

On dispersion relations and hadronic light-by-light scattering contribution to the muon anomalous magnetic moment

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We discuss the use of dispersion relations for the evaluation of the pseudoscalar contributions to the muon anomalous magnetic moment. We point out that, in the absence of experimental data, reconstruction of light-by-light scattering amplitudes from their absorptive parts is ambiguous and requires additional theoretical input. The need for an additional input makes dispersive computations of the hadronic light-by-light scattering contribution to $g-2$ akin to phenomenological models, in spite of pretense to the contrary. In particular, we argue that the recent proposal [1], based on the dispersive approach, satisfies short distance constraints at the expense of unjustifiably large deviations from the chiral limit.

The measured value of the muon anomalous magnetic moment a_μ [2] disagrees with the theoretical prediction for this quantity, computed within the Standard Model, by slightly more than three standard deviations or $\mathcal{O}(240) \times 10^{-11}$. If new experiment [3], currently underway at FNAL, will confirm the results of the previous measurements, a smaller error of the FNAL measurement will increase the discrepancy to about 5σ . To claim an even larger significance of the deviation will require either a shift in the measured value of a_μ or a reduction of the theory error that, currently, is close to 50×10^{-11} .

Moreover, since significant contribution to the muon anomalous magnetic moment arises from kinematic regions where low-energy hadronic interactions are important, a continuous scrutiny of theoretical assumptions, which are behind the predicted value of $g-2$, is essential. Among the various Standard Model contributions to $g-2$, the so-called hadronic light-by-light scattering contribution is the one that is the most worrisome. This is so because experimental information about hadronic contributions to the Green's function of three electromagnetic currents in the background of a soft magnetic field

$$(2\pi)^4 \delta^4(q_1 + q_2 + q_3 - k) T^{\mu\nu\alpha}(q_1, q_2, q_3) = - \int d^4x d^4y d^4z \times e^{-i(q_1x + q_2y + q_3z)} \langle 0 | T j_{\text{em}}^\mu(x) j_{\text{em}}^\nu(y) j_{\text{em}}^\alpha(z) | \gamma(k, \epsilon) \rangle, \quad (1)$$

is, at best, very limited. In Eq.(1), the soft magnetic field is modeled by a transition from a vacuum to a soft photon with momentum k and polarization vector ϵ ; only linear terms in the soft photon momentum k must be retained in Eq.(1). Also, we note that throughout the paper we consider all momenta to be outgoing.

Original computations of the hadronic light-by-light tensor Eq.(1) and its contributions to the muon anomalous magnetic moment were performed [4] using mod-

els where interactions of photons with photons were described by exchanges of relatively light hadrons; such features of QCD as the large- N_c counting [5] and the chiral limit were employed for guidance [6].

Nevertheless, it felt very desirable to develop a way to evaluate hadronic light-by-light scattering contribution to the muon anomalous magnetic moment that is based on first principles, to reduce the model dependence as much as possible. In (recent) years, two approaches to this problem emerged. One approach is based on the idea that the hadronic tensor in Eq.(1) can be unambiguously predicted in certain kinematic limits and that models for hadronic light-by-light scattering contribution should conform to these predictions. A particularly useful application of this approach in connection with the muon magnetic anomaly, is the asymmetric limit $q_1^2 \sim q_2^2 \gg q_3^2$, where q_3^2 can be either larger than or comparable to Λ_{QCD}^2 [7]. By combining these limits, we arrived at an unambiguous prediction for the longitudinal contribution to the Green's function in the chiral limit. We have also used the perturbative regime and the operator product expansion (OPE) to constrain the transversal contribution as well. We employed pion, ρ -, ω - and a_1 -mesons to construct a minimal model for the longitudinal and transversal structure functions which satisfies short-distance and non-perturbative constraints in the asymmetric kinematic limit [7].

Another way to compute the hadronic light-by-light scattering tensor in Eq.(1) from first principles involves dispersion relations [1, 8] that represent $T^{\mu\nu\alpha}$ as integrals of its absorptive parts over certain kinematic variables. While formally exact, dispersion relations become useful in practice if the absorptive parts of the corresponding Green's functions can be directly obtained from experimental data. This is exactly what happens in a simpler case of the hadronic vacuum polarization where data on $e^+e^- \rightarrow$ hadrons annihilation cross section, in dependence on energy, can be used to determine the hadronic contribution to the Green's function

$$\Pi^{\mu\nu}(q) = i \int d^4x e^{-iqx} \langle 0 | T j_{\text{em}}^\mu(x) j_{\text{em}}^\nu(0) | 0 \rangle, \quad (2)$$

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using dispersion relations. The situation with the light-by-light tensor is obviously more complicated since not only are the dispersion relations more complex, but also the absorptive parts are not quite known experimentally. Therefore, although dispersion representations of Green's functions in general and in case of the hadronic tensor in Eq.(1) in particular, are undoubtedly correct, it is not possible to use them in case of the hadronic light-by-light scattering contribution without additional theoretical input required to construct absorptive parts. Because of that, it is our opinion, that the use of dispersion relations *per se* does not add much to a better understanding of the hadronic light-by-light contribution to $g - 2$, compared to original computations [4, 7].

It is instructive to compare the two approaches by studying contributions of pseudoscalar mesons to the hadronic light-by-light scattering tensor Eq.(1). In the dispersive approach,¹ where one looks for a discontinuity with respect to a variable q_3^2 [1], this contribution, effectively, arises from the process $\gamma^*(q_1)\gamma^*(q_2) \rightarrow \pi_0^*(q_3) \rightarrow \gamma^*(q_3)\gamma(k)$. The result is proportional to

$$F_\pi(q_1^2, q_2^2) \frac{1}{q_3^2 - m_\pi^2} F_\pi(q_3^2, 0), \quad (3)$$

where the pion transition form factors refer to a process $\gamma^*(q_1)\gamma^*(q_2) \rightarrow \pi^0$. It is interesting to point out that the second pion form factor in Eq.(3), $F_\pi(q_3^2, 0)$, is conventionally interpreted as a transition of a photon with momentum q_3^2 and another on-shell photon to a pion on the mass shell. However, in case of $g - 2$ the required kinematics is different since the on-shell photon is, actually, *soft*. This implies that, for $q_3^2 \neq m_\pi^2$, $F_\pi(q_3^2, 0)$ can not be a regular pion transition form factor. In line with this observation, the dependence of this form factor on q_3^2 is ambiguous within the dispersive approach since

$$F_\pi(q_1^2, q_2^2) \frac{1}{q_3^2 - m_\pi^2} F_\pi(q_3^2, 0) = F_\pi(q_1^2, q_2^2) \times \left[\frac{F_\pi(m_\pi^2, 0)}{q_3^2 - m_\pi^2} + \frac{F_\pi(q_3^2, 0) - F_\pi(m_\pi^2, 0)}{q_3^2 - m_\pi^2} \right]. \quad (4)$$

Clearly, the second term in square brackets on the r.h.s. of Eq.(8) is non-singular at $q_3^2 = m_\pi^2$ and, therefore, can not be associated with the pion pole, whereas the first term does not have an ambiguous transition form factor anymore. The second term in Eq.(8) can not be obtained from the pion pole discontinuity and requires and requires additional information about Green's functions $\gamma^*\gamma^* \rightarrow \gamma^*\gamma$.

It is said sometimes that the dispersion relation in Eq.(3) cannot be derived in the *reduced* kinematics, when the photon k is soft. Instead, Eq.(3) should be obtained

as a $q_4 \rightarrow 0$ limit of a dispersion representation of the full $2 \rightarrow 2$ process $\gamma^*(q_1)\gamma^*(q_2) \rightarrow \gamma^*(q_4)\gamma(q_3)$. A dispersive reconstruction of the π_0 pole contribution to the full amplitude gives

$$F_\pi(q_1^2, q_2^2) \frac{1}{(q_3 + q_4)^2 - m_\pi^2} F_\pi(q_3^2, q_4^2), \quad (5)$$

with obvious constraints on $q_{1,\dots,4}$ in the form factors $(q_1 + q_2)^2 = m_\pi^2$ and $(q_3 + q_4)^2 = m_\pi^2$. If we take the soft limit $q_4 \rightarrow 0$, the first constraint remains unaffected and covers large phase space of possible values of q_1 and q_2 , while the second one immediately implies $q_3^2 = m_\pi^2$, turning the form factor $F_\pi(q_3^2, q_4^2)$ into a constant $F_\pi(m_\pi^2, 0)$. Hence, independent of where one starts, the second form factor in Eq.(3) does not follow from the dispersive reconstruction of the π_0 -pole contribution to the hadronic light-by-light scattering tensor in the kinematics relevant for muon $g - 2$.

Additional information such as subtraction terms that is *naturally* missed by the dispersive reconstruction of the pseudoscalar pole contribution can be obtained by studying the light-by-light tensor in a particular kinematic limit where *full non-perturbative description* of an important part of the hadronic tensor in the chiral limit can be achieved. Indeed, as explained in Ref.[7], it is beneficial to study an asymmetric kinematic limit $q_1^2 \approx q_2^2 \gg q_3^2$. In that limit the OPE of the product of two electromagnetic currents is given by the axial-vector current j_α^5 . Then, the light-by-light amplitude is reduced to a triangle amplitude that describes a transition of an axial-vector current to a soft photon and vector current, $j_\rho^5 \rightarrow \gamma^*(q_3)\gamma(k)$. This transition amplitude can be decomposed into contributions with definite SU(3) quantum numbers, corresponding to isovector, octet and singlet axial-vector currents. We write

$$T_{\mu\nu\alpha}(q_1, q_2, q_3) = \frac{8}{\hat{q}^2} \epsilon_{\mu\nu\delta\rho} \hat{q}^\delta \sum_{a=3,8,0} W^{(a)} T_{\rho\alpha}^{(a)}(q), \quad (6)$$

where $\hat{q} = (q_1 - q_2)/2$, $q = q_3$ and

$$T_{\alpha\mu}^{(a)} = w_L^{(a)}(q^2) q_\alpha q^\sigma \tilde{f}_{\sigma\mu} + w_T^{(a)}(q^2) \left(q^2 \tilde{f}_{\alpha\mu} - q_\mu q^\sigma \tilde{f}_{\alpha\sigma} - q_\alpha q^\sigma \tilde{f}_{\sigma\mu} \right). \quad (7)$$

In Eq.(7) we used $\tilde{f}_{\alpha\beta} = 1/2 \epsilon_{\alpha\beta} f^{\alpha\beta}$ and $f_{\alpha\beta} = \epsilon_\alpha k_\beta - k_\alpha \epsilon_\beta$.

The amplitude $T_{\alpha\mu}$ at this point is defined *non-perturbatively* as a matrix element of the time-ordered product of the axial and a vector current between the vacuum and the soft photon

$$T_{\alpha\mu}^{(a)}(q) = \int d^4x e^{iqx} \langle 0 | T j_\alpha^{5(a)}(x) j_\mu^{\text{em}}(0) | \gamma(k, \epsilon) \rangle. \quad (8)$$

By matching Eq.(6) to the short-distance limit of the light-by-light scattering amplitude one finds

$$w_L^{(3)}(q^2) = 2w_T^{(3)}(q^2), \quad w_L^{(3)}(q^2) = -\frac{2}{q^2}, \quad (9)$$

¹ We restrict ourselves to the π^0 contribution; discussion of other pseudoscalars is largely identical.

for the isovector contribution. Perturbatively, in the chiral limit there are no corrections to the longitudinal structure function $w_L^{(3)}$, thanks to the Adler-Bardeen theorem [9], and to the transversal one $w_T^{(3)}$, due a peculiar relation between $w_T^{(3)}$ and $w_L^{(3)}$ in the chiral limit discovered in [10].

Furthermore, it is possible to compute non-perturbative corrections to the Green's function in Eq.(8) by performing an operator product expansion. It follows from this analysis [7], that the longitudinal form factor w_L *does not receive any corrections in the chiral limit* whereas there are non-perturbative corrections to the transversal form factor. This information has been used to construct models for w_L and w_T using small number of light mesons that contribute in a pseudoscalar and vector channels. The form factors read [7]

$$\begin{aligned} w_L^{(3)}(q^2) &= -\frac{2}{q^2 - m_\pi^2}, \\ w_T^{(3)}(q^2) &= \frac{1}{m_{a_1}^2 - m_\rho^2} \left[\frac{m_\rho^2}{q^2 - m_\rho^2} - \frac{m_{a_1}^2}{q^2 - m_{a_1}^2} \right]. \end{aligned} \quad (10)$$

The mass of the pion is added to $w_L^{(3)}$ to go beyond the chiral limit at small $q \sim m_\pi$. One may be concerned that this step does not properly account for possible chiral-violating effects in $w_L^{(3)}$ at $q^2 \gg m_\pi^2$. Such concerns are, however, unfounded; see the discussion at the end of this paper.

The model Eq.(10) was recently criticized in Ref. [1] on the basis that it “distorts” the low-energy “dispersive” formula shown in Eq.(3) where the “distortion” means the absence of the second form factor $F_\pi(q_3^2, 0)$ in the “right” version of Eq.(3). It was suggested in Ref. [1] that one can “repair” the model of Ref. [7] by allowing for an infinitely large number of excited pions to contribute to the light-by-light scattering amplitude. The constraint $w_L(q^2) = -2/q^2$ at $q^2 \gg \Lambda_{\text{QCD}}^2$ is then enforced by a particular choice of form factors.

In what follows we would like to explain why the two claims made in Ref. [1], i.e. the incompatibility of Eq.(10) with dispersion relations and the possibility to “correct” this incompatibility by allowing infinite number of exchanges in the longitudinal form factor, are strongly questionable.

To address the problem of incompatibility with dispersive reconstruction, we note that, as already pointed out after Eq.(8) the dispersion relations in q_3^2 are ambiguous and the presence or absence of the form factor $F_\pi(q_3^2, 0)$ is a question about non-pole rather than an absorptive or pole part. These additional terms do not follow from dispersion relations applied to the pion pole and require additional information. Given this ambiguity, it is impossible to discuss the issue of an incompatibility seriously.

Moreover, in the asymmetric kinematics $q_1^2 \sim q_2^2 \gg q_3^2$, the dispersion reconstruction in the variable q_3^2 should be done for a transition amplitude shown in Eq.(8) which, essentially, describes a mixing of vector and vector-axial

currents in a constant magnetic field. To provide a dispersion reconstruction of this Green's function we simply saturate it in the spirit of the vector dominance by allowing vector and axial currents to “mix” into lightest mesons with relevant quantum numbers. These mesons then have *point-like* interactions with the soft photon and the remaining current.

The lightest mesons that we consider are the pion and the a_1 axial-vector meson, that can mix into the axial current, and the ρ , ω mesons that can mix into the vector current. Ignoring the (tiny) mass difference between ρ and ω mesons, we can write their contribution to the Green's function as

$$\begin{aligned} T_{\alpha\mu}^{(3)(\rho)}(q^2) &= c_L^{(\rho)} q_\alpha q^\sigma \tilde{f}_{\sigma\nu} \frac{-g_\mu^\nu}{q^2 - m_\rho^2} \\ &+ c_T^{(\rho)} \left(q^2 \tilde{f}_{\alpha\nu} - q_\nu q^\sigma \tilde{f}_{\alpha\sigma} - q_\alpha q^\sigma \tilde{f}_{\sigma\nu} \right) \frac{-g_\mu^\nu}{q^2 - m_\rho^2}, \end{aligned} \quad (11)$$

where $c_{L,T}^{(\rho)}$ are unknown constants. Note, that we discarded $q_\mu q_\nu / m_\rho^2$ term in the numerator of the ρ propagator in Eq.(11), since the amplitude Eq.(8) is constructed in such a way that the vector current is conserved.

A similar contribution of an a_1 axial-vector meson reads

$$\begin{aligned} T_{\alpha\mu}^{(3)(a_1)}(q^2) &= c_L^{(a_1)} \frac{-g_\alpha^\beta + (q_\alpha q^\beta / m_{a_1}^2)}{q^2 - m_{a_1}^2} q_\beta q^\sigma \tilde{f}_{\sigma\mu} \\ &+ c_T^{(a_1)} \frac{-g_\alpha^\beta}{q^2 - m_{a_1}^2} \left(q^2 \tilde{f}_{\beta\mu} - q_\mu q^\sigma \tilde{f}_{\beta\sigma} - q_\beta q^\sigma \tilde{f}_{\sigma\mu} \right) \end{aligned} \quad (12)$$

Note that in this case $q_\alpha q_\beta / m_{a_1}^2$ can be discarded for the transversal part but it plays an important role in the longitudinal part. Indeed, we obtain

$$\frac{-g_{\alpha\beta} + (q_\alpha q_\beta / m_{a_1}^2)}{q^2 - m_{a_1}^2} q^\beta = \frac{q_\alpha}{m_{a_1}^2}, \quad (13)$$

which implies that the pseudo-vector meson does not contribute to the longitudinal part of the Green's function *in a dispersive sense* since the vector-axial pole disappeared.

Adding a pion-pole contribution to the longitudinal form factor leads to the following result for the dispersive reconstructed structure function

$$\begin{aligned} w_L^{(3)}(q^2) &= -\frac{2}{q^2} - \frac{c_L^{(\rho)}}{q^2 - m_\rho^2}, \\ w_T^{(3)}(q^2) &= -\frac{c_T^{(\rho)}}{q^2 - m_\rho^2} - \frac{c_T^{(a_1)}}{q^2 - m_{a_1}^2}. \end{aligned} \quad (14)$$

At this point, the constants $c_{L,T}^{(\rho,a_1)}$ are arbitrary. For example, by choosing $c_L^{(\rho)} = -2$, we obtain

$$w_L(q^2) = -\frac{2}{q^2} \frac{-m_\rho^2}{q^2 - m_\rho^2}. \quad (15)$$

Hence, this choice of an “effective” coupling constant introduces a “pion form factor” into a longitudinal structure function $w_L(q^2)$. Such a choice will be in accord with Eq.(3) and would imply a stronger suppression of the longitudinal contribution for $q_3^2 \gg m_\rho^2$ than what is allowed by perturbative and non-perturbative matching.

However, it is quite obvious from Eq.(11) that other choices are possible and are, in fact, better motivated. As we proved in Ref. [7], the longitudinal structure function $w_L(q^2) = -2/q^2$ is *exact* in the chiral limit and it is not renormalized by either perturbative or non-perturbative corrections. Clearly, the choice $c_L^{(\rho)} = -2$ leads to $w_L(q^2)$ that violates this assertion. To comply with it, we have chosen $c_L^{(\rho)} = 0$. It is also seen from Eq.(11) that the function $w_T(q^2)$ can be reconstructed independently of the longitudinal one. The coefficients $c_T^{(\rho, a_1)}$ in those cases are fixed by requiring that at large values of q^2 the transversal function w_T matches perturbative asymptotic and that non-perturbative corrections at large q^2 are consistent with the operator product expansion [7].

One may be wondering if models that include more resonances can satisfy the asymptotic behavior required by short-distance QCD [1]. To analyze this question in a simple setting, suppose that we include yet another ρ -meson into a dispersive reconstruction of the longitudinal function $w_L^{(3)}$. We find

$$w_L^{(3)}(q^2) = -\frac{2}{q^2} - \frac{c_L^{(\rho)}}{q^2 - m_\rho^2} - \frac{c_L^{(\rho_1)}}{q^2 - m_{\rho_1}^2}. \quad (16)$$

The large- q^2 asymptotic of the short-distance constraint requires $c_L^\rho = -c_L^{\rho_1}$. Then $w_L^{(3)}$ becomes

$$w_L^{(3)}(q^2) = -\frac{2}{q^2} - c_L^\rho \frac{m_\rho^2 - m_{\rho_1}^2}{(q^2 - m_\rho^2)(q^2 - m_{\rho_1}^2)}, \quad (17)$$

and c_L^ρ remains unconstrained. To constrain it, we note that the longitudinal structure function should be equal to $w_L^{(3)} = -2/q^2$ *non-perturbatively* in the chiral limit. Hence, we obtain the constraint

$$\lim_{m_{u,d,s} \rightarrow 0} c_L^\rho = 0. \quad (18)$$

We note that the model of Ref.[1] *violates* the above equation and claims, effectively, that $c_L^\rho \sim 1$ also in the chiral limit.

In order to account for the violation of chiral symmetry at $q \sim m_\pi$ we added a pion mass to the $1/q^2$ pole in

$w_L^{(3)}$, $1/q^2 \rightarrow 1/(q^2 - m_\pi^2)$, see Eq.(10). This modification introduces a deviation from the asymptotic behavior of w_L and implies

$$c_L^\rho \sim \frac{m_\pi^2}{m_\rho^2}. \quad (19)$$

One may wonder if this estimate is reasonable. We addressed this question in Ref. [11] using the OPE for the longitudinal structure functions. It was shown there that non-perturbative corrections to $w_L^{(3)}$ are generated by an operator $\mathcal{O}_{\alpha\beta} = -i\bar{q}\sigma_{\alpha\beta}\gamma_5 q$ and are proportional to quark masses. The matrix element of the operator $\mathcal{O}_{\alpha\beta}$ between the photon and the vacuum can be estimated by expressing it through the magnetic susceptibility of the quark condensate. Numerically, it leads to the change $\Delta w_L^{(3)}$ in the longitudinal $w_L^{(3)}$,

$$\frac{\Delta w_L^{(3)}}{w_L^{(3)}} = \frac{(0.18 \text{ GeV})^2}{q^2}, \quad (20)$$

which supports introduction of the pion mass as the *only* source of the chiral symmetry violation and does not leave any room for the proposal of Ref. [1].

It is clear that our arguments concern a special kinematic region $q_1^2 \sim q_2^2 \gg q_3^2$ and one may ask if this kinematic region is sufficient for the evaluation of the hadronic light-by-light scattering contribution to $g - 2$ with sufficient accuracy. However, there is no doubt that (a) this region provides the largest contribution to a_μ^{hlbl} ; (b) it allows for an *exact* non-perturbative analysis of the longitudinal structure function in the chiral limit and (c) it supplies strong evidence that corrections to the chiral limit are *small*. It is our view that the above points provide a motivation for using the asymmetric kinematic limit as a diagnostic tool to check the validity of different models. Unfortunately, the model of Ref. [1], whose authors purport [1] to remove the “biggest systematic uncertainty due to short-distance constraints” with the result “that the asymptotic part of the hadronic light-by-light tensor is under sufficient control for the first release from the Fermilab experiment,” fails to pass the test.

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