HARD SCATTERING AND ELECTROWEAK CORRECTIONS AT THE LHC

J.H. Kühn

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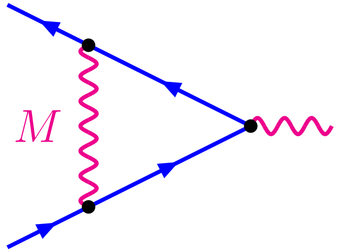
I. Introduction

"Typical" size of electroweak corrections: $\frac{\alpha_{\text{weak}}}{\pi} \approx 10^{-2}$

new aspects at LHC: $\sqrt{\hat{s}} \approx$ 1-2TeV $\gg M_{W,Z}$

strong enhancement of negative corrections

one-loop example: massive U(1)



$$\Rightarrow \quad \text{Born} * \left[1 + \frac{\alpha}{4\pi}\left(-\ln^2\frac{s}{M^2} + 3\ln\frac{s}{M^2} - \frac{7}{2} + \frac{\pi^2}{3}\right)\right]$$

$\frac{s}{M^2}$	$-\ln^2\frac{s}{M^2}$	$+3\ln\frac{s}{M^2}$	$-\frac{7}{2}+\frac{\pi^2}{3}$	Σ	$* 4\,\frac{\alpha_{\text{weak}}}{4\pi}$
$\left(\frac{1000}{80}\right)^2$	-25.52	$+15.15$	-0.21	-10.6	-13%
$\left(\frac{2000}{80}\right)^2$	-41.44	$+19.31$	-0.21	-22.3	-27%

(four-fermion cross section $\Rightarrow$ factor 4)

- leading $\log^2$ multiplied by (charge)$^2 = I(I+1) = \begin{cases} 3/4 & I = 1/2 \\ 2 & I = 1 \end{cases}$
  $\Rightarrow$ further enhancement for W-pairs by nearly factor 2.

- important subleading logarithms (NLL+...)

- One-loop up to $\mathcal{O}(30\%) \rightarrow$ two-loop terms may be relevant

- interplay between electroweak and QCD corrections

- important differences between fermions and electroweak gauge bosons

- important differences between long. and transverse gauge bosons ($I = 1/2$ vs. $I = 1$)

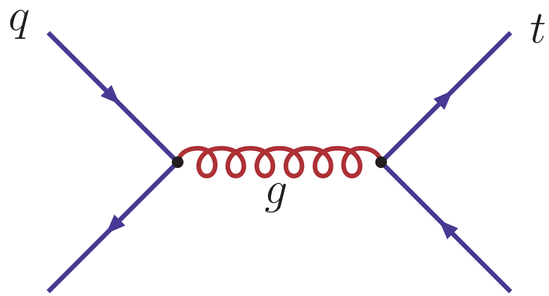
II. One-Loop Results

1. Top and Bottom Pair Production

JK, Scharf, Uwer, Eur. Phys. J. C45(2006), C51 (2007)37, PRD 82 (2010) 013007

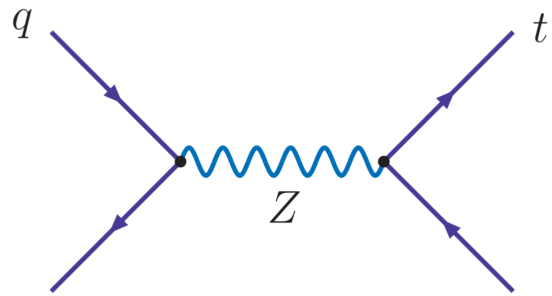
(Related Results:
Beenakker, Denner, Hollik, Mertig, Sack, Wackeroth (1994)
Bernreuther, Fücker, Si
Moretti, Holten, Ross)

$q\,\bar{q} \to t\,\bar{t}$:



$\sim \mathcal{O}(\alpha_s)$

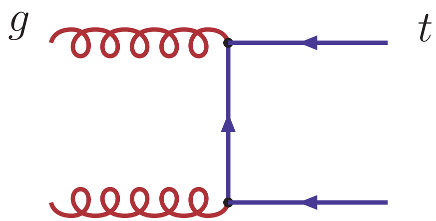
no interference with

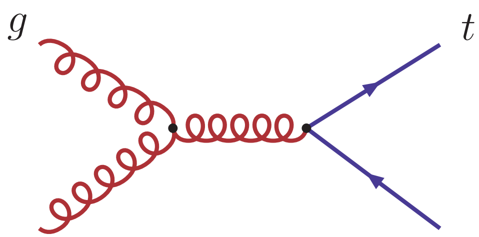


$\sim \mathcal{O}(\alpha_{\mathrm{weak}})$

(more problematic for $qq \to qq$!)

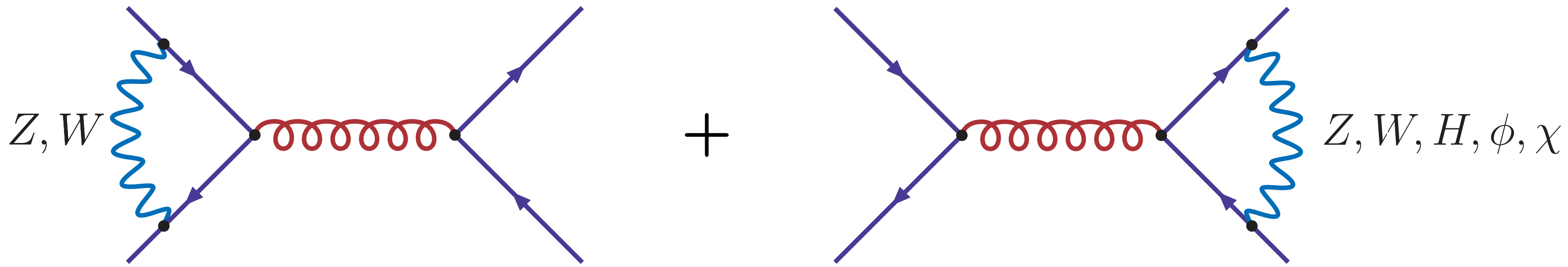
$g\,g \to t\,\bar{t}$:

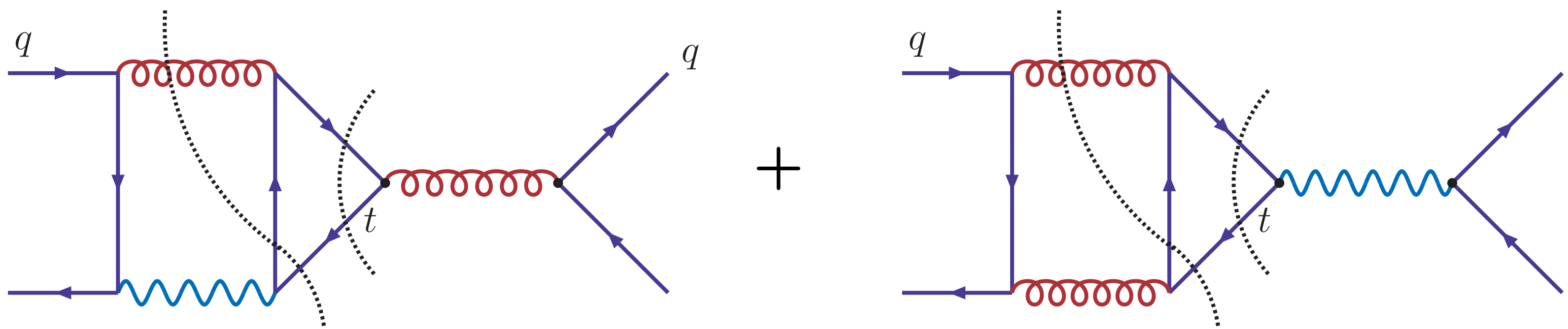




$\sim \mathcal{O}(\alpha_s)$

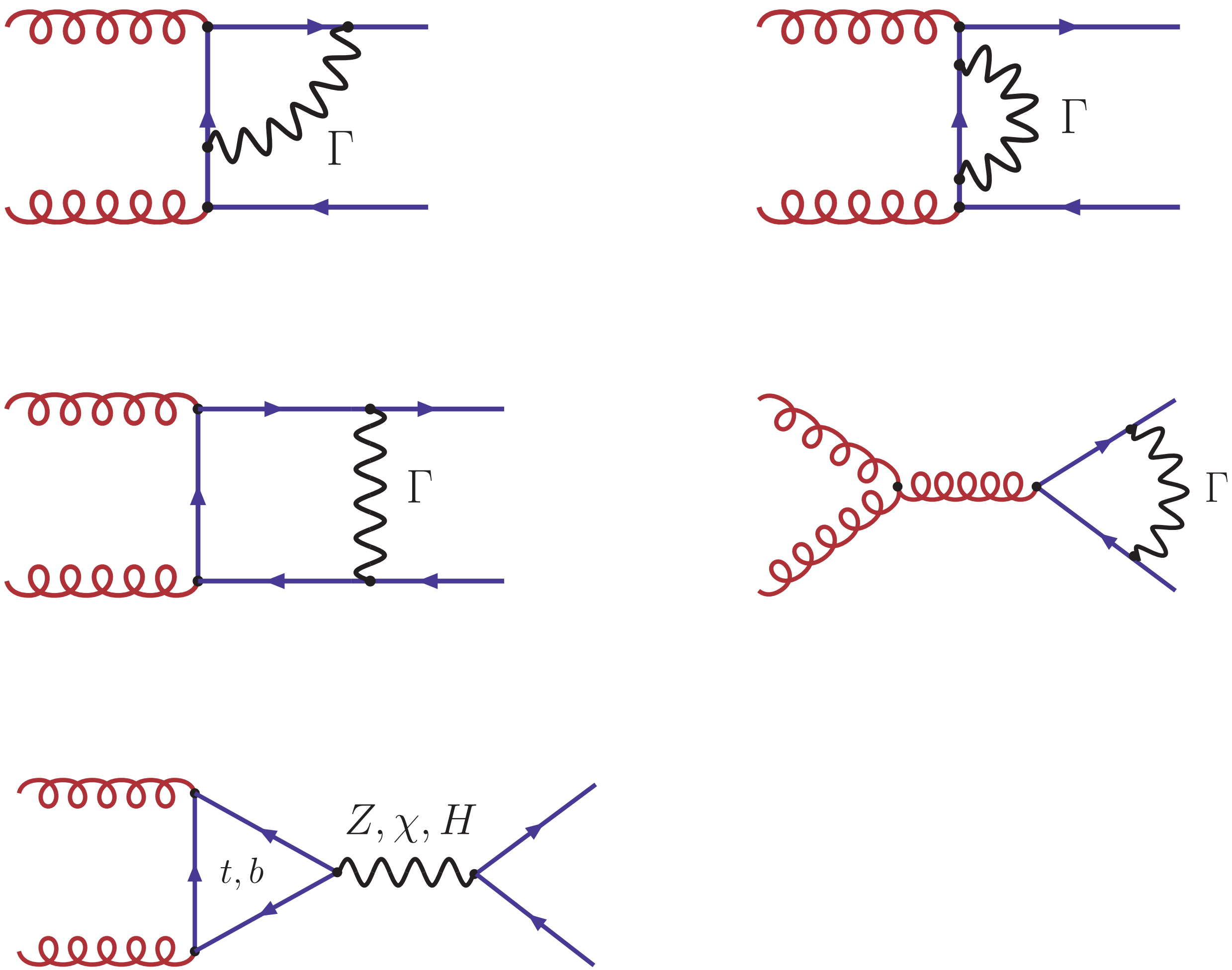
$\mathcal{O}(\alpha_s^2 \alpha_{\text{weak}})$ weak corrections ($q\,\bar{q} \to t\,\bar{t}$)





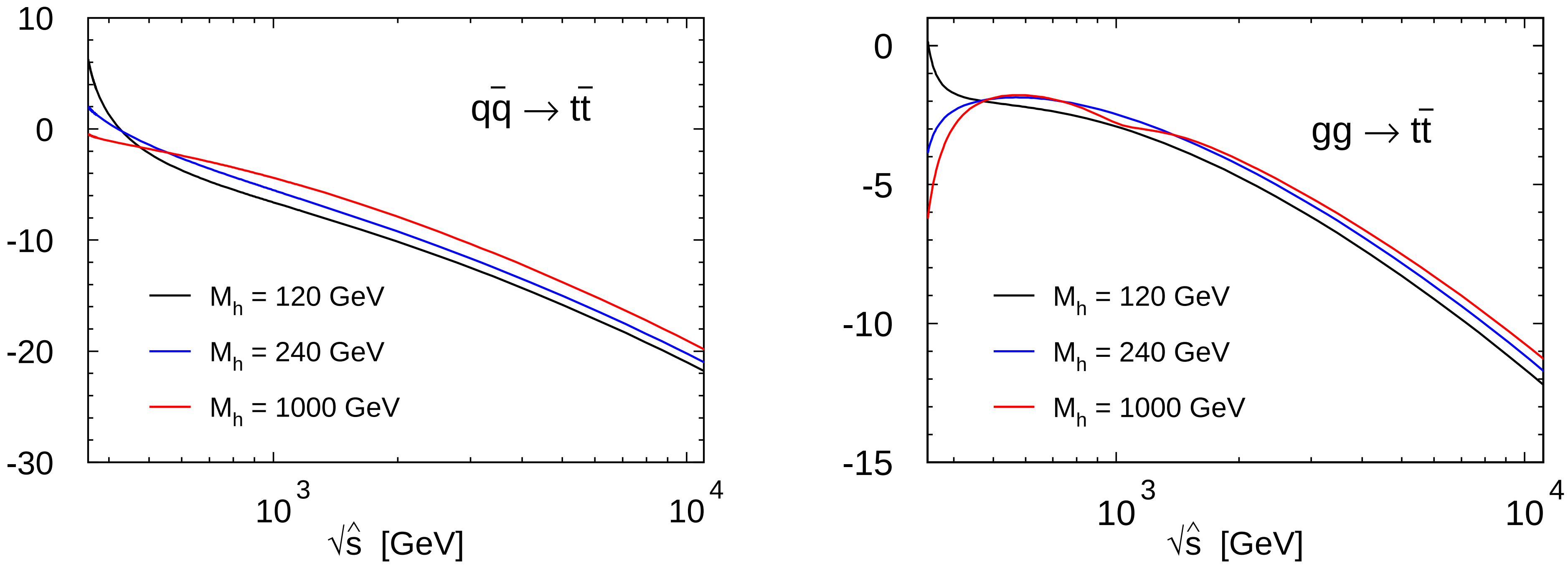
cuts of second group individually IR-divergent

$\mathcal{O}(\alpha_s^2 \alpha_{\text{weak}})$ weak corrections $(g\,g \to t\,\bar{t}\,)$



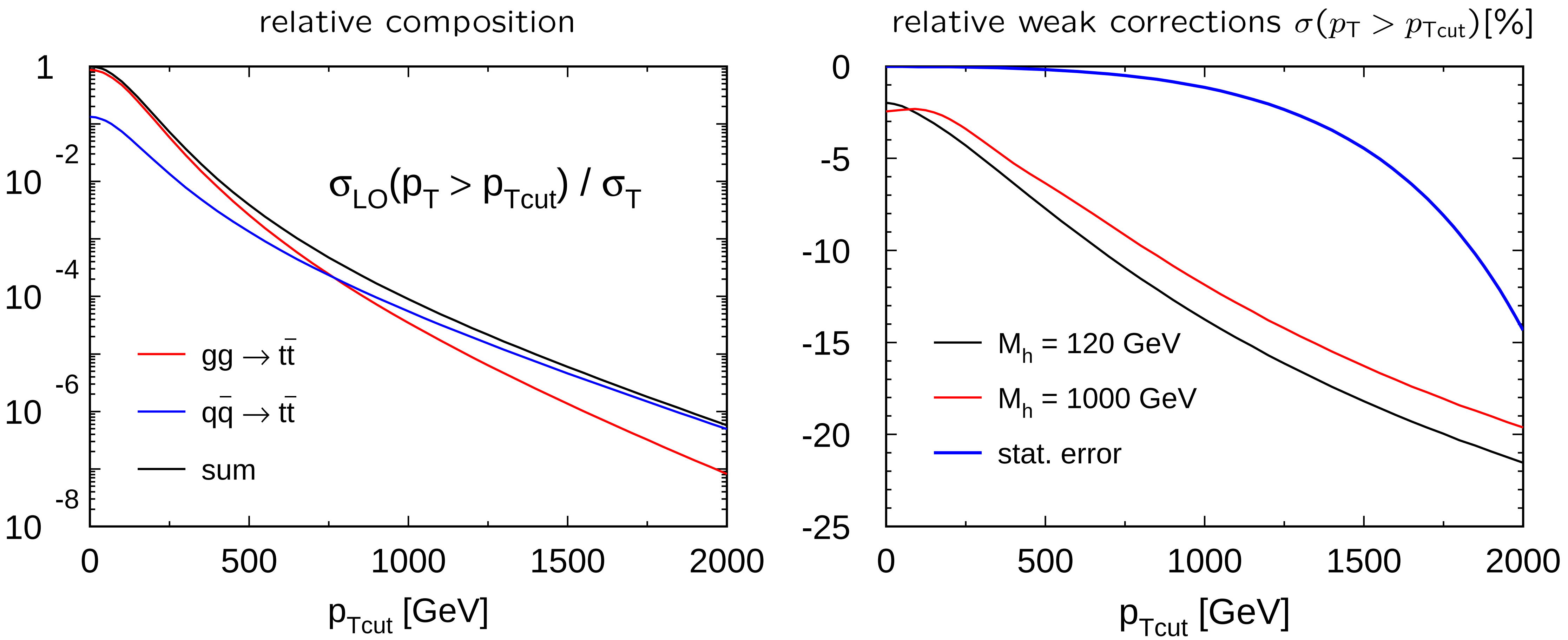
large corrections for large $\sqrt{\hat{s}}$

sizable M_h-dependence

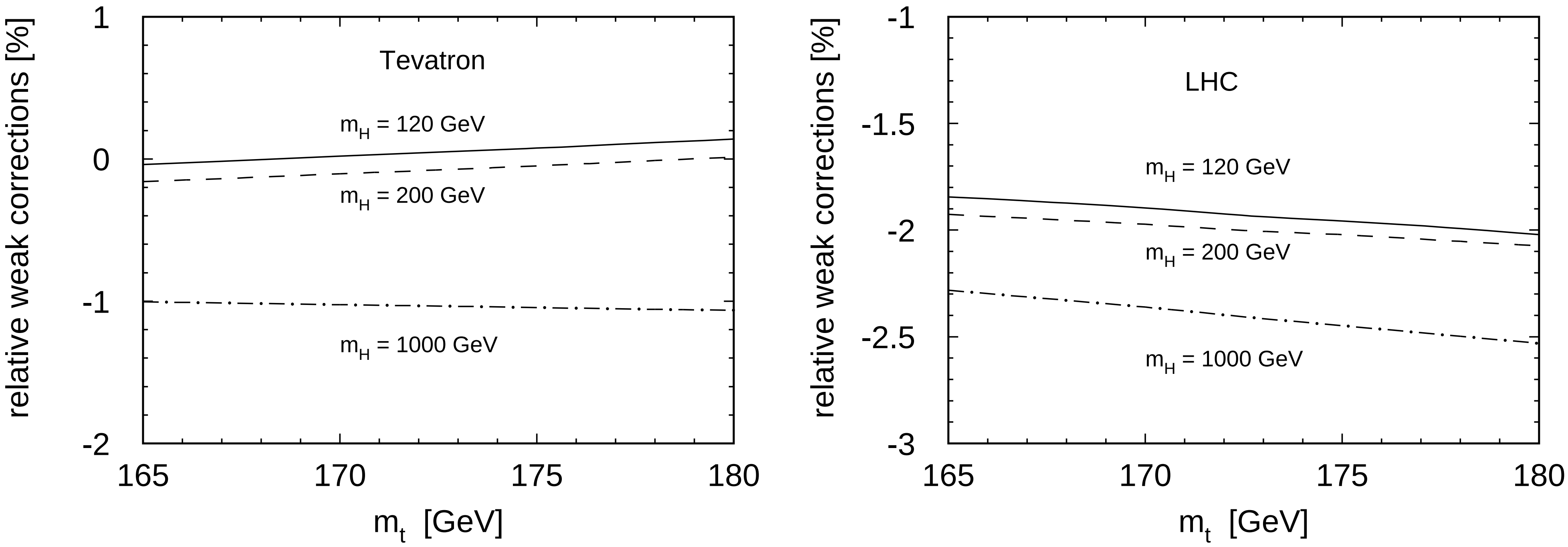


(relative weak corrections [%])

Transverse momentum dependence

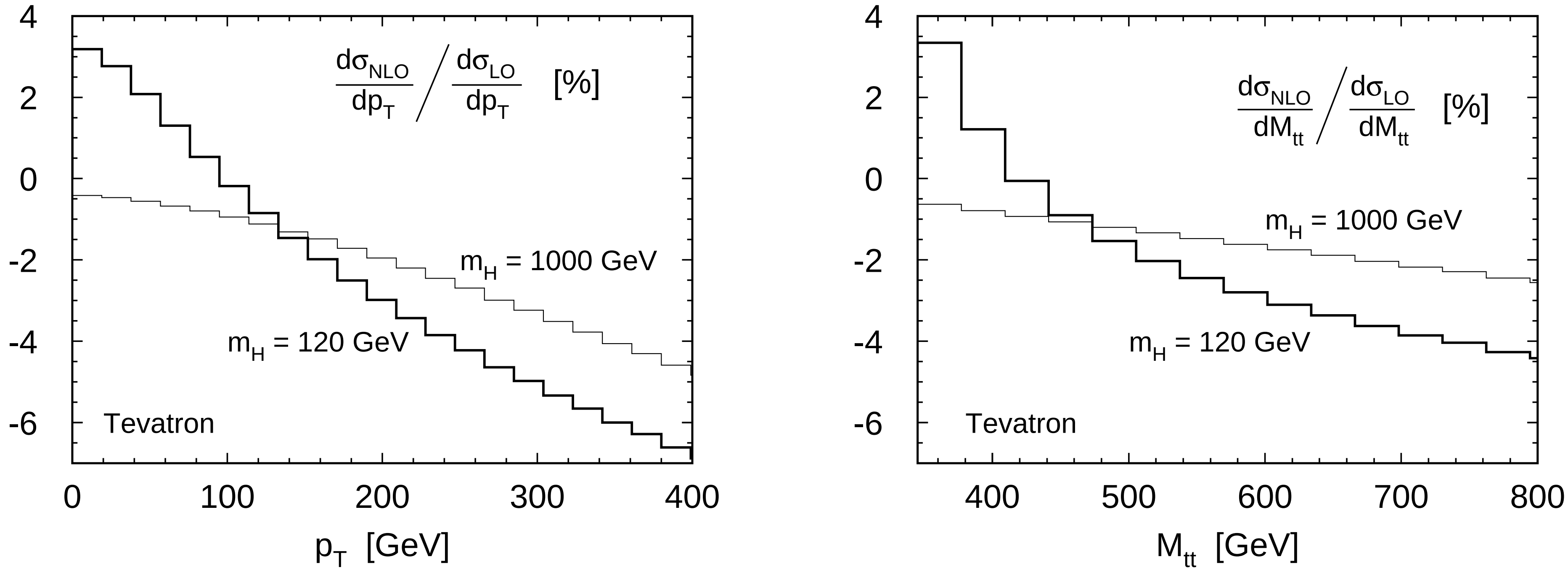


Total cross sections



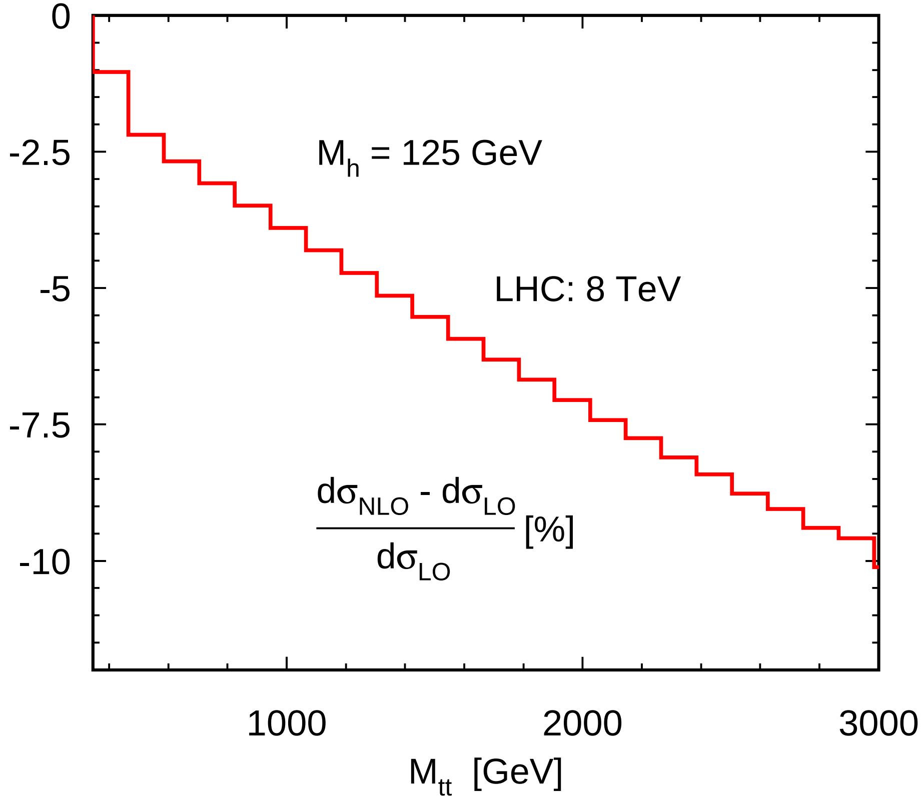
Left: Weak corrections to top-quark pair production at the Tevatron and **Right:** at the LHC for three different Higgs masses ($m_H = 120$ GeV (full line), $m_H = 200$ GeV (dashed), $m_H = 1000$ GeV (dash-dotted)).

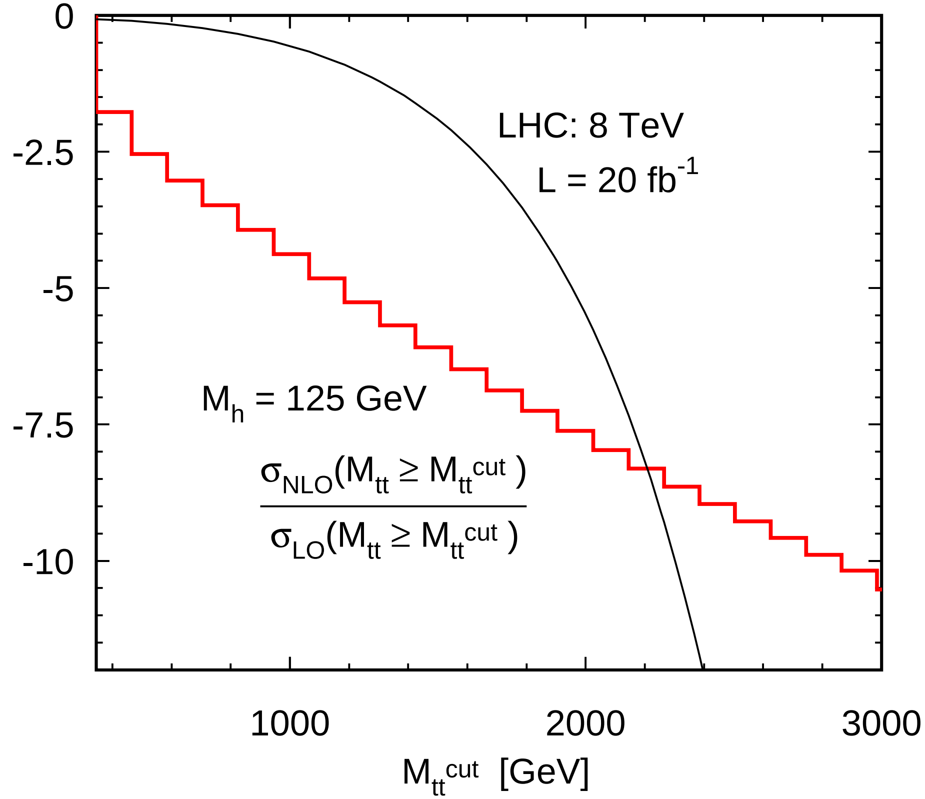
Differential distributions



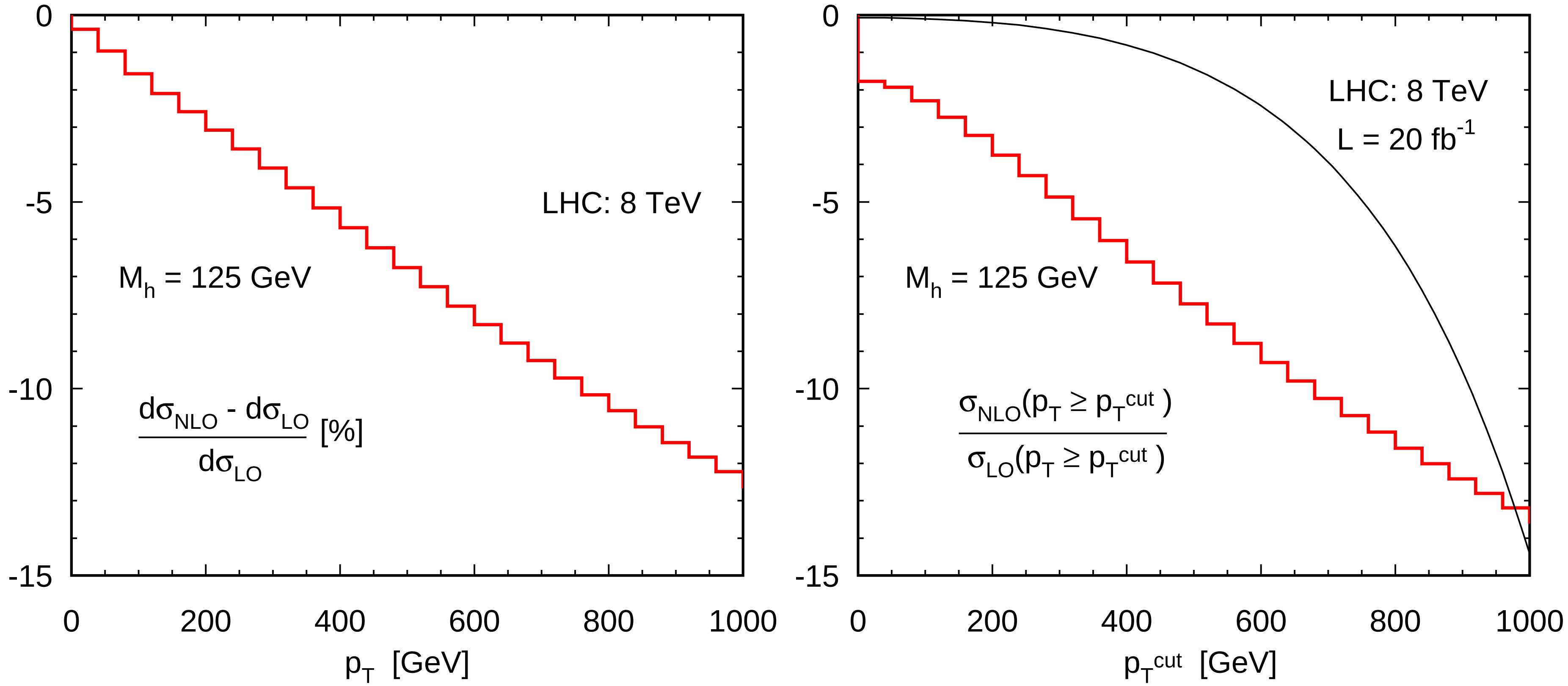
The relative corrections to the p_T and $M_{t\bar{t}}$ distribution for the Tevatron for $m_H = 120$ GeV (bold histogram) and $m_H = 1000$ GeV (thin histogram).

LHC at 8 TeV, top-pair invariant mass



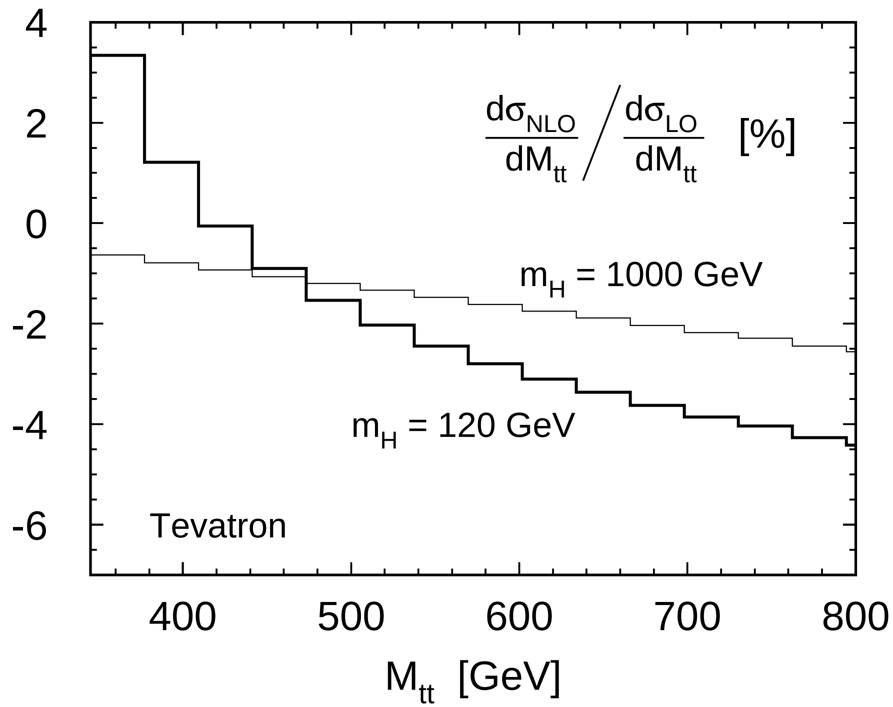


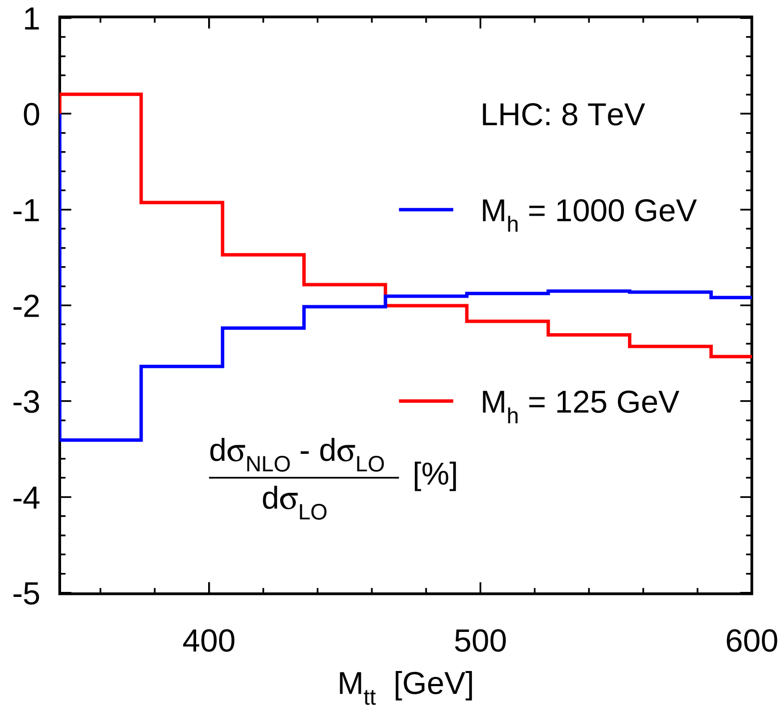
LHC at 8 TeV, transverse momentum



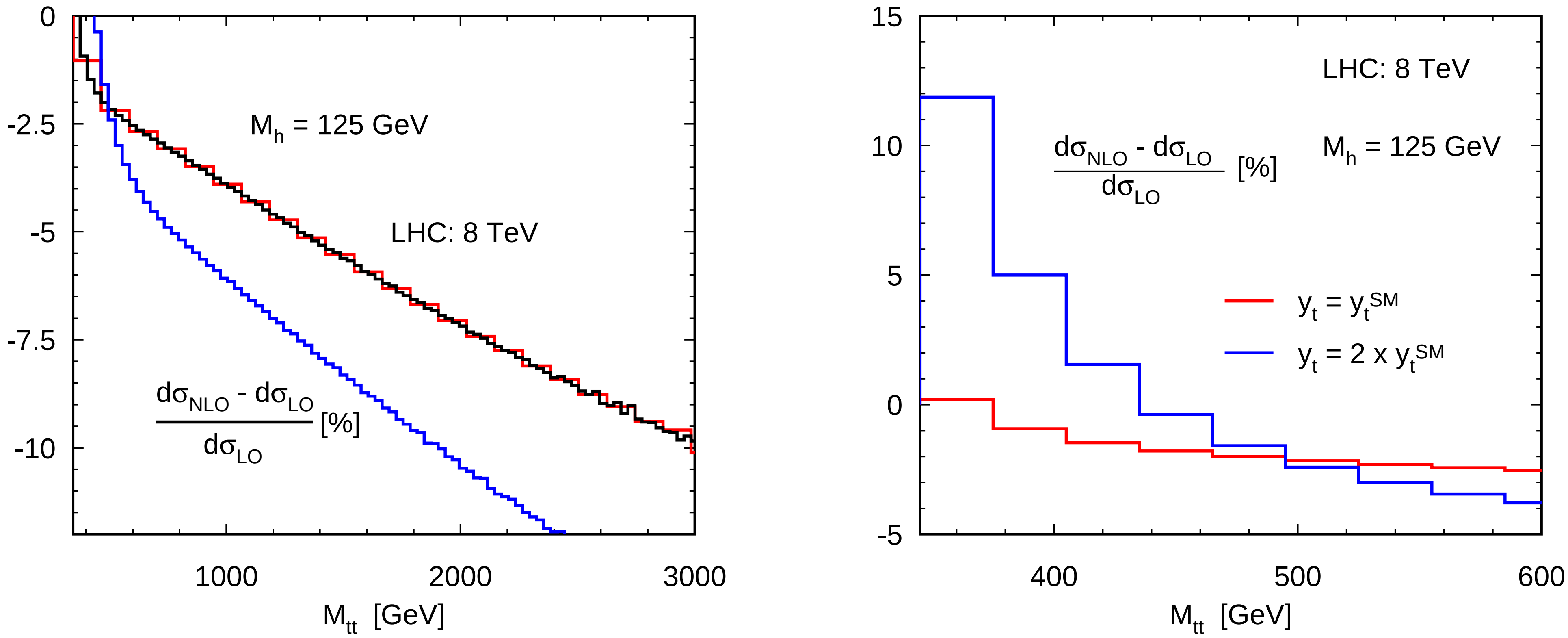
Threshold behaviour: Tevatron and LHC

$\Rightarrow$ $m_{\rm H}$ and Yukawa coupling



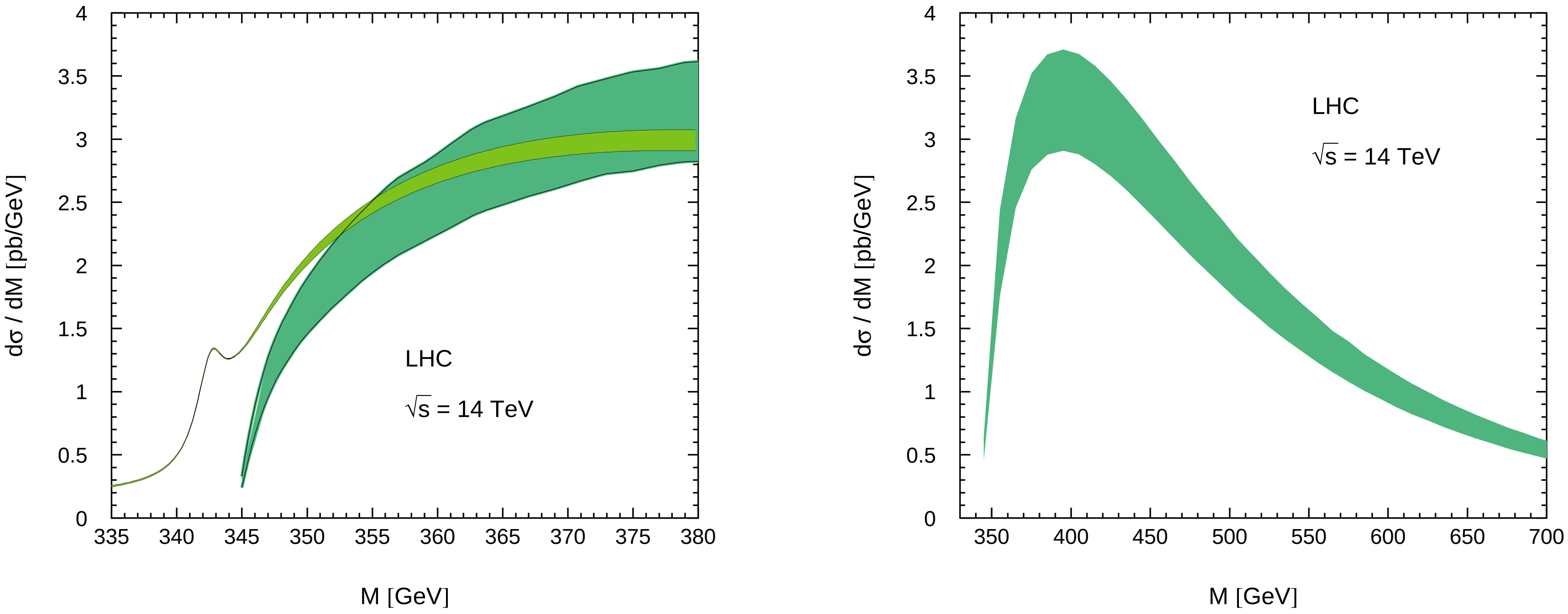


dependence on Higgs mass



dependence on Yukawa coupling: $Y_{\rm top} \Rightarrow 2 * Y_{\rm top}$

Threshold behaviour



Left: Invariant mass distribution dσ/dM from NRQCD and for a fixed NLO for LHC with $\sqrt{s} = 14$ TeV. The bands are due to scale variation for $\mathcal{L} \otimes F$ from m_t to $4m_t$.
Right: Invariant mass distribution dσ/dM from NLO calculation for LHC with $\sqrt{s} = 14$ TeV.

- analytical & numerical results available
  (earlier partial results by Beenakker *et al.*, some disagreements)
  independent evaluation by Bernreuther, Fücker, Si $\Rightarrow$ agreement
  Moretti, Holten, Ross $\Rightarrow$ some disagreement ?

- $(\text{box contribution})_{\text{up-quark}} = -(\text{box contribution})_{\text{down-quark}}$
  $\Rightarrow$ suppression

- box contribution moderately $\hat{s}$-dependent

- corrections strongly increasing with $\hat{s}$, angular dependent; ($M_{\text{tt}}/2$ vs p_{T})

- sizable M_{h}-dependence, large effect close to threshold
  $\Rightarrow$ determination of Yukawa coupling

$b\bar{b}$ production: similar to $t\bar{t}$

Additional contributions from:

- $gb \to gb$: (single b-tag) through crossing

- $bb \to bb$: s and t exchange;
  terms of $\mathcal{O}(\alpha_s \alpha_w)$ small
  corrections irrelevant

- $qq \to qq$ **etc.:** s and t exchange;
  terms of $\mathcal{O}(\alpha_s \alpha_w)$ contribute
  new terms of $\mathcal{O}(\alpha_s^2 \alpha_w)$;
  interplay between QCD and EW corrections
  $\Rightarrow$ Moretti et al.

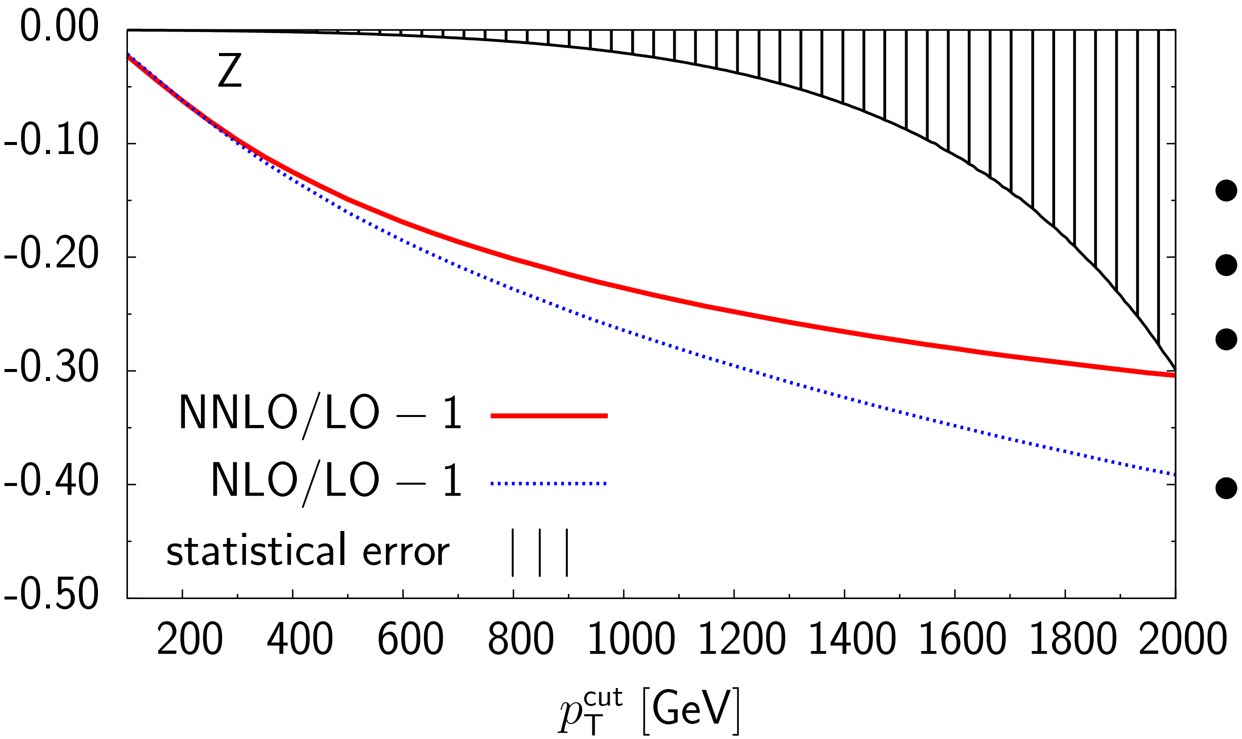
II. 2. V + jet, V=W,Z,γ

JK, Kulesza, Pozzorini, Schulze

Denner, Dittmaier, Kasprzik, Mück

$\Rightarrow$ talk by Kasprzik

Complete one loop calculation
NLL approximation at two loops



- one-loop $\sim 30\%$ at $p_T \sim 1$TeV
- two-loop relevant above 1 TeV
- important angular-dependent logarithmic terms
- experiment: p_T up to 2 TeV

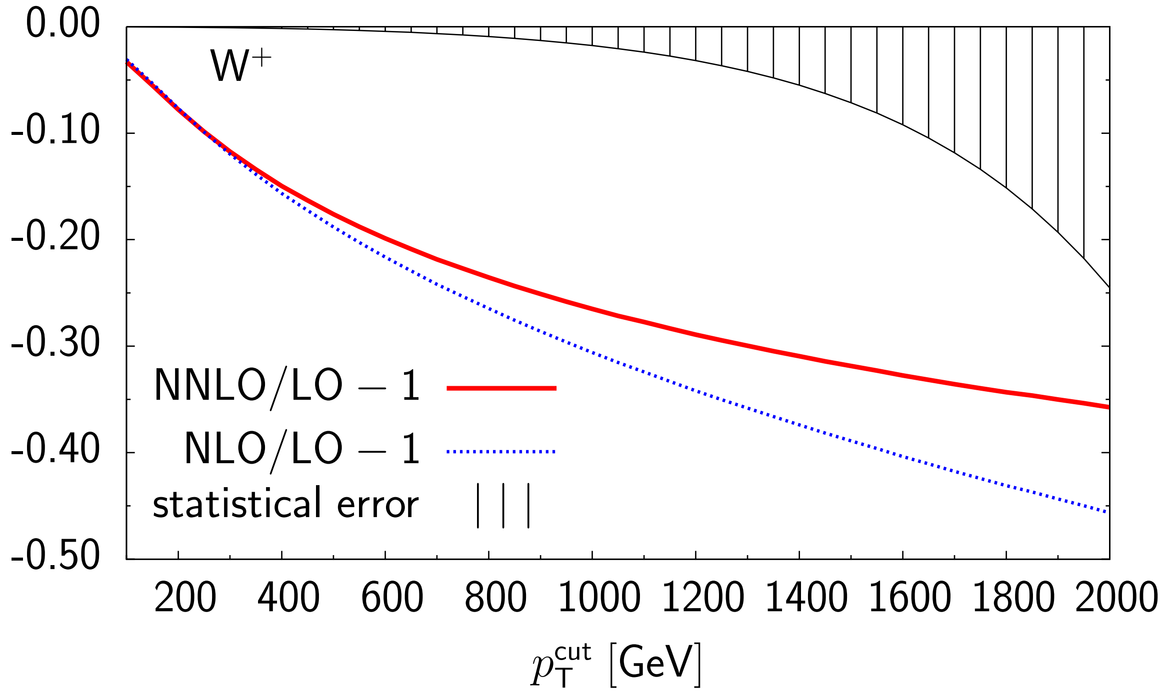
Relative **NLO** and **NNLO** corrections w.r.t. the **LO** and statistical error for the unpolarized integrated cross section for $pp \to Zj$ at $\sqrt{s} = 14$ TeV.

(Similarly, but smaller by a factor 2 for jet+γ)

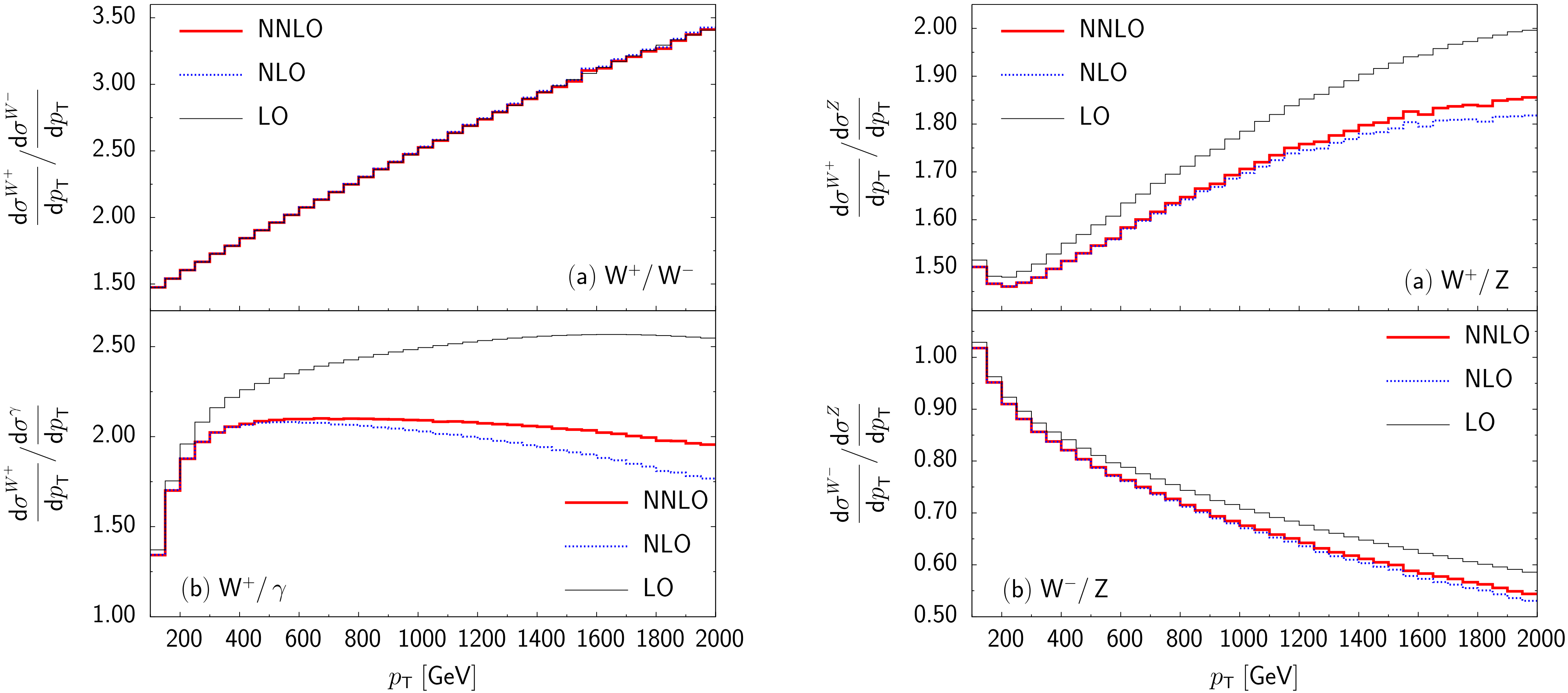
W production

additional complications:

- photon radiation as necessary part of virtual corrections (gauge invariance)
- IR singularities must be compensated by real radiation
- $p_{\mathrm{T}}(W) = p_{\mathrm{T}}(\text{jet}) + p_{\mathrm{T}}(\gamma)$



(related results: Dittmaier, Kasprzik, ...)



ratios are less sensitive to QCD corrections

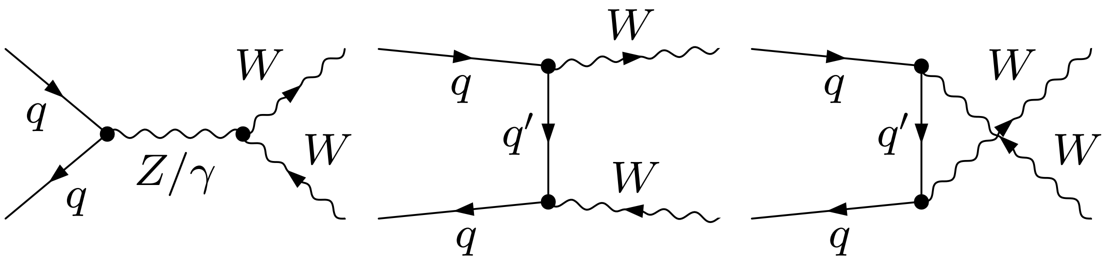
II. 3. W^+W^-, $W^\pm Z$, ZZ

Two Approaches:

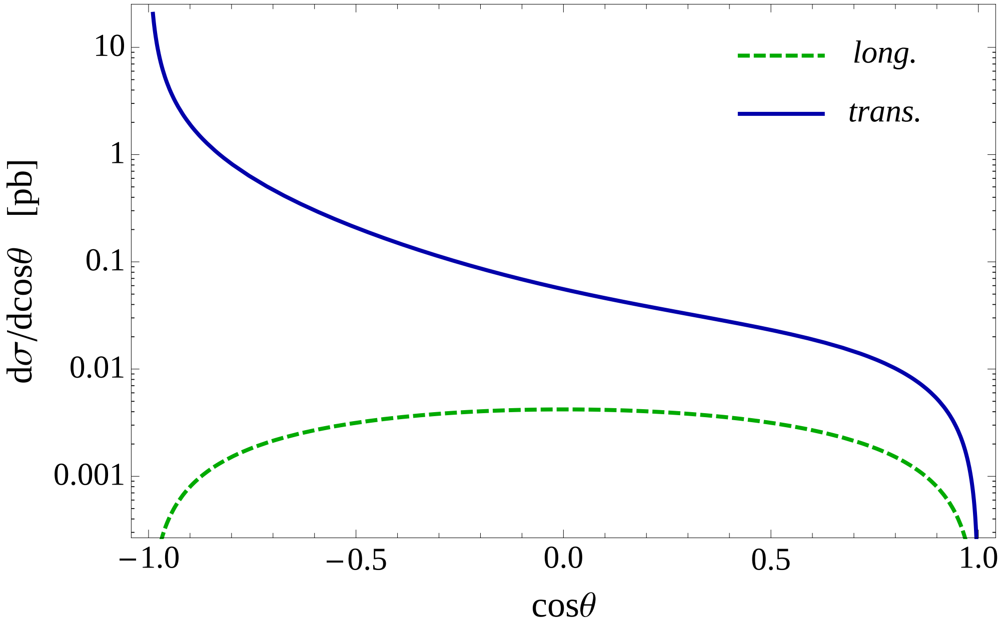
- dominant, logarithmically enhanced terms via evolution equation & separation of QED
  $\Rightarrow$ one- and two-loop terms in NNLL
  J.H.K., Metzler, Penin, Uccirati: JHEP 1106 (2011) 143
  related work based on SCET: Manohar,...

- one-loop calculation, including $M_W^2/\hat{s}$ terms and real radiation: full NLO
  Bierweiler, Kasprzik, J.H.K., Uccirati
  related work: logarithmically enhanced terms only, including W decays
  Accomando, Denner, Kaiser

Leading Order

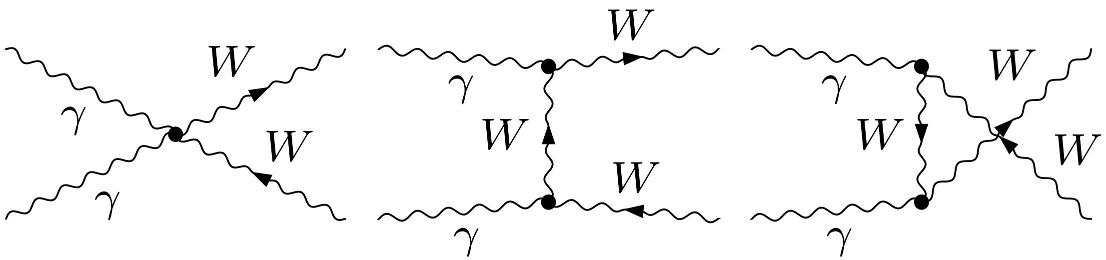


$u\bar{u} \to W^+W^-$, $\sqrt{s} = 1$ TeV

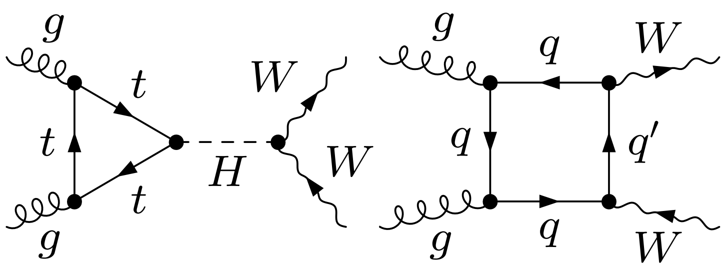


- Strong enhancement for $\Theta \to 180°$
- dominance of transverse W

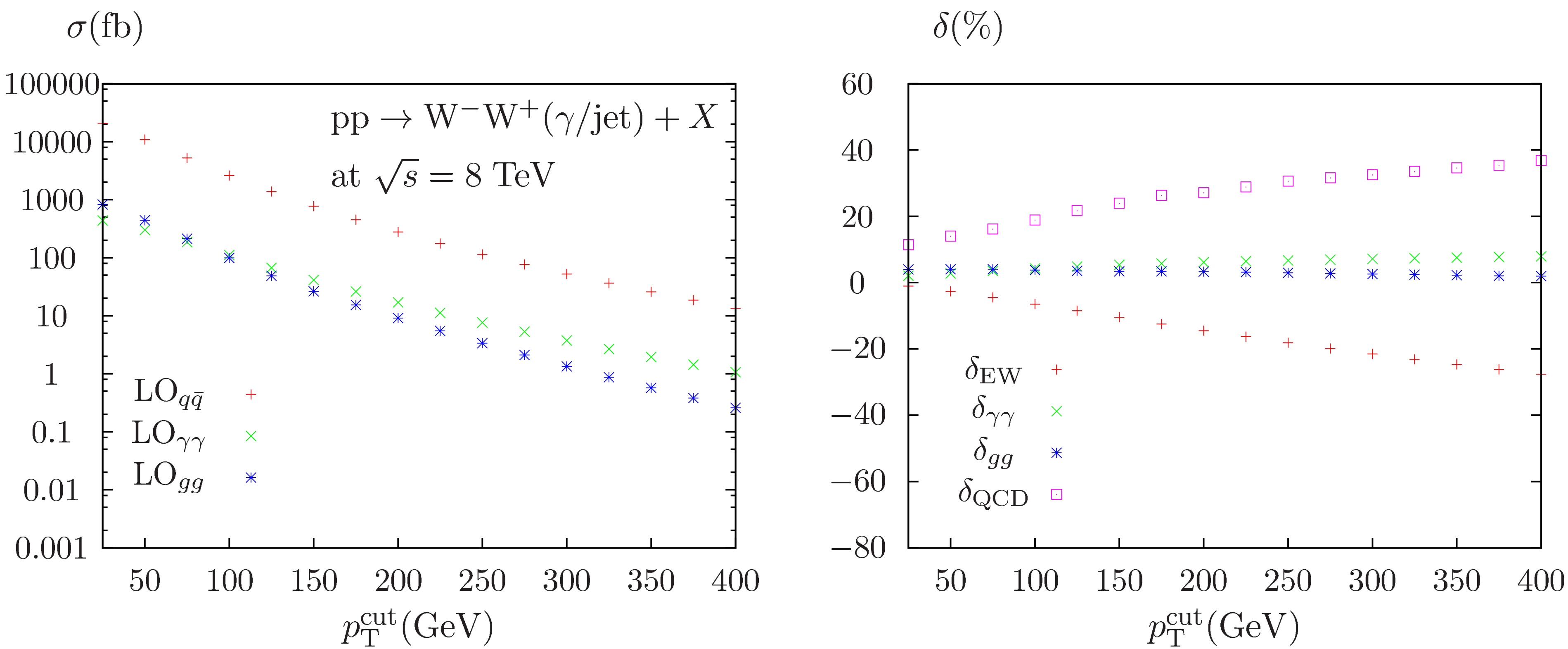
Also included: $\gamma\gamma \to WW$



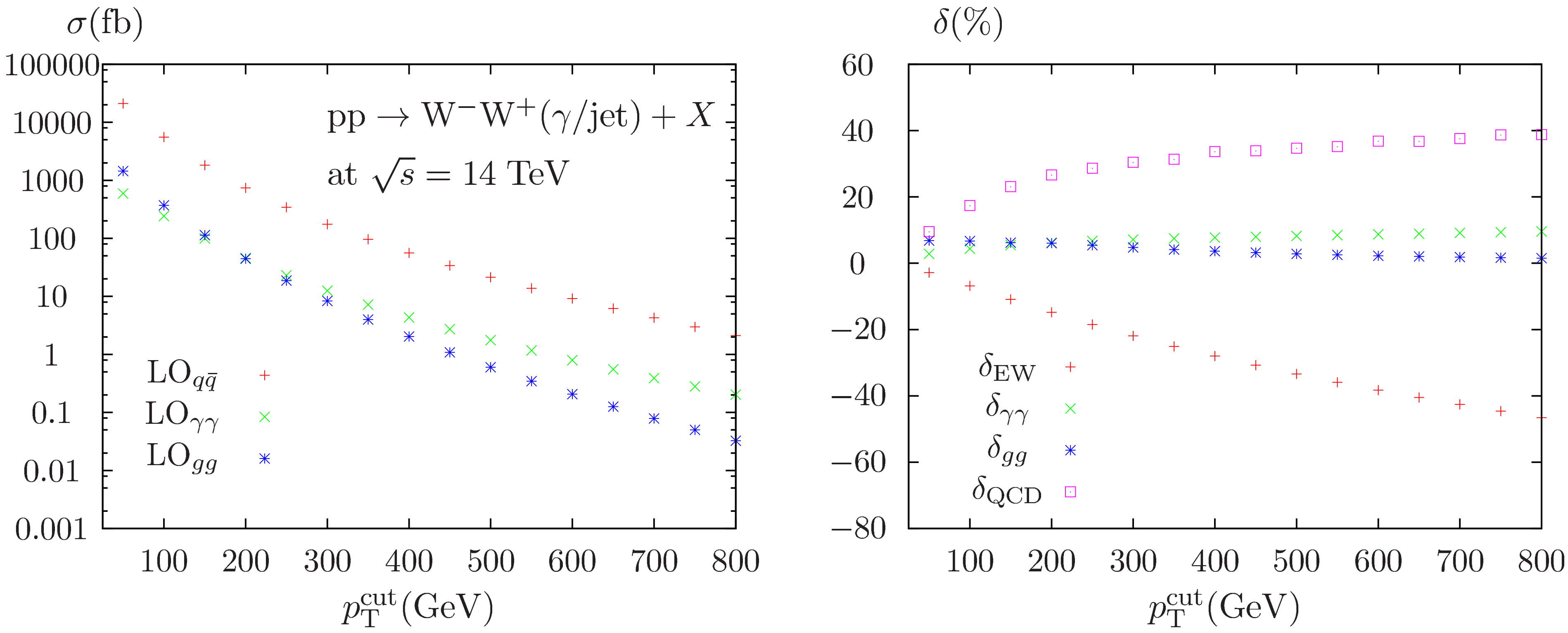
also included: $gg \to WW$

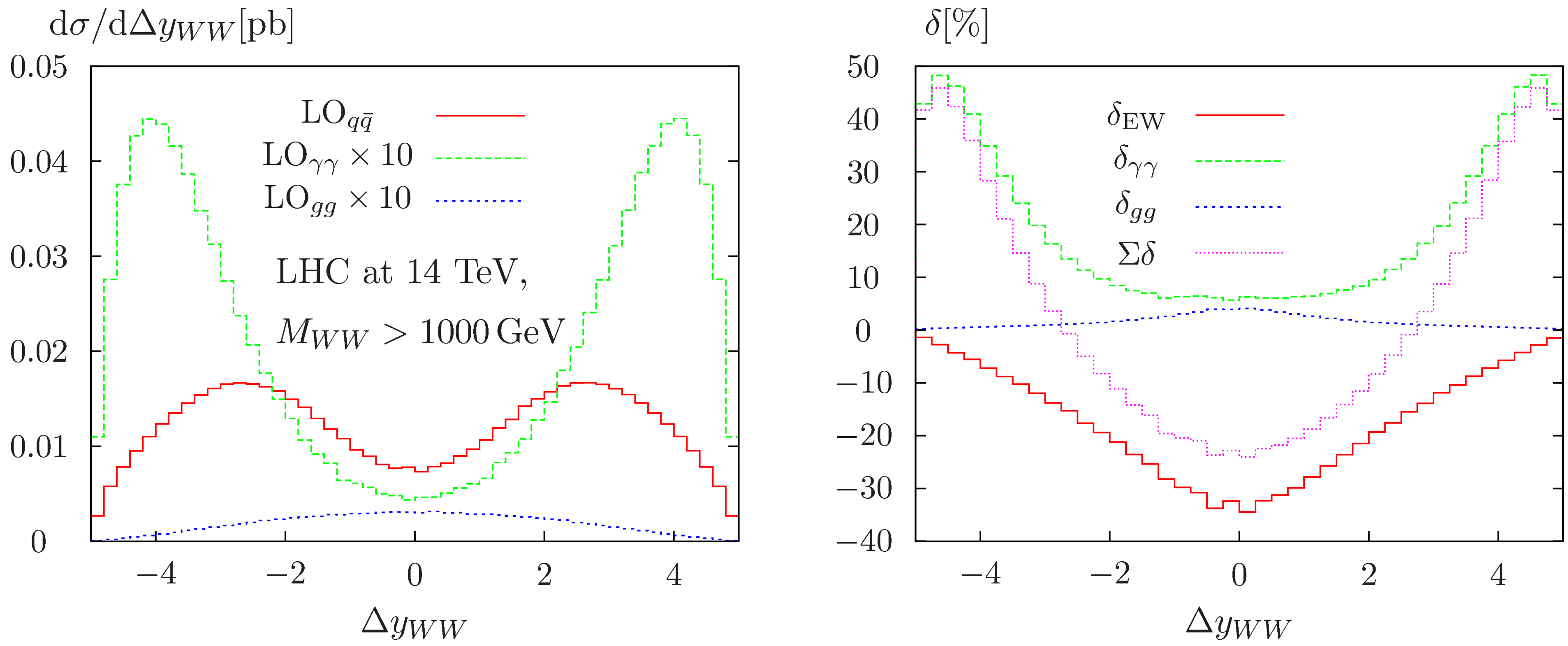


Total cross sections LHC8, $p_T > p_{T,\mathrm{cut}}$



Total cross sections LHC14, $p_T > p_{T,\mathrm{cut}}$





Differential LO cross sections for the W-boson rapidity gap with a minimal invariant mass of 1000 GeV at the LHC14. On the right-hand-side, the corresponding relative rates due to photon- and gluon-induced channels w.r.t. the $q\bar{q}$-contributions are shown, as well as the EW corrections.

Drell-Yan process:

$pp \to l^+l^- + X$ at $\sqrt{s} = 14\text{TeV}$

M_{ll}/GeV	50–∞	100–∞	200–∞	500–∞	1000–∞	2000–∞
σ_0/pb	738.733(6)	32.7236(3)	1.48479(1)	0.0809420(6)	0.00679953(3)	0.000303744(1)
$\sigma_0\|_{FS/PS}$/pb	738.773(6)	32.7268(3)	1.48492(1)	0.0809489(6)	0.00680008(3)	0.000303767(1)
$\delta_{\gamma\gamma,0}$/%	0.17	1.15	4.30	4.92	5.21	6.17
$\delta^{\text{rec}}_{q\bar{q},\text{phot}}$/%	−1.81	−4.71	−2.92	−3.36	−4.24	−5.66
$\delta^{\mu^+\mu^-}_{q\bar{q},\text{phot}}$/%	−3.34	−8.85	−5.72	−7.05	−9.02	−12.08
$\delta^{\mu^+\mu^-}_{\text{multi}-\gamma}$/%	$0.073^{+0.027}_{-0.024}$	$0.49^{+0.18}_{-0.15}$	$0.17^{+0.06}_{-0.05}$	$0.23^{+0.07}_{-0.06}$	$0.33^{+0.09}_{-0.08}$	$0.54^{+0.13}_{-0.12}$
$\delta_{q\bar{q},\text{weak}}$/%	−0.71	−1.02	−0.14	−2.38	−5.87	−11.12
$\delta_{\text{h.o.weak}}$/%	0.030	0.012	−0.23	−0.29	−0.31	−0.32
$\delta^{(2)}_{\text{Sudakov}}$/%	−0.00046	−0.0067	−0.035	0.23	1.14	3.38
$\delta_{q/\bar{q}\gamma,\text{phot}}$/%	−0.11	−0.21	0.38	1.53	1.91	2.34
$\delta^{\text{rec}}_{\gamma\gamma,\text{phot}}$/%	−0.0060	−0.032	−0.11	−0.14	−0.16	−0.23
$\delta^{\mu^+\mu^-}_{\gamma\gamma,\text{phot}}$/%	−0.011	−0.058	−0.22	−0.30	−0.39	−0.59
$\delta_{\gamma\gamma,\text{weak}}$/%	0.000045	0.00056	−0.025	−0.14	−0.31	−0.64
δ_{QCD}/%	4.0(1)	13.90(6)	26.10(3)	21.29(2)	8.65(1)	−11.93(1)

Dittmaier, Huber (arXiv: 0911.2329v2 [hep-ph])

III. QED corrections: QED and PDFs

QED and EW one-loop corrections can be separated in some cases:

- Drell-Yan process: $q\bar{q} \xrightarrow{\gamma^*/Z} \mu^+\mu^-$

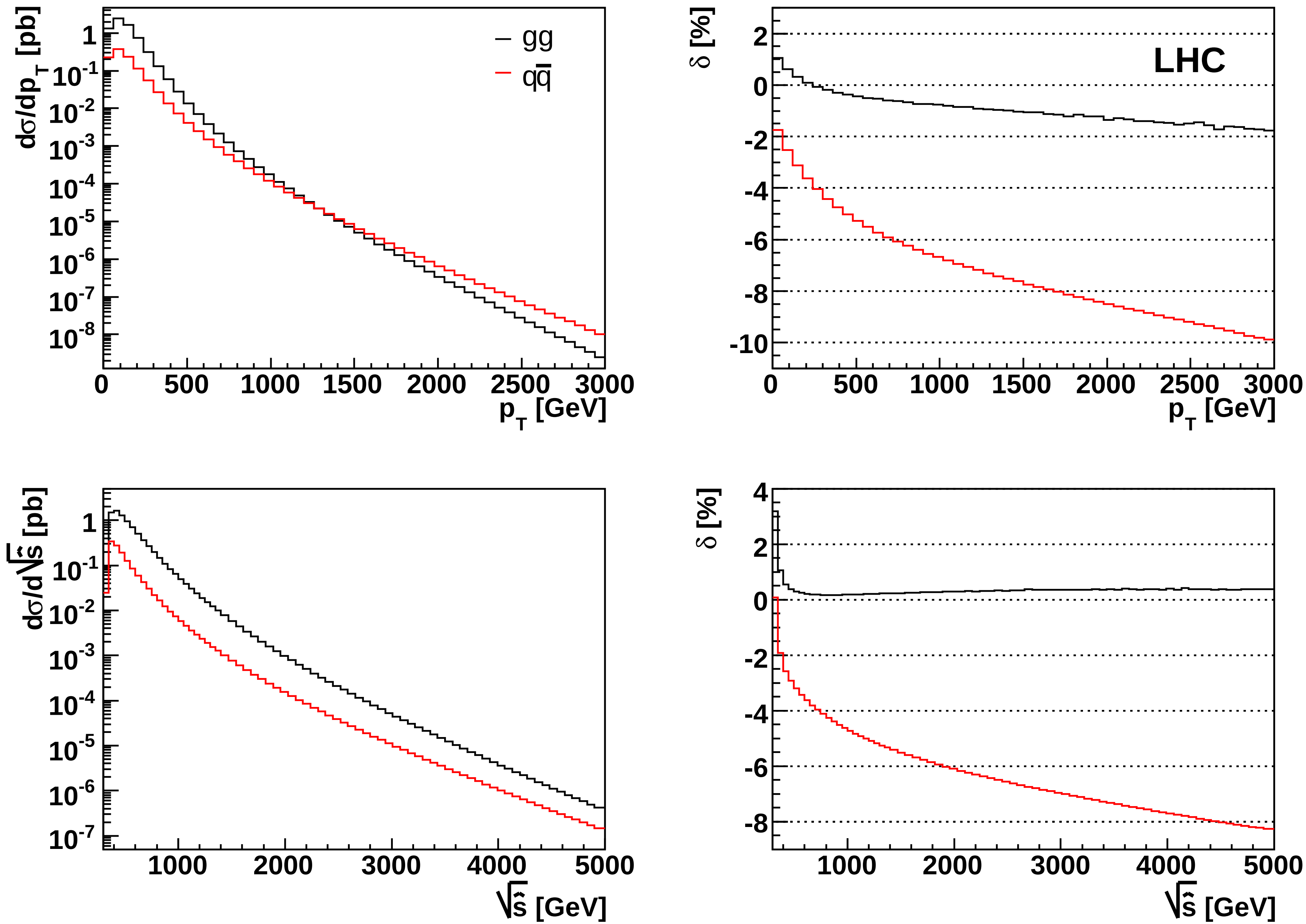
- $\gamma\gamma$ or ZZ production: $q\bar{q} \longrightarrow \gamma\gamma$ or ZZ

- top pair production: $q\bar{q} \longrightarrow$ t$\bar{\text{t}}$ or $gg \longrightarrow$ t$\bar{\text{t}}$

not for $qg \longrightarrow Wq'$, $q\bar{q} \longrightarrow W^+W^-, \ldots$

naively estimated to be small: $\mathcal{O}\left(\frac{\alpha}{\pi}\right) \leq 1\%$

Results for $t\bar{t}$ (Hollik, Kollar)



Hollik, Kollar (arXiv:0708.1697[hep-ph])

Results for $t\bar{t}$ (Hollik, Kollar)

Process	σ_{tot} without cuts [pb]	
	Born	correction
$u\bar{u}$	34.25	-1.41
$d\bar{d}$	21.61	-0.228
$s\bar{s}$	4.682	-0.0410
$c\bar{c}$	2.075	-0.0762
gg	407.8	2.08
$g\gamma$		4.45
pp	470.4	4.78

Production cross section:

MRST2004qed

($\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha)$ improved)

Comments:

- large contributionfrom $\gamma g \longrightarrow t\bar{t}$
  strongly dependent on $f_{\gamma/P}$
- collinear singularities absorbed in PDF,
  but: calculation without QCD corrections, PDF with $\mathcal{O}(\alpha_s)$.

Drell-Yan (Dittmaier, Huber)

admixture from $\gamma\gamma \longrightarrow \mu^+\mu^-$

$M_{\mu\mu}$	$\langle 50, \infty \rangle$	$\langle 100, \infty \rangle$	$\langle 200, \infty \rangle$	$\langle 500, \infty \rangle$	$\langle 1000, \infty \rangle$
$\delta_{\gamma\gamma}/\%$	0.17	1.15	4.30	4.92	5.21

(MRST2004qed, $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha)$ corrections, include photon PDFs.)

$\gamma\gamma \longrightarrow W^+W^-$**:** ($\Rightarrow$ talk by Bierweiler)

- prediction strongly dependent on PDF
  (MRST2004qed $\widehat{=}$ educated guess)
- $\sigma(\gamma\gamma \longrightarrow W^+W^-) \xrightarrow{s\to\infty} \frac{8\,\pi\,\alpha^2}{M_W^2}$
  (strongly enhancement in forward-backward direction)

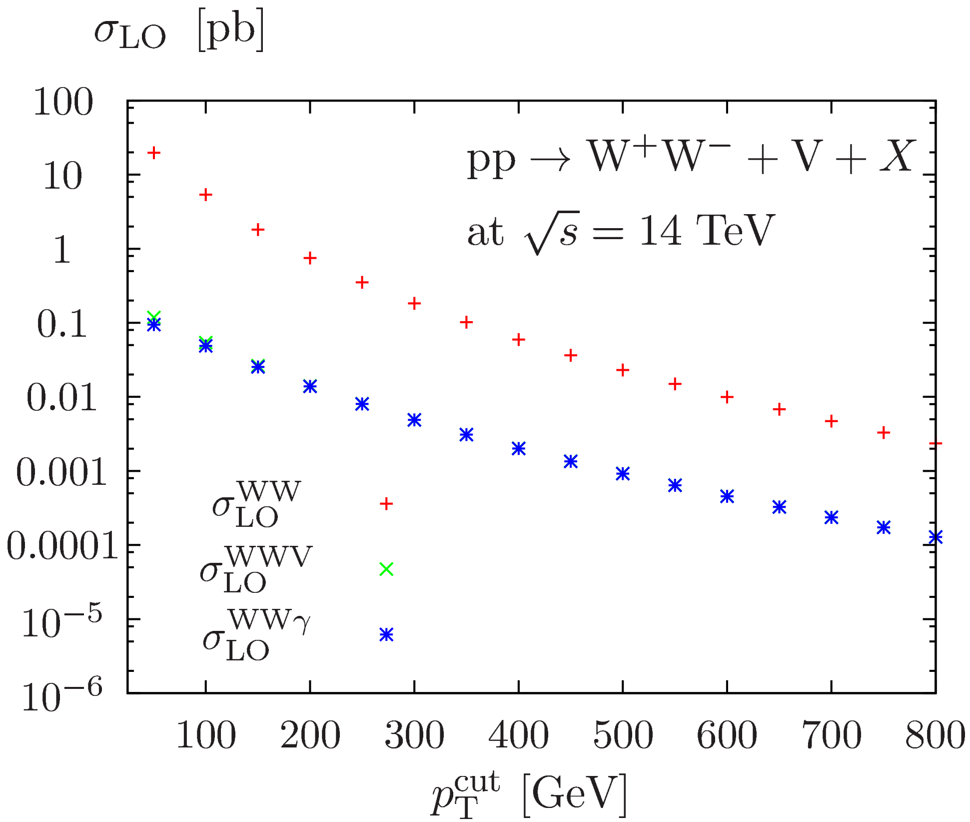
IV. Real W, Z Radiation: Compensation?

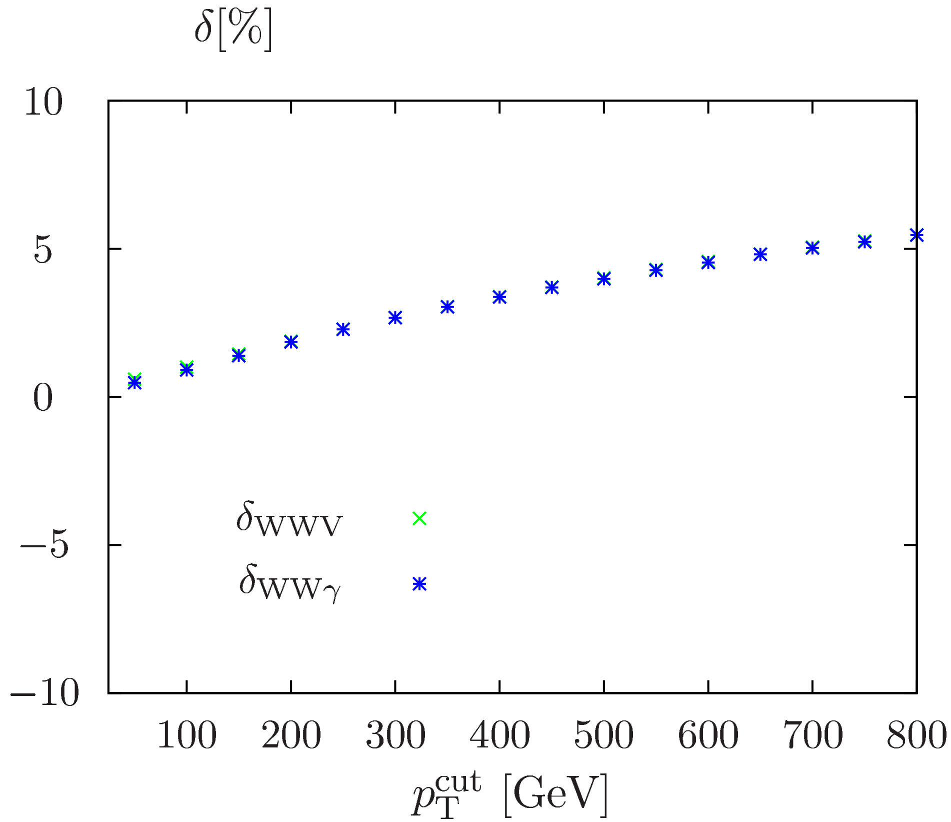
- soft and/or collinear radiation may (partly) compensate or overcompensate virtual corrections:

- model study (Bell, J.K., Rittinger arXiv:1004.4117; EPJC)
  strong dependence on cuts!
  asymptotic energies (multi-TeV)

- MC simulation with decays for $t\bar{t}$ (Baur)
  partial compensation

- semi-realistic evaluation (on-shell W, Z) (Bierweiler, Kasprzik, J.K.)
  $q\bar{q} \longrightarrow W^+W^-(\gamma)$ (Born + one-loop)
  vs. $q\bar{q} \longrightarrow W^+W^-Z$
  $q\bar{q} \longrightarrow W^+W^-W^+$ + c.c.





Aim: real radiation taken care of by MC $\Rightarrow$ different final states:

$\mathrm{t\bar{t}}$: $q\bar{q} \longrightarrow \mathrm{t\bar{t}}Z$; $q\bar{q} \longrightarrow \mathrm{t\bar{b}}W$; ...

W^+W^-: $q\bar{q} \longrightarrow W^+W^-Z$, ...

V. Two Loop Results (Sudakov Logarithms)

one-loop: $\sim 30\%$

$\Rightarrow$ two-loop: $\sim$?

(Vast amount of literature since $\sim$ 2000)

Karlsruhe (Jantzen, J.K., Metzler, Penin, Smirnov, Uccirati)

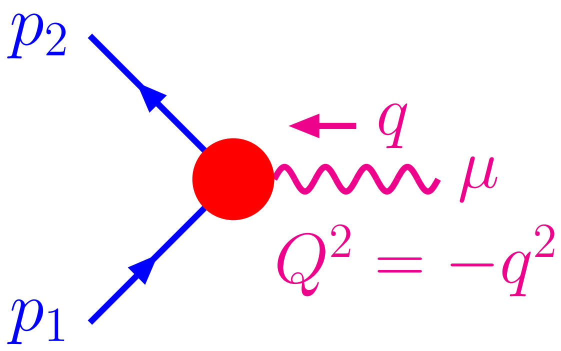
Fadin, Lipatov, Martin, Melles

PSI (Denner, Melles, Pozzorini, ...)

Ciafaloni, ...

Manohar, ...

A) Form Factor and Evolution Equations



Born:

$$\mathcal{F}_{\text{Born}} = \bar{\psi}(p_2)\gamma_\mu \psi(p_1)$$

$$\frac{\partial}{\partial \ln Q^2} \mathcal{F} = \left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x} \gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2)) \right] \mathcal{F}$$

Collins, Sen

$$\Rightarrow \; \mathcal{F} = \mathcal{F}_{\text{Born}}\, F_0(\alpha(M^2)) \exp\left\{ \int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x} \left[\int_{M^2}^{x} \frac{\mathrm{d}x'}{x'} \gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2)) \right] \right\}$$

aim: N^4LL $\Rightarrow$ corresponds to all terms of the form:

$$\alpha^n \Big[\underset{\text{LL}}{\mathcal{L}^{2n}} + \underset{\text{NLL}}{\mathcal{L}^{2n-1}} + \underset{\text{NNLL}}{\mathcal{L}^{2n-2}} + \underset{\text{N}^3\text{LL}}{\mathcal{L}^{2n-3}} + \underset{\text{N}^4\text{LL}}{\mathcal{L}^{2n-4}} \Big]$$

$$\mathcal{L} \equiv \ln(Q^2/M^2)$$

NNLL requires running of α (i.e. β_0 and β_1) and:

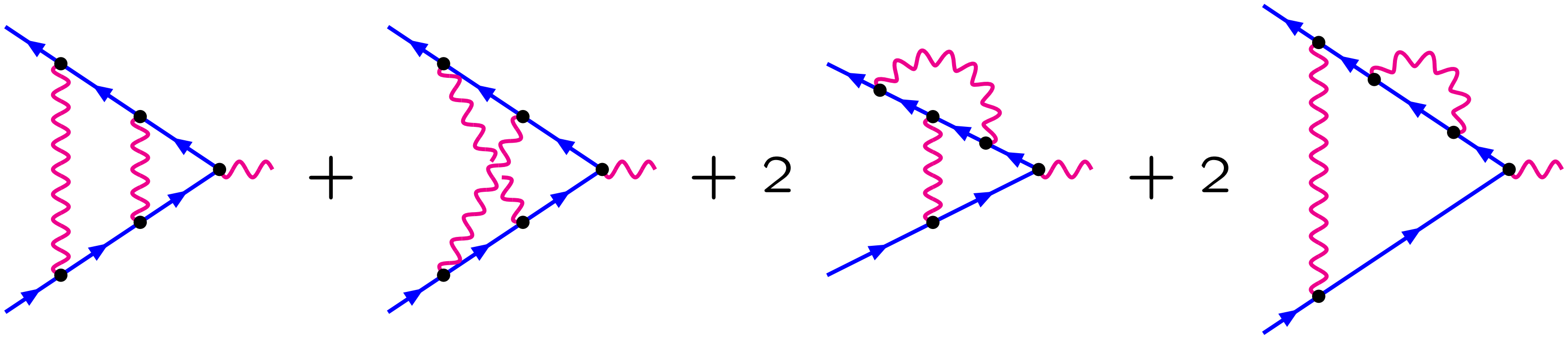
$\zeta(\alpha)$, $\xi(\alpha)$, $F_0(\alpha)$	up to $\mathcal{O}(\alpha)$	(one-loop)
$\gamma(\alpha)$	up to $\mathcal{O}(\alpha^2)$	(massless two loop)

N^3LL requires two-loop calculation in high-energy limit including linear logarithms (available for non-abelian theory)

N^4LL requires complete two-loop calculation in high-energy limit (available for abelian theory)

B) Two-Loop Results: Massive U(1) Model

$$\mathcal{F}_\alpha(M,Q) = \mathcal{F}_{\text{Born}}\left[1 + \frac{\alpha}{4\pi} f^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 f^{(2)} + \ldots\right]$$



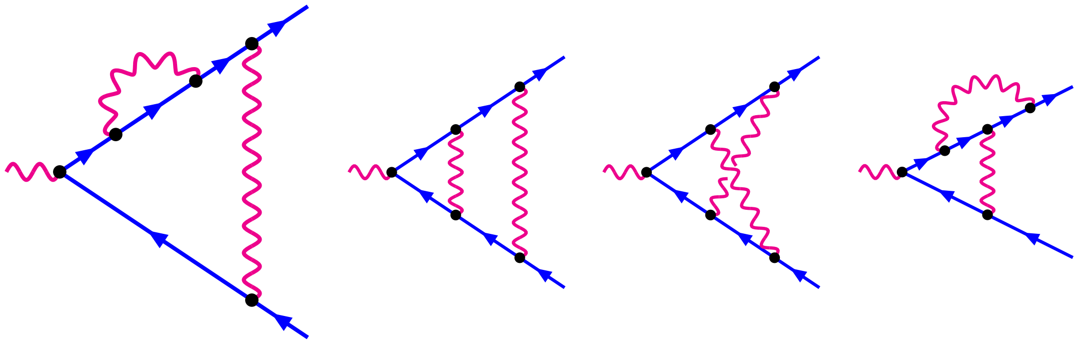
$$f^{(2)} = \frac{1}{2}\mathcal{L}^4 - 3\,\mathcal{L}^3 + \left(8 + \frac{2}{3}\pi^2\right)\mathcal{L}^2 - \left(9 + 4\pi^2 - 24\zeta_3\right)\mathcal{L}$$

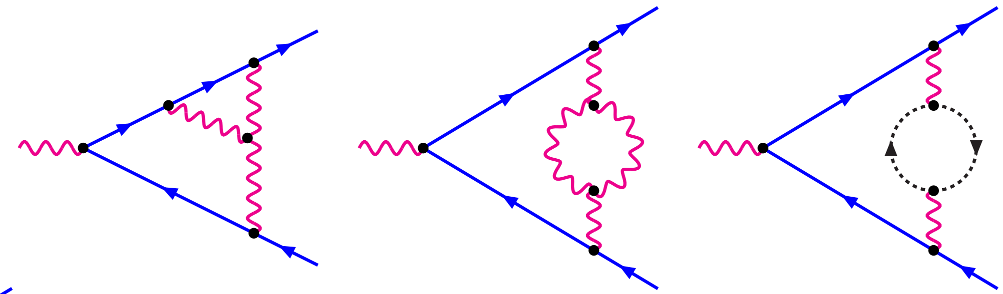
$$+\frac{25}{2} + \frac{52}{3}\pi^2 + 80\zeta_3 - \frac{52}{15}\pi^4 - \frac{32}{3}\pi^2 \ln^2 2 + \frac{32}{3}\ln^4 2 + 256\,\text{Li}_4\left(\frac{1}{2}\right)$$

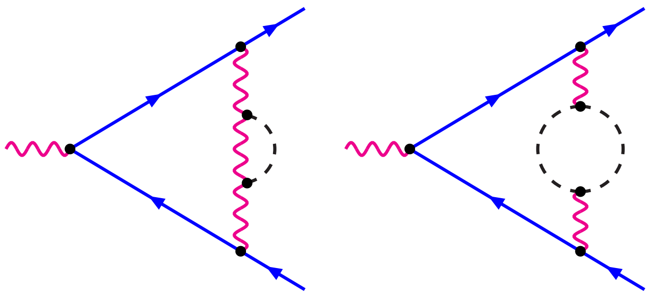
$\mathcal{L} \equiv \ln(Q^2/M^2)$

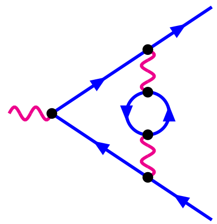
C) Massive SU(2) form factor in 2-loop approximation

2-loop vertex diagrams (massless fermions, massive bosons):

Abelian (C_F^2): 

non-Abelian ($C_F C_A$): last 2 + 

Higgs: 

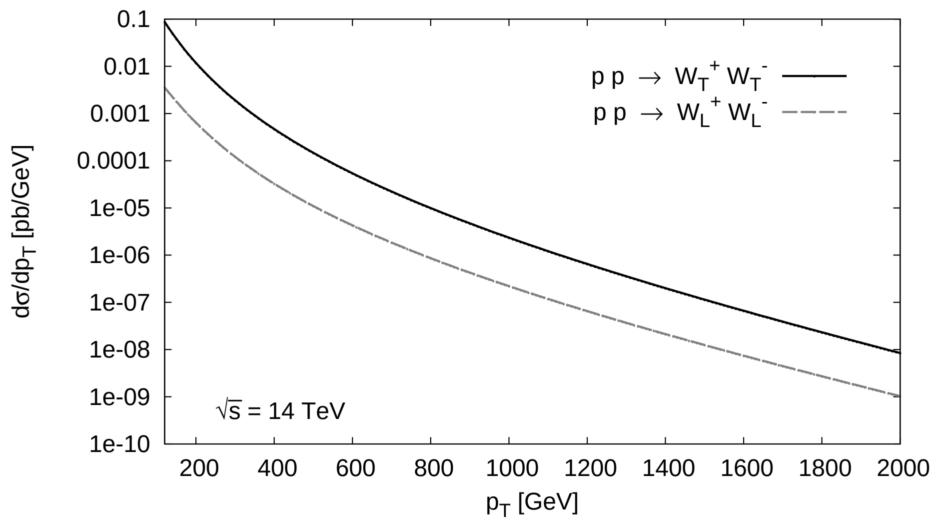
fermion ($C_F T_F n_f$): 

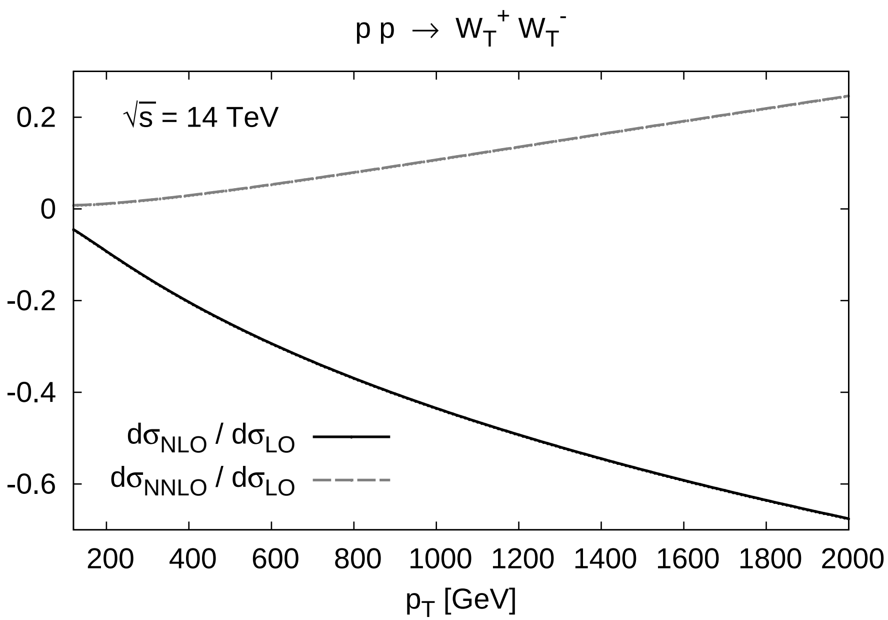
+ 1-loop×1-loop corrections + renormalization

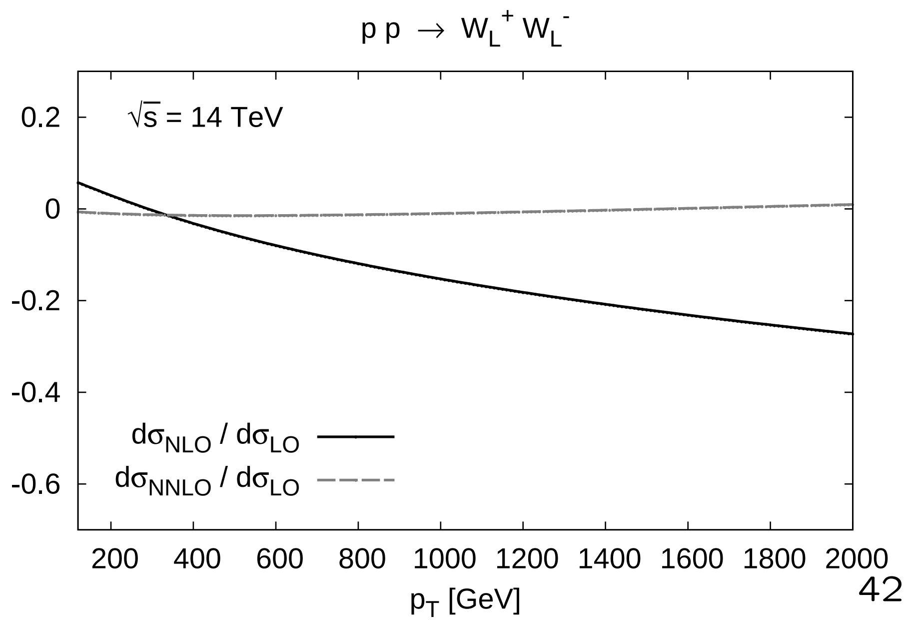
$$f_2 = +\frac{9}{32}\mathcal{L}^4 - \frac{19}{48}\mathcal{L}^3 - \left(-\frac{7}{8}\pi^2 + \frac{463}{48}\right)\mathcal{L}^2$$
$$+ \left(\frac{39}{2}\frac{\mathrm{Cl}_2\left(\frac{\pi}{3}\right)}{\sqrt{3}} + \frac{45}{4}\frac{\pi}{\sqrt{3}} - \frac{61}{2}\zeta_3 - \frac{11}{24}\pi^2 + 29\right)\mathcal{L}$$

Extensions to: 4-fermion scattering (detailed discussion: Penin et al.)
gauge boson pair production (J.K., Metzler, Penin, Uccirati)

Complication: massless photon! $\Rightarrow$ QED subtracted







VI. Open Questions:

- QCD $\otimes$ EW: ambiguities

$$(1+\delta_{\text{QCD}})(1+\delta_{\text{EW}}) - (1+\delta_{\text{QCD}}+\delta_{\text{EW}}) = \delta_{\text{QCD}'}\,\delta_{\text{EW}}$$

  e.g.:

  $\delta_{\text{QCD}} \sim 40\%$; $\delta_{\text{EW}} \sim 30\%$

  first steps: virtual corrections for $q\bar{q} \longrightarrow Z$ of $\mathcal{O}(\alpha_w \alpha_s)$ available:

  J.K., Veretin (no Sudakov logs!)

- definition of final state:

  do we consider W (hard) $\oplus$ g (hard) $\oplus$ W(soft) part of W-pair production?

- Can we discriminate top $\longrightarrow Wb$ or $W \longrightarrow q\bar{q}'$, $Z \longrightarrow q\bar{q}$ from fat QCD jets?