Singlet Contributions to the Vector Correlator

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in collaboration with

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based on P. Baikov, K. Chetyrkin, and J. K., Nucl.Phys.Proc.Suppl.205-206:237-241,2010; P. Baikov, K. Chetyrkin, J. K. and J. Rittinger, in preparation

Outline

- status of vector (VV) correlator in massless QCD: singlet versus nonsinglet
- tool-box
- singlet contribution $\mathcal{O}(\alpha_s{}^4)$ for a generic gauge group
- QED β-function in five loops
- phenomenological implications for $\sigma_{tot}(e^+e^- \to hadrons)$
- Gross-Llewellyn Smith sum rule in $\mathcal{O}(\alpha_s{}^4)$ and test of the constraints on the singlet part of the Adler function coming from the Crewther relation
- Conclusions

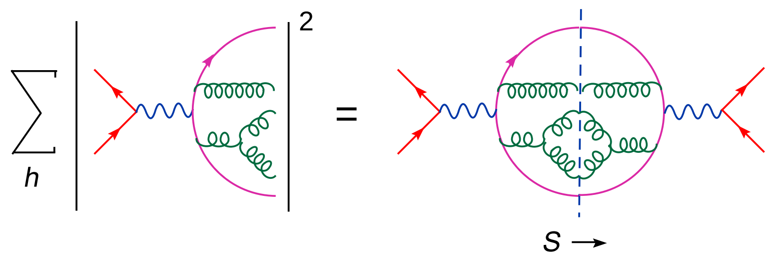
“gold plated” (Bjorken, 1979) QCD observables:

$$R_Z = \Gamma(Z_0 \to hadrons)/\sigma(Z_0 \to \mu^+\mu^-)$$

$$R_\tau = \Gamma(\tau \to hadrons + \nu_\tau)/\Gamma(\tau \to l + \bar{\nu}_l + \nu_\tau)$$

$$R(s) = \sigma_{tot}(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$$

(via unitarity) $R(s) \approx \Im\,\Pi(s - i\delta)$



$$\Pi(Q^2) \approx \int e^{iqx} \langle 0|T[\ j^v_\mu(x) j^v_\mu(0)\ ]|0\rangle dx$$

$R(s) \leftrightarrow D(Q) \quad \Longleftarrow$ Adler function $\equiv Q^2 \dfrac{d}{d\,Q^2}\Pi(q^2) = Q^2 \displaystyle\int \frac{R(s)}{(s+Q^2)^2}\,ds$

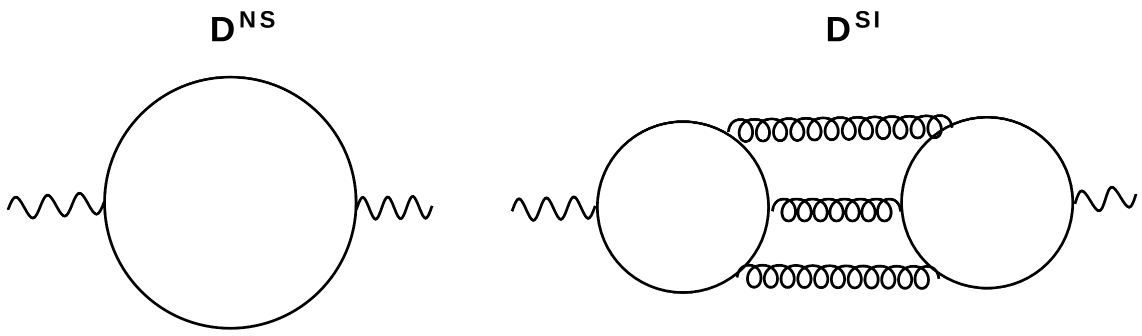
$$R(s) = 1 + \sum_{i\geq 1} r_i\, a_s(s)^i, \quad D = 1 + \sum_{i\geq 1} d_i\, a_s(Q)^i, \quad (a_s \equiv \alpha_s/\pi, \mu = Q, Q^2 \equiv -q^2)$$

• status of theory (in the massless limit) •

$$R^{NS} = 3\sum_i Q_i^2 \left(1 + \frac{\alpha_s}{\pi} + \# \left(\frac{\alpha_s}{\pi}\right)^2 + \# \left(\frac{\alpha_s}{\pi}\right)^3 + \# \left(\frac{\alpha_s}{\pi}\right)^4 + \cdots \right)$$

parton model	QED Källen+ Sabry 1955	Chetyrkin, Kataev, Tkachov; Dine, Sapirstein; Celmaster 1979	Gorishny, Kataev, Larin; Surguladze, Samuel 1991 Chetyrkin /gen. gauge/ 1996	Baikov, Chetyrkin, Kühn 2008; Baikov, Chetyrkin, Kühn 2010 (Feynman Gauge only)

$$R^{SI} = \left(\sum_i Q_i\right)^2 \left(\# \left(\frac{\alpha_s}{\pi}\right)^3 + \boxed{?? \left(\frac{\alpha_s}{\pi}\right)^4} + \cdots \right)$$



Recall non-singlet results (PLR 101 (2008) 012002):

$$R^{NS} = 1 + a_s + (1.9857 - 0.1152\, n_f)\, a_s^2$$

$$(-6.63694 - 1.20013 n_f - 0.00518 n_f^2)\, a_s^3$$

$$\boldsymbol{+(-156.61 + 18.77\, n_f - 0.7974\, n_f^2 + 0.0215\, n_f^3)\, a_s^4}$$

Impact on α_s from $Z-$decays:

$$\boldsymbol{\mathcal{O}(\alpha_s^3): \quad \alpha_s(M_Z)^{NNLO} = 0.1185 \pm 0.0026^{\mathrm{exp}} \pm 0.002^{\mathrm{th}}}$$

Including the α_s^4 term leads to an increase of $\boldsymbol{\delta\alpha_s(M_Z) = 0.0005}$ and to *four-fold* decrease of the theory error!

$$\boldsymbol{\mathcal{O}(\alpha_s^4): \quad \alpha_s(M_Z)^{NNNLO} = 0.1190 \pm 0.0026^{\mathrm{exp}} \pm 0.0005^{\mathrm{th}}}$$

Impact on α_s from τ-decays (FO and CI): α_s decreased by $\delta\alpha_s(M_Z) = 0.0016$

$$\delta\alpha_s(M_Z)^{NNNLO} = 0.1202 \pm 0.0019^{\mathrm{exp}}$$

Massless Correlators: Technicalities

Π related to the corresponding absorptive part $R(s)$ through

$$R^{jj}(s) \approx \Im\,\Pi^{jj}(s - i\delta)$$

RG equation

$$\Pi^{jj} = Z^{jj} + \Pi^{\mathrm{B}are}(-Q^2, \alpha_s^{\mathrm{B}are})$$

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(a_s)\frac{\partial}{\partial a_s}\right)\Pi = \gamma^{jj}(a_s)$$

extremely useful for determining the absorptive part of Π^{jj}

For Π at $\mathbf{5}$ loop

$$\frac{\partial}{\partial \log(\mu^2)} \Pi = \gamma^{jj}(a_s) - \left(\beta(a_s) \frac{\partial}{\partial a_s} \right) \Pi$$

anom.dim. at $\mathcal{O}(\alpha_s^4)$
5 loop integrals

4-loop integrals at $\mathcal{O}(\alpha_s^3)$ only contribute
due to the factor of $\beta(a_s)\frac{\partial}{\partial a_s} = \mathcal{O}(a_s)$

- to find Log-dependent part of Π at 5-loops one needs 5-loop anomalous dimension γ^{jj} and 4-loop Π (BUT! including its constant part)

- 5-loop anom.dim. reducible to 4-loop p-integrals

Tool-box for massless correlators at α_s^4:

- IRR / Vladimirov, (78)/ + IR R^* -operation /Chetyrkin, Smirnov (1984)/ + resolved combinatorics /Chetyrkin, (1997)/
- reduction to Masters: "direct and automatic" construction of CF's through $1/D$ expansion within the Baikov's representation for Feynman integrals (Phys. Lett. B385 (1996) 403; B474 (2000) 385; Nucl.Phys.Proc.Suppl.116:378-381,2003)
- computing: MPI-based (PARFORM) as well as thread-based (TFORM) versions of FORM
  Vermaseren, Retey, Fliegner, Tentyukov, ...(2000 – ...)

Results

Singlet contribution to the Adler function (Last missing term!)

$$D^{SI}(Q^2) = d_R \left(\sum_{i=3}^{\infty} d_i^{SI}\, a_s^i(Q^2) \right)$$

$$d_3^{SI} = \frac{d^{abc}d^{abc}}{d_R}\left(\frac{11}{192} - \frac{1}{8}\zeta_3\right),\ d_4^{SI} = \frac{d^{abc}d^{abc}}{d_R}\left(C_F\, d_{4,1}^{SI} + C_A\, d_{4,2}^{SI} + T\, n_f\, d_{4,3}^{SI}\right)$$

$$d_{4,1}^{SI} \;=\; = -\frac{13}{64} - \frac{\zeta_3}{4} + \frac{5\zeta_5}{8}, \quad d_{4,3}^{SI} \;=\; \frac{-149}{576} - \frac{13}{32}\zeta_3 + \frac{5}{16}\zeta_5 + \frac{1}{8}\zeta_3^2$$

$$d_{4,2}^{SI} \;=\; -\frac{3893}{4608} + \frac{169}{128}\zeta_3 - \frac{45}{64}\zeta_5 - \frac{11}{32}\zeta_3^2$$

Phenomenological implications for $\sigma_{tot}(e^+e^- \to hadrons)$

Numerically:

$$
\begin{aligned}
R(s) \;=\; & 3\sum_f Q_f^2 \left\{1 + a_s + a_s^2\,(1.986 - 0.1153 n_f)\right. \\
+\; & \left. a_s^3\left(-6.637 - 1.200 n_f - 0.00518 n_f^2\right)\right\} \\
-\; & \left(\sum_f Q_f\right)^2 \left(1.2395\, a_s^3 + \underline{\left(-17.8277 + 0.57489 n_f\right)\, a_s^4}\right)
\end{aligned}
$$

for n_f=5

$$
\frac{11}{3}\left[1 + a_s + a_s^2\, 1.409 - 12.767\, a_s^3 - 79.98\, a_s^4\right] + \frac{1}{9}\left[-1.240\, a_s^3 - 14.95\, a_s^4\right]
$$

Extra suppression factor $\frac{3}{99} \approx 0.03$!

QED β-function in five loops

By a proper change of color factors we arrive at the full Adler function of QED in five loops $\Longrightarrow$ the QED β-function;

for a QED with one charged fermion we get $(A \equiv \frac{e^2}{16\pi^2})$

$$\beta^{QED} = \frac{4}{3} A + 4 A^2 - \frac{62}{9} A^3 - A^4 \left(\frac{5570}{243} + \frac{832}{9} \zeta_3 \right)$$

Gorishny, Kataev, Larin, Surguladze, 1991

$$-A^5 \boxed{\left(\frac{195067}{486} + \frac{800}{3}\zeta_3 + \frac{416}{3}\zeta_4 - \frac{6880}{3}\zeta_5 \right)}$$

Numerically $(\mathbf{A} = \frac{\alpha}{4\pi} \approx \mathbf{5.81} \cdot \mathbf{10^{-4}})$

$$\beta^{QED} = \frac{4}{3} A \left(1 + 3\, A - 5.1667\, A^3 - 100.534\, A^4 + 1129.51\, A^5 \right)$$

To check reduction to masters, two more calculations in $\mathcal{O}({\alpha_s}^4)$, all for *general gauge group!*:

1. Perturbative factor $C^{Bjp}(a_s)$ in Bjorken sum rule:

$$\int_0^1 [g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)]dx = \frac{1}{6}|\frac{g_A}{g_V}|\, C^{Bjp}(a_s)$$

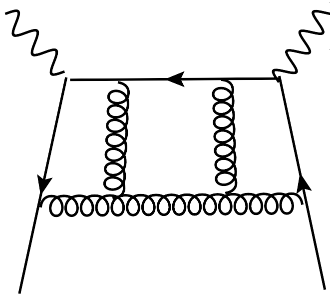
2. Perturbative factor $C_{GLS}(a_s)$ in Gross-Llewellyn Smith sum rule:

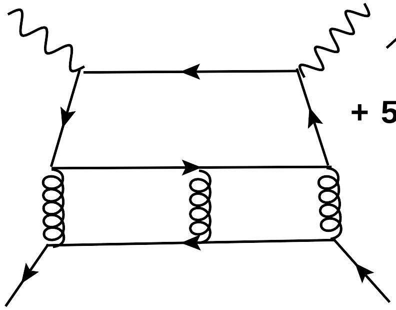
$$\frac{1}{2}\int_0^1 F_3^{\nu p+\overline{\nu}p}(x,Q^2)dx = 3\, C_{GLS}(a_s)$$

Both sum rules are unambiguous QCD predictions /modulo higher twists!/ confrontable with data

Typical diagrams at ${\alpha_s}^3$ (computed in early nineties /Larin & Vermaseren/),

Bjp *and* GLS (non-singlet) $\quad$ GLS *only* (singlet) $\quad$ (1)





$$C^{NS}_{GLS} \equiv C_{BJp} = 1 - a_s + a_s^2 [-4.583 + 0.3333\, n_f] + a_s^3 [-41.44 + 7.607\, n_f - 0.1775\, n_f^2] + a_s^4 [-479.4 + 123.4\, n_f - 7.697\, n_f^2 + 0.1037\, n_f^3]$$

$$C^{SI}_{GLS} = 0.4132\, n_f\, a_s^3 + a_s^4\, n_f\, (5.80157 - 0.233185\, n_f)$$

Note: $C^{SI}_{GLS} \ll C^{NS}_{GLS}$ as expected ($\overline{\text{MS}}$-scheme):

(Generalized) Crewther relation for D^{NS}

$$\boldsymbol{C^{Bjp}(a_s)\, D^{NS}(a_s) = 1 + \frac{\beta(a_s)}{a_s}\Big[K^{NS} = K_1\, a_s + K_2\, a_s^2 + K_3\, a_s^3 + \ldots\Big]} \quad (\star)$$

with $\frac{\beta(\alpha_s)}{\alpha_s} \equiv -\beta_0 a_s + \ldots, \quad \beta_0 = \frac{11}{12}C_A - \frac{T_f\, n_f}{3}$

$(\star)$ implies **6** constraints on 12 color structures

$$C_F^4\,,\ C_F^3 C_A\,,\ C_F^2 C_A^2\,,\ C_F C_A^3\,,\ C_F^3 T_F n_f\,,\ C_F^2 C_A T_F n_f\,,$$
$$C_F C_A^2 T_F n_f\,,\ C_F^2 T_F^2 n_f^2\,,\ C_F C_A T_F^2 n_f^2\,,\ C_F T_F^3 n_f^3\,,\ d_F^{abcd} d_A^{abcd}\,,\ n_f d_F^{abcd} d_F^{abcd}$$

appearing at $\mathcal{O}(\alpha_s^4)$ in the difference

$$\boldsymbol{D^{NS} - 1/C^{Bjp}}$$

All 6 constraints are met identically! (which means $6 \cdot 7 = 42$ separate constraints on coefficients of $\zeta_3, \zeta_3^2, \ldots$

Crewther relation between $D = D^{NS} + D^{SI}$ and C_{GLS}

$$\left(D^{NS} + d_3^{SI} a_s^3 + d_4^{SI} a_s^4\right) \left(C_{GLS}^{NS} + c_3^{SI} a_s^3 + c_4^{SI} a_s^4\right) =$$
$$1 + \frac{\beta(\alpha_s)}{\alpha_s}\Big[K^{NS} + a_s^3 K_3^{SI} n_f \frac{d^{abc}d^{abc}}{d_R}\Big] \quad \star$$

with $\frac{\beta(\alpha_s)}{\alpha_s} \equiv -\beta_0 a_s + \ldots, \quad \beta_0 = \frac{11}{12} C_A - \frac{T_f}{3}$

$$d_3^{SI} = n_f \frac{d^{abc}d^{abc}}{d_R} d_{3,1}^{SI}, \quad d_4^{SI} = n_f \frac{d^{abc}d^{abc}}{d_R}\left(C_F d_{4,1}^{SI} + C_A d_{4,2}^{SI} + T_F d_{4,3}^{SI}\right)$$

$$c_3^{SI} = n_f \frac{d^{abc}d^{abc}}{d_R} c_{3,1}^{SI}, \quad c_4^{SI} = n_f \frac{d^{abc}d^{abc}}{d_R}\left(C_F c_{4,1}^{SI} + C_A c_{4,2}^{SI} + T_F c_{4,3}^{SI}\right)$$

rhs of $\star$ depends on only 1 unknown parameter, K_3^{SI}, thus 3-1 =2 constraints on three coefficients in d_4^{SI}

Obvious solution of these constraints reads:

$$d^{SI}_{4,1} = -\frac{3}{2}c^{SI}_{3,1} - c^{SI}_{4,1} = -\frac{13}{64} - \frac{\zeta_3}{4} + \frac{5\zeta_5}{8}$$

$$d^{SI}_{4,2} = -c^{SI}_{4,2} + \frac{11}{12}K^{SI}_{3,1} \qquad d^{SI}_{4,3} = -c^{SI}_{4,3} + \frac{1}{3}K^{SI}_{3,1}$$

All 2 constraints are met identically! (which means 2*7=14 separate constraints on coefficients of front of $\zeta_3, \zeta_3^2, \zeta_4, \zeta_4\zeta_3, \zeta_5, \zeta_7, \frac{n}{m}$)

CONCLUSIONS

- Singlet and non-singlet parts of the Adler function and the perturbative factors $C(a_s)$ of Bjorken and GLS sum rule have been both analytically evaluated for generic gauge group at $\boldsymbol{\mathcal{O}(\alpha_s{}^4)}$
- The generalized Crewther relation puts 42 constraints on the non-singlet result and 14 constraints on the difference $\boldsymbol{d_4^{SI} - C_4^{GLS}}$ which **are all** fulfilled!
- Numerically, the singlet contribution to the VV-correlator is tiny