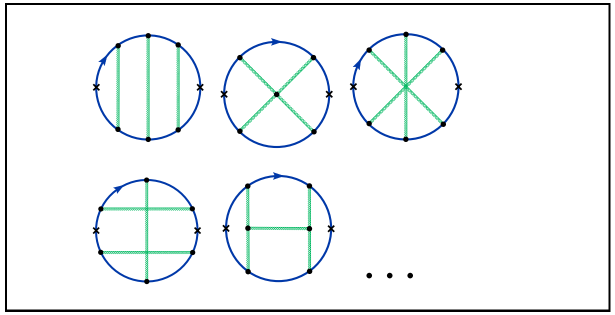
n_f^2-contributions



n_f^1-contributions



n_f^0-contributions



 : heavy quarks,  : light quarks,

n_f: number of active quarks

$\Rightarrow$ About 700 Feynman-diagrams

$\boxed{\bar{C}_0 \text{ and } \bar{C}_1 \text{ in order } \alpha_s^3 \text{ (four loops!)}}$ (2006)

Reduction to master integrals

(Chetyrkin, JK, Sturm; Boughezal, Czakon, Schutzmeier)

$\bar{C}_2$ and $\bar{C}_3$ (2008)

(Maier, Maierhöfer, Marquard, A. Smirnov)

All master integrals known analytically and double checked.

(Schröder + Vuorinen, Chetyrkin et al., Schröder + Steinhauser,

Laporta, Broadhurst, Kniehl et al.)

$\bar{C}_4 - \bar{C}_{10}$: extension to higher moments by Padé method, using analytic information from low energy ($q^2 = 0$), threshold ($q^2 = 4m^2$), high energy ($q^2 = -\infty$) (Kiyo, Maier, Maierhöfer, Marquard, 2009)

Relation to measurements

$$\mathcal{M}_n^{\text{th}} \equiv \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\bigg|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n$$

Perturbation theory: $\bar{C}_n$ is function of α_s and $\ln\frac{m_c^2}{\mu^2}$

dispersion relation:

$$\Pi_c(q^2) = \frac{q^2}{12\pi^2}\int \mathrm{d}s\,\frac{R_c(s)}{s(s-q^2)} + \text{subtraction}$$

$$\Rightarrow \mathcal{M}_n^{\text{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}}R_c(s)$$

constraint: $\mathcal{M}_n^{\text{exp}} = \mathcal{M}_n^{\text{th}}$

$$\boxed{\Rightarrow m_c = \tfrac{1}{2}\left(\tfrac{9}{4}Q_c^2\bar{C}_n/\mathcal{M}_n^{\text{exp}}\right)^{1/(2n)}}$$

qualitative considerations

$$\mathcal{M}_n = \int_{threshold} \frac{d\,s}{s^{n+1}} R_c(s) \sim \text{dimension } (m_c)^{-2n}$$

- depends moderately on α_s!
- $\Pi(q^2)$ is an analytic function with branch cut from $(2m_D)^2$ to ∞.
- averaging over resonances reduces influence of long distances

  (binding effects).
- $\Pi(q^2 = 0)$ and its derivatives at $q^2 = 0$ are short distance quantities.

  $\Rightarrow$ pQCD is applicable.

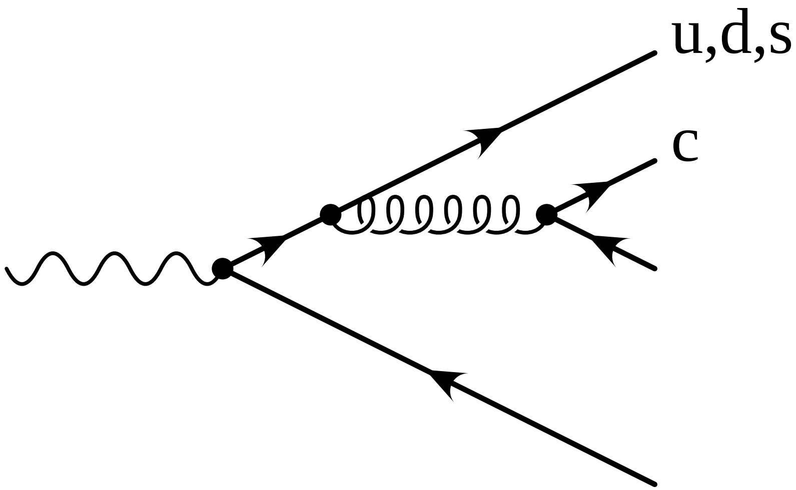
I. 2. Experimental Analysis: m_c

$$\mathcal{M}_n^{\mathrm{exp}} \equiv \int \frac{ds}{s^{n+1}} R_{\mathrm{charm}}(s)$$

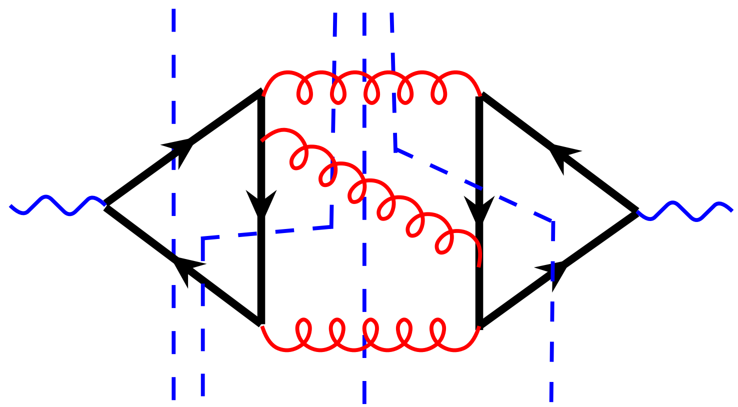
$$\boxed{\Rightarrow m_c = \frac{1}{2}\left(\frac{9}{4} Q_c^2 \bar{C}_n / \mathcal{M}_n^{\mathrm{exp}}\right)^{1/(2n)}}$$

- $\bar{C}_n$ from calculation

- $\mathcal{M}_n^{\mathrm{exp}}$ from experiment



u,d,s
c



Ingredients (charm)

experiment:

- $\Gamma_e(J/\psi,\psi')$ from BES & CLEO & BABAR (PDG)
- ψ(3770) and $R(s)$ from BES
- $\alpha_s = 0.1187 \pm 0.0020$

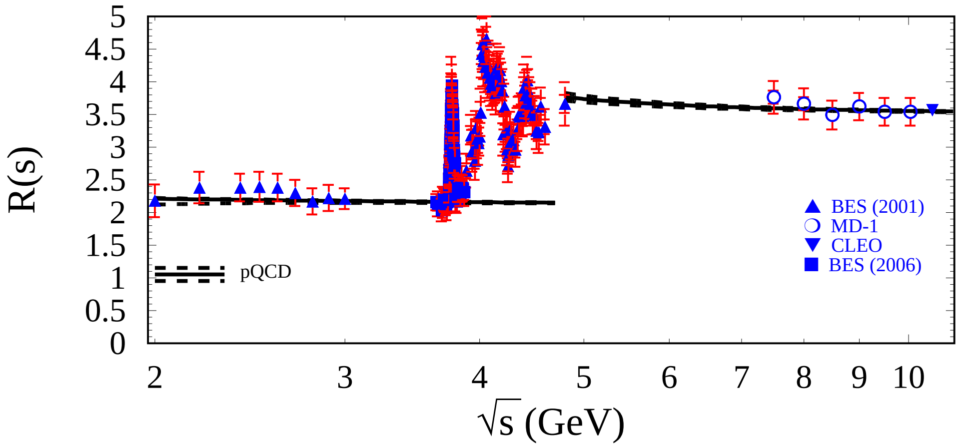
theory:

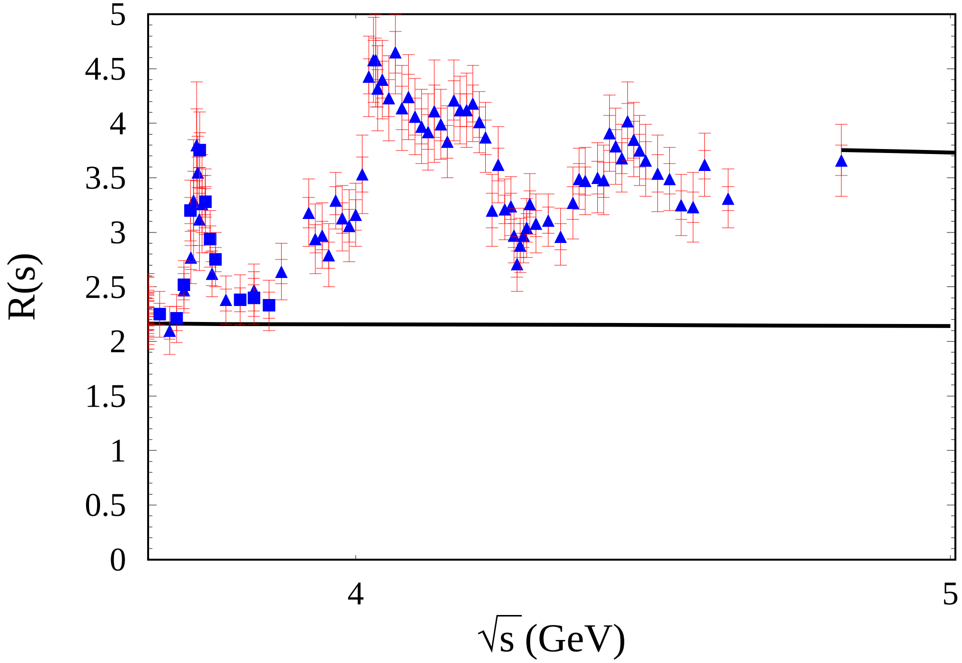
- N^{3}LO for $n = 1, 2, 3, 4$
- include condensates

$$\delta\mathcal{M}_n^{\mathrm{np}} = \frac{12\pi^2 Q_c^2}{(4m_c^2)^{(n+2)}} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle a_n$$

(including NLO-terms)

- estimate of non-perturbative terms
  (oscillations, based on Shifman)
- careful extrapolation of R_{uds}
- careful definition of R_c


R(s)
√s (GeV)
pQCD
BES (2001)
MD-1
CLEO
BES (2006)


R(s)
√s (GeV)

Contributions from

- narrow resonances: $R = \frac{9\Pi M_R \Gamma_e}{\alpha^2(s)} \delta(s - M_R^2)$ (PDG)
- threshold region ($2\,m_D$ – 4.8 GeV) (BESS)
- perturbative continuum ($E \geq 4.8$ GeV) (Theory)

n	$\mathcal{M}_n^{\mathrm{res}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\mathrm{thresh}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\mathrm{cont}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\mathrm{exp}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\mathrm{np}}$ $\times 10^{(n-1)}$
1	0.1201(25)	0.0318(15)	0.0646(11)	0.2166(31)	−0.0001(2)
2	0.1176(25)	0.0178(8)	0.0144(3)	0.1497(27)	0.0000(0)
3	0.1169(26)	0.0101(5)	0.0042(1)	0.1312(27)	0.0007(14)
4	0.1177(27)	0.0058(3)	0.0014(0)	0.1249(27)	0.0027(54)

Different relative importance of resonances vs. continuum for $n = 1, 2, 3, 4$.

Results (m_c)

PRD 80: (2009) 074010

n	$m_c(3\,\text{GeV})$	exp	α_s	μ	np	total
1	986	9	9	2	1	13
2	976	6	14	5	0	16
3	978	5	15	7	2	17
4	1004	3	9	31	7	33

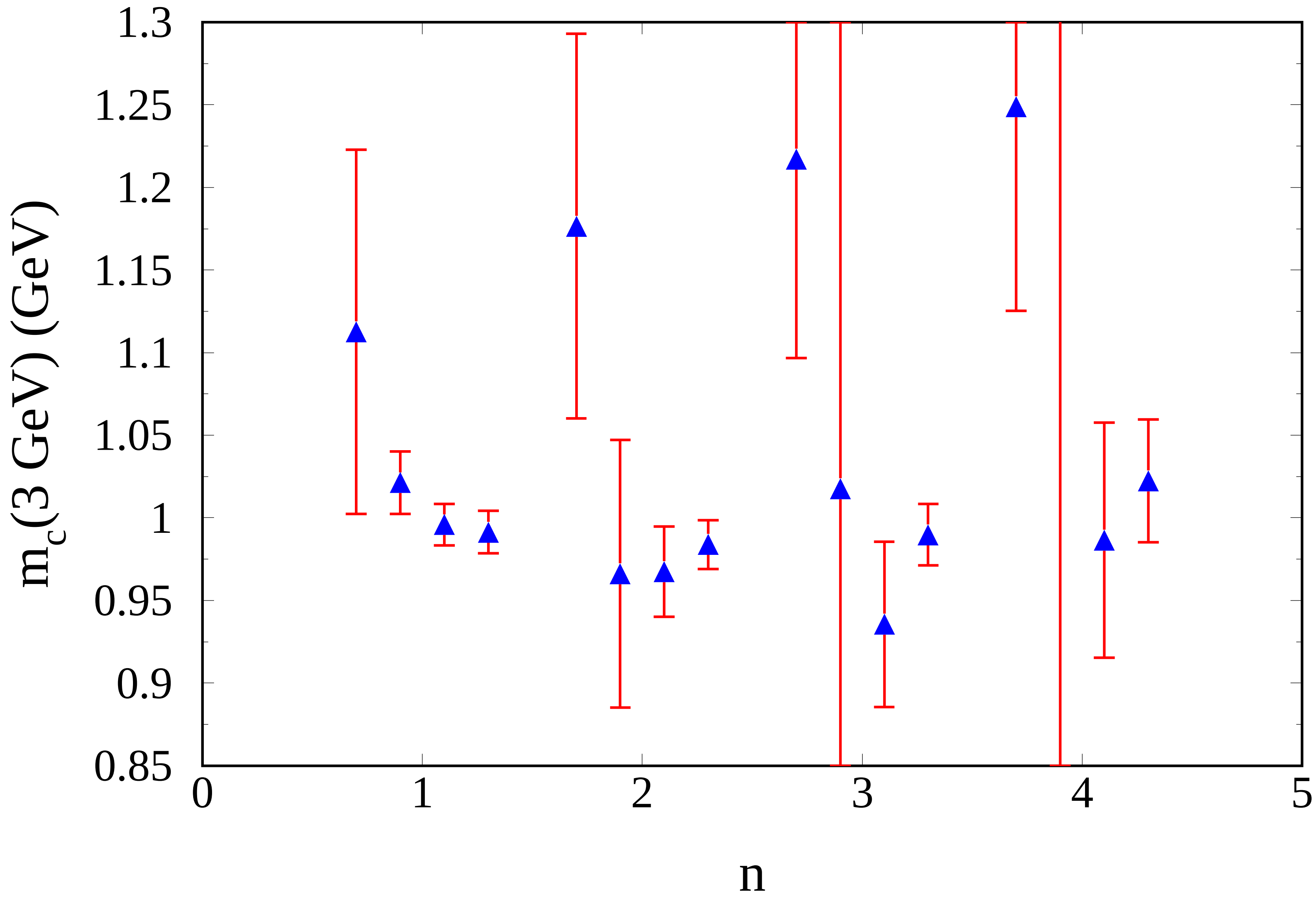
Remarkable consistency between $n = 1, 2, 3, 4$

and stability ($\mathcal{O}(\alpha_s^2)$ vs. $\mathcal{O}(\alpha_s^3)$);

prefered scale: $\mu = 3\,\text{GeV}$,

conversion to $m_c(m_c)$:

- $m_c(3\,\text{GeV}) = 986 \pm 13\,\text{MeV}$
- $m_c(m_c) = 1279 \pm 13\,\text{MeV}$

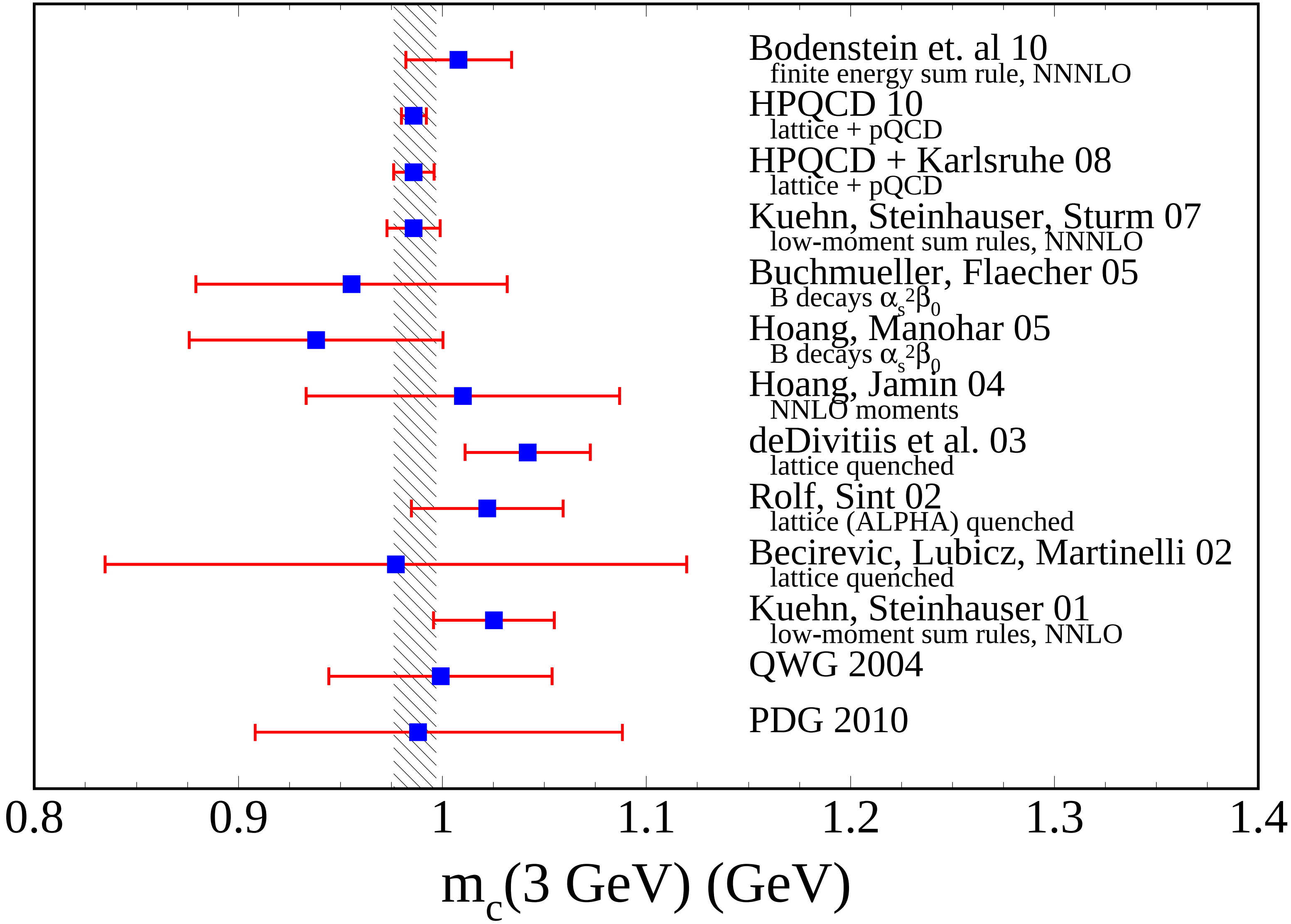

1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
1
2
3
4
5
m_c(3 GeV) (GeV)
n

Perturbative stability

$$
\begin{aligned}
m_c &= \frac{1}{2}\left(\frac{9Q_c^2}{4}\frac{\bar{C}_n^{\mathrm{Born}}}{\mathcal{M}_n^{\mathrm{exp}}}\right)^{\frac{1}{2n}}\left(1+r_n^{(1)}\alpha_s+r_n^{(2)}\alpha_s^2+r_n^{(3)}\alpha_s^3\right) \\
&\propto 1-\begin{pmatrix} 0.328 \\ 0.524 \\ 0.618 \\ 0.662 \end{pmatrix}\alpha_s-\begin{pmatrix} 0.306 \\ 0.409 \\ 0.510 \\ 0.575 \end{pmatrix}\alpha_s^2-\begin{pmatrix} 0.262 \\ 0.230 \\ 0.299 \\ 0.396 \end{pmatrix}\alpha_s^3, \qquad (1)
\end{aligned}
$$

error from next order $\leq r_n^{max}\alpha_s^4 <$ 2 to 3 permille

(smaller than μ-variation!)



Bodenstein et. al 10
finite energy sum rule, NNNLO
HPQCD 10
lattice + pQCD
HPQCD + Karlsruhe 08
lattice + pQCD
Kuehn, Steinhauser, Sturm 07
low-moment sum rules, NNNLO
Buchmueller, Flaecher 05
B decays $\alpha_s^2\beta_0$
Hoang, Manohar 05
B decays $\alpha_s^2\beta_0$
Hoang, Jamin 04
NNLO moments
deDivitiis et al. 03
lattice quenched
Rolf, Sint 02
lattice (ALPHA) quenched
Becirevic, Lubicz, Martinelli 02
lattice quenched
Kuehn, Steinhauser 01
low-moment sum rules, NNLO
QWG 2004
PDG 2010
0.8
0.9
1
1.1
1.2
1.3
1.4
m_c(3 GeV) (GeV)

Potential improvements in analysis

1) Combined fit to lowest three moments

   $\Rightarrow$ optimal usage of experimental information,

   $\Rightarrow$ requires dedicated analysis of correlation.

2) More “aggressive” choice for $\delta\alpha_s$:

$$\alpha_s = 0.1189 \pm 0.002 \quad \Rightarrow \quad \alpha_s = 0.1184 \pm 0.0007 \text{ (PDG 2012)}$$

3) Theory error from perturbation series ($2-3$ permille)

   instead of μ-variation

   $\Rightarrow$ δm_c below 10 MeV

Potential experimental improvements

1.) narrow resonances ($J/\Psi, \Psi'$) dominate:

$\Gamma_e(1S) = 5.55 \pm 0.14 \pm 0.02$ keV; $\Gamma_e(2S) = 2.35 \pm 0.04$ keV

improvement? correlations?

2.) threshold region: improvement at BESS?

subtraction of $u\bar{u}$, $d\bar{d}$, $s\bar{c} \Rightarrow$ precise reference point below charm threshold needed

3.) continuum above 4.8 GeV:

missing data substituted by theory $\Rightarrow$ small error

excellent agreement with data

improved calibration at ~ 4.5 GeV desirable

$\rightarrow$ crucial information over wide range

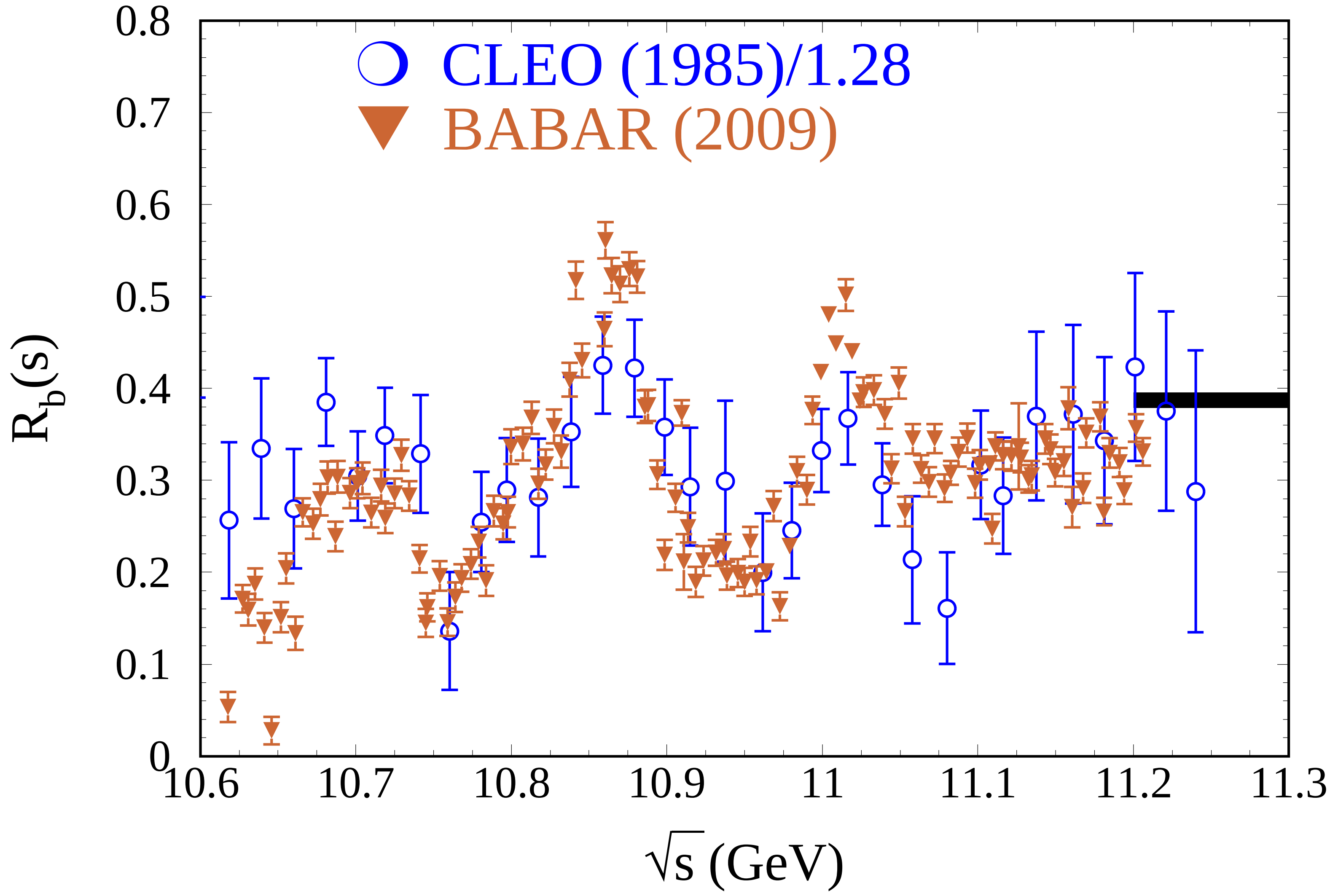
In total $\delta m_c \approx 7$ MeV may be within reach

Experimental Ingredients for m_b

Contributions from

- narrow resonances ($\Upsilon(1S)$ – $\Upsilon(4S)$) (PDG)
- threshold region (10.618 GeV – 11.2 GeV) (BABAR 2009)
- perturbative continuum ($E \geq 11.2$ GeV) (Theory)
- different relative importance of resonances vs. continuum for $n = 1,2,3,4$

n	$\mathcal{M}_n^{\mathrm{res},(1S-4S)} \times 10^{(2n+1)}$	$\mathcal{M}_n^{\mathrm{thresh}} \times 10^{(2n+1)}$	$\mathcal{M}_n^{\mathrm{cont}} \times 10^{(2n+1)}$	$\mathcal{M}_n^{\mathrm{exp}} \times 10^{(2n+1)}$
1	1.394(23)	0.287(12)	2.911(18)	4.592(31)
2	1.459(23)	0.240(10)	1.173(11)	2.872(28)
3	1.538(24)	0.200(8)	0.624(7)	2.362(26)
4	1.630(25)	0.168(7)	0.372(5)	2.170(26)


CLEO (1985)/1.28
BABAR (2009)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$R_b(s)$
10.6
10.7
10.8
10.9
11
11.1
11.2
11.3
$\sqrt{s}$ (GeV)

BELLE?

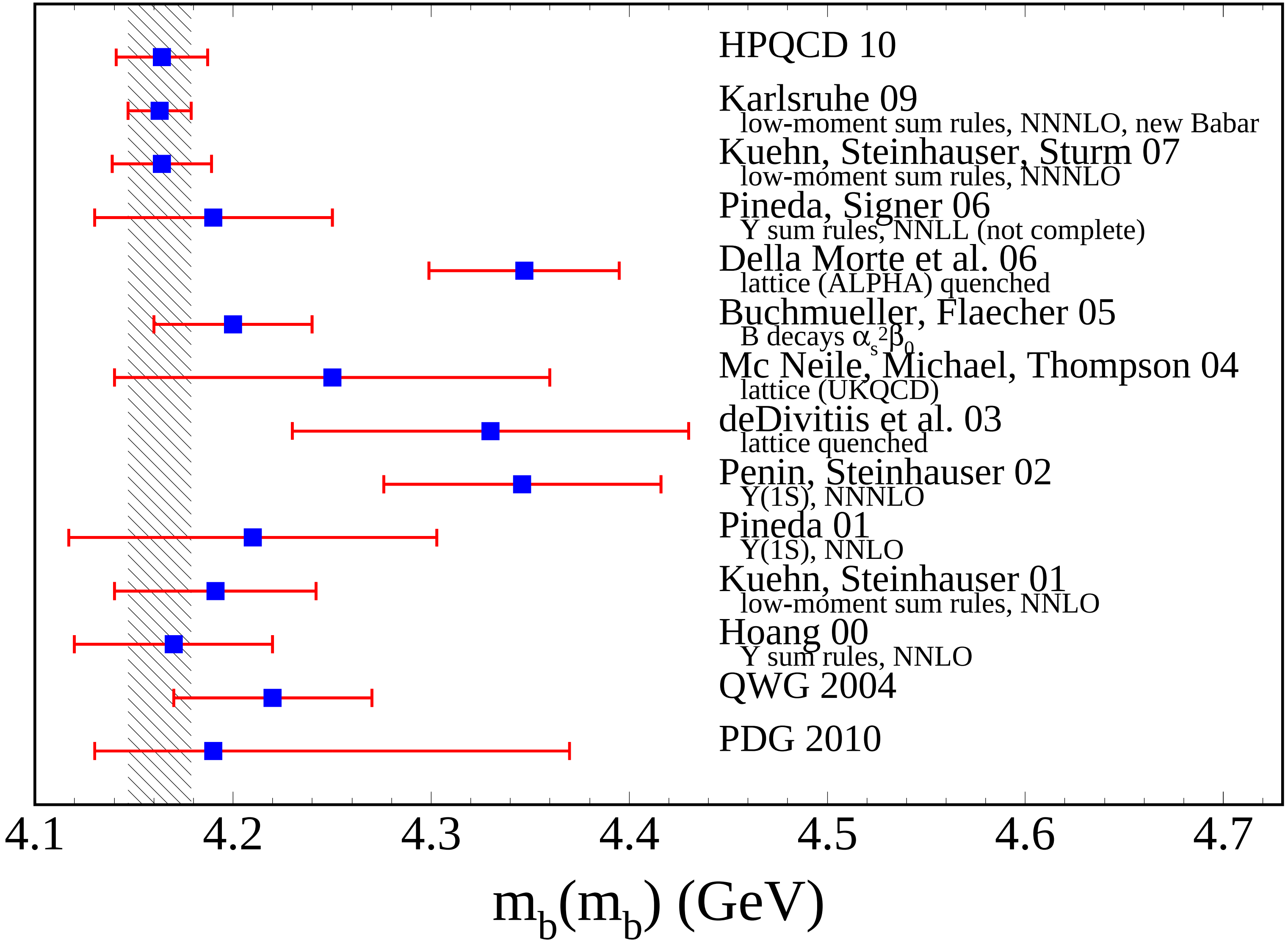
n	$m_b(10\,\text{GeV})$	exp	α_s	μ	total	$m_b(m_b)$
1	3597	14	7	2	16	4151
2	3610	10	12	3	16	4163
3	3619	8	14	6	18	4172
4	3631	6	15	20	26	4183

Consistency ($n = 1, 2, 3, 4$) and stability ($\mathcal{O}(\alpha_s^2)$ vs. $\mathcal{O}(\alpha_s^3)$);

- $m_\mathrm{b}(10\,\text{GeV}) = 3610 \pm 16\,\text{MeV}$
- $m_\mathrm{b}(m_\mathrm{b}) = 4163 \pm 16\,\text{MeV}$

potential improvements:

- $\delta\alpha_s$
- combined analysis of 3 lowest moments
- improved data


HPQCD 10
Karlsruhe 09
low-moment sum rules, NNNLO, new Babar
Kuehn, Steinhauser, Sturm 07
low-moment sum rules, NNNLO
Pineda, Signer 06
Υ sum rules, NNLL (not complete)
Della Morte et al. 06
lattice (ALPHA) quenched
Buchmueller, Flaecher 05
B decays $\alpha_s^2\beta_0$
Mc Neile, Michael, Thompson 04
lattice (UKQCD)
deDivitiis et al. 03
lattice quenched
Penin, Steinhauser 02
Υ(1S), NNNLO
Pineda 01
Υ(1S), NNLO
Kuehn, Steinhauser 01
low-moment sum rules, NNLO
Hoang 00
Υ sum rules, NNLO
QWG 2004
PDG 2010
4.1
4.2
4.3
4.4
4.5
4.6
4.7
$m_b(m_b)$ (GeV)

α_s-dependence

$$m_c(3\text{ GeV}) = \left(986 - \frac{\alpha_s - 0.1189}{0.002} \cdot 9 \pm 10\right)\text{ MeV}$$

$$m_b(10\text{ GeV}) = \left(3610 - \frac{\alpha_s - 0.1189}{0.002} \cdot 12 \pm 11\right)\text{ MeV}$$

$$m_b(m_b) = \left(4163 - \frac{\alpha_s - 0.1189}{0.002} \cdot 12 \pm 11\right)\text{ MeV}$$

$$m_b(M_Z) = \left(2835 - \frac{\alpha_s - 0.1189}{0.002} \cdot 27 \pm 8\right)\text{ MeV}$$

$$m_b(161.8\text{ GeV}) = \left(2703 - \frac{\alpha_s - 0.1189}{0.002} \cdot 28 \pm 8\right)\text{ MeV}$$

$$m_c(3\,\text{GeV}) = 986(13)\ \text{MeV}$$

$$m_b(10\,\text{GeV}) = 3610(16)\ \text{MeV}$$
$$m_b(m_b) = 4163(16)\ \text{MeV}$$

Improvement by factor 2 conceivable

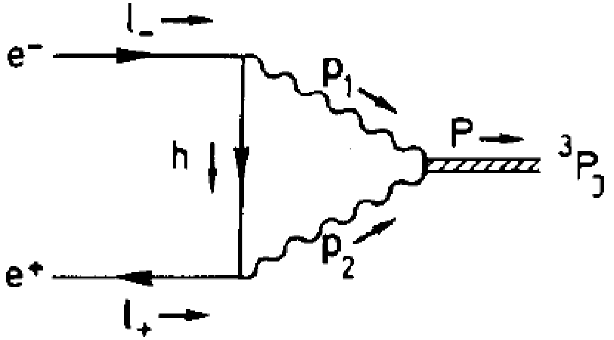
II. Resonant Production of χ_{c1} and χ_{c2}

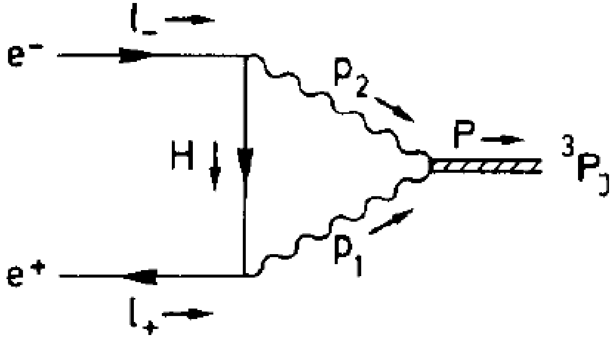
Concept

direct coupling of χ_{cJ} to e^+e^-

$$\begin{aligned} \mathfrak{A}(e^+e^- \to \chi_0) &= O(m_e/M_\chi) \approx 0 \\ \mathfrak{A}(e^+e^- \to \chi_1) &= g_1 \bar{\mathrm{v}} \gamma_5 \not{\varepsilon}\, \mathrm{u} \\ \mathfrak{A}(e^+e^- \to \chi_2) &= g_2 \bar{\mathrm{v}} \gamma^\mu \mathrm{u}\, \varepsilon_{\mu\nu} (l_+^\nu - l_-^\nu)/M_\chi \end{aligned}$$

through two virtual photons





quarkonium: short distance calculation

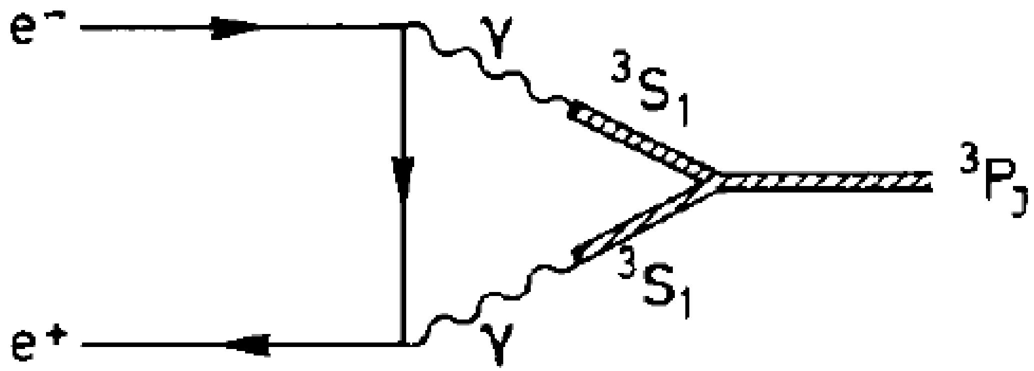
$$g_1 = -\frac{\alpha^2}{M^2} 32\, a \ln \frac{2b}{M}$$

$$g_2 = +\frac{\alpha^2}{M^2} 32\, a \left[\sqrt{2} \ln \frac{2b}{M} + \frac{\sqrt{2}}{3} (i\pi + \ln 2 - 1) \right]$$

with $M \equiv M_\chi$, $a = \sqrt{\frac{3}{4\pi m_c}} \Phi'(0)$,

$b =$ "binding energy" $\rightarrow b = \pm 0.5$ GeV

vector dominance:



Results: $\Gamma(\chi_J \to e^+e^-)$ in eV

J	1	2
quarkonium		
$b = +0.5$ GeV	0.023	0.013
$b = -0.5$ GeV	0.17	0.27
vector dominance	0.46	0.014
unitarity limit	0.044	0.0023

consider $J = 1$ and use $\Gamma = 0.1$ eV

Strategy:

R-scan around $M(\chi_1) = 3.511$ GeV

A) all final states

$$\Rightarrow R_{\text{peak}} = \frac{9\pi}{2\alpha\sqrt{2\pi}} \frac{\Gamma_1}{\Delta M} c_{\text{rad}} \approx 0.05 \frac{\Gamma_1[\text{eV}]}{\Delta M[\text{MeV}]}$$

$\Delta M =$ energy spread $= 1.1$ MeV; $\quad c_{\text{rad}} \approx 0.5$

$\boxed{R_{\text{peak}} = 0.005 \text{ above background of } R = 2}$

($\Rightarrow 100\,\text{pb}^{-1}$ per point with 5 points $\Rightarrow$ 3 sigma)

B) selected final state $\chi_1 \to \gamma J/\psi (\to \mu^+\mu^-, e^+e^-)$

(Br $\approx 0.35 \cdot 0.12 \approx 0.054$)

background from $e^+e^- \to \gamma J/\psi$ (radiative return)

similar requirement on luminosity

SUMMARY

precise charm and bottom quark masses can be further improved through:

- improved $\Gamma(J/\psi \to e^+e^-)$ etc.
- improved R_{charm} and R_{bottom}
- improved α_s
- combined analysis of lowest three moments

direct resonant production of χ_{1c} is possible

- through a dedicated scan around 3.511 GeV
- perhaps through radiative return

  $\to$ improved insight in charmonium dynamics