Gauge and Yukawa coupling beta functions of two-Higgs-doublet models to three-loop order

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README for ancillary files

This file contains the description of the ancillary files to [1]. We provide seven files containing the analytic results given in Mathematica syntax:
`gaugebeta.m, invbeta.m, lambeta.m`
`rules.m, specialbasisbeta.m, yukawabeta.m, Z.m`

The files `gaugebeta.m`, `yukawabeta.m` and `lambeta.m` contain the beta functions for the gauge couplings, Yukawa matrices and quartic couplings. In `Z.m` we provide renormalization constants needed for computing the gauge and Yukawa beta functions. Rules to obtain the beta functions and renormalization constants for the general 2HDM, for four $\mathbb{Z}_2$-symmetric 2HDMs and for the SM can be found in `rules.m`.

We also provide the beta functions for the eleven flavour invariants $I_1, ..., I_{11}$ defined in Section 2 in `invbeta.m` and for the special basis discussed in Section 4.3 in `specialbasisbeta.m`.

In the following we provide some examples on how to obtain the beta functions and renormalization constants in a specific model, list all the quantities given in the individual files and define all symbols appearing in the final expressions.

1 Example

The gauge coupling beta functions in the SM are obtained via:

```
<< "gaugebeta.m";
<< "rules.m";

betaSMalpha1 = betaalpha1 //. TypeSpecSM;
betaSMalpha2 = betaalpha2 //. TypeSpecSM;
betaSMalpha3 = betaalpha3 //. TypeSpecSM;
```

For the Yukawa matrices and the quartic couplings the expressions are obtained in analogy, For example, for the type II 2HDM we have:

```
<< "yukawabeta.m";
<< "lambeta.m";
<< "rules.m";

betaIIYd = betaYd2 //. TypeSpecTypeII;
betaIIlam1122 = betalam1122 //. TypeSpecTypeII;
```

2 Quantities defined in the files

`gaugebeta.m` contains the three-loop gauge coupling beta functions:

- `betaalpha1`: β_1
- `betaalpha2`: β_2
- `betaalpha3`: β_3

`yukawabeta.m` contains the three-loop Yukawa matrix beta functions:

- `betaYu1`: $\beta_{u,1}$
- `betaYu2`: $\beta_{u,2}$
- `betaYd1`: $\beta_{d,1}$
- `betaYd2`: $\beta_{d,2}$
- `betaYl1`: $\beta_{l,1}$
- `betaYl2`: $\beta_{l,2}$

`lambeta.m` contains the one-loop quartic coupling beta functions:

- `betalam1111`: $\beta_{11,11}$
- `betalam2222`: $\beta_{22,22}$
- `betalam1122`: $\beta_{11,22}$
- `betalam1221`: $\beta_{12,21}$
- `betalam1212`: $\beta_{12,12}$

- `betalam2121`: $\beta_{21,21}$
- `betalam1112`: $\beta_{11,12}$
- `betalam1121`: $\beta_{11,21}$
- `betalam1222`: $\beta_{12,22}$
- `betalam2122`: $\beta_{21,22}$

`Z.m` contains the following renormalization constants up to three loops:

- `Zalpha1`: Z_{α_1}
- `Zalpha2`: Z_{α_2}
- `Zalpha3`: Z_{α_3}
- `ZB`: Z_B, wave function renormalization constant of the $U(1)$ gauge boson
- `ZW`: Z_W, wave function renormalization constant of the $SU(2)$ gauge boson
- `ZcW`: Z_{c_W}, wave function renormalization constant of the $SU(2)$ ghost
- `ZWcW`: $Z_{Wc_WC_W}$, wave function renormalization constant of the $SU(2)$ gauge-boson-ghost-vertex
- `ZG`: Z_G, wave function renormalization constant of the $SU(3)$ gauge boson
- `ZcG`: Z_{c_G}, wave function renormalization constant of the $SU(3)$ ghost
- `ZGcG`: $Z_{Gc_GC_G}$, wave function renormalization constant of the $SU(3)$ gauge-boson-ghost-vertex
- `ZQL`: Z_L^Q, wave function renormalization constant of the left-handed quark doublet
- `ZLL`: Z_L^L, wave function renormalization constant of the left-handed lepton doublet
- `ZuR`: Z_R^u, wave function renormalization constant of the right-handed up-type quark singlet
- `ZdR`: Z_R^d, wave function renormalization constant of the right-handed down-type quark singlet
- `ZlR`: Z_R^l, wave function renormalization constant of the right-handed charged lepton singlet
- `ZPhi11`: $Z_{1,1}^{\Phi}$, $1,1$ component of the wave function renormalization matrix of the scalar doublets

- `ZPhi12`: $Z^{\Phi}_{1,2}$, $1,2$ component of the wave function renormalization matrix of the scalar doublets
- `ZPhi21`: $Z^{\Phi}_{2,1}$, $2,1$ component of the wave function renormalization matrix of the scalar doublets
- `ZPhi22`: $Z^{\Phi}_{2,2}$, $2,2$ component of the wave function renormalization matrix of the scalar doublets
- `ZQuP1`: $\sum_{\beta}\sum_{b=1}^{2} Z^{uu\Phi}_{1b,\alpha\beta} Y^{u}_{b,\beta\alpha'}$
- `ZQuP2`: $\sum_{\beta}\sum_{b=1}^{2} Z^{uu\Phi}_{2b,\alpha\beta} Y^{u}_{b,\beta\alpha'}$
- `ZQdp1`: $\sum_{\beta}\sum_{b=1}^{2} Z^{dd\Phi}_{1b,\alpha\beta} Y^{d}_{b,\beta\alpha'}$
- `ZQdp2`: $\sum_{\beta}\sum_{b=1}^{2} Z^{dd\Phi}_{2b,\alpha\beta} Y^{d}_{b,\beta\alpha'}$
- `ZLlp1`: $\sum_{\beta}\sum_{b=1}^{2} Z^{ll\Phi}_{1b,\alpha\beta} Y^{l}_{b,\beta\alpha'}$
- `ZLlp2`: $\sum_{\beta}\sum_{b=1}^{2} Z^{ll\Phi}_{2b,\alpha\beta} Y^{l}_{b,\beta\alpha'}$

`rules.m` contains the rules to specify beta functions and renormalization constants to the individual models:

- `TypeSpecSM`: rules to specify to the SM
- `TypeSpecTypeI`: rules to specify to the Type I 2HDM
- `TypeSpecTypeII`: rules to specify to the Type II 2HDM
- `TypeSpecTypeX`: rules to specify to the Type X 2HDM
- `TypeSpecTypeY`: rules to specify to the Type Y 2HDM
- `TypeSpecTypeIII`: rules to specify to the general 2HDM

`invbeta.m` contains the three-loop beta functions for the quark sector invariants:

- `SMbetai1`: β_{I_1} in the SM
- `THDMIbetai1`: β_{I_1} in the Type I/X 2HDM
- `THDMIIbetai1`: β_{I_1} in the Type II/Y 2HDM

The other ten invariants are defined in analogy. Note that, since we neglect the lepton Yukawa interactions for the running of the quark sector invariants, the beta functions in Type I and X, as well as in Type II and Y agree with each other.

`typeinv.m` contains the three-loop Yukawa beta functions in the special basis:

Symbol	Description
`fourpiinv`	$1/(4\pi)$
`epsilon`	DREG parameter ϵ, $d = 4 - 2\epsilon$
`z3`	$\zeta(3)$
`Ng`	Number of fermion generations n_G
`alpha1`, `xiB`	$U(1)$ gauge coupling α_1 and gauge parameter ξ_B
`alpha2`, `xiW`	$SU(2)$ gauge coupling α_2 and gauge parameter ξ_W
`alpha3`, `xi`	$SU(3)$ gauge coupling α_3 and gauge parameter ξ
`lam[i,j,k,l]`	Quartic couplings $\hat{\lambda}_{ij,kl}$
`lam`	SM quartic coupling $\hat{\lambda}$
`Yu`,`Yd`,`Yl`	Yukawa matrices Y^u, Y^d, Y^l in $\mathbb{Z}_2$-symmetric models
`Yud`,`Ydd`,`Yld`	hermitian conjugates of the above Yukawa matrices
`Yu[i]`,`Yd[i]`,`Yl[i]`	Yukawa matrices Y^u_i, Y^d_i, Y^l_i in the general 2HDM
`Yud[i]`,`Ydd[i]`,`Yld[i]`	hermitian conjugates of the above Yukawa matrices
`MatrixProduct`	Function encapsulating products of Yukawa matrices
`ytp`	top Yukawa coupling in the special basis $y_1^{t\,\prime}$
`ybp1`	bottom Yukawa coupling in the special basis $y_1^{b\,\prime}$
`ybp2`	bottom Yukawa coupling in the special basis $y_2^{b\,\prime}$
`lamp1111`	$\hat{\lambda}'_{11,11}$, quartic coupling in front of $\left(\Phi_1^{\dagger\prime}\Phi_1{}'\right)^2$ in the special basis, other quartic couplings are defined in analogy
`i1`	Quark Yukawa invariant I_1, the other 10 are named analogously

Table 1: Symbols appearing in the final results.

- `betaytb`: $\beta_{y_1^{t\prime}}$
- `betaybp1`: $\beta_{y_1^{b\prime}}$
- `betaybp2`: $\beta_{y_2^{b\prime}}$

3 Conventions and symbols

When defining the Yukawa couplings in $\mathbb{Z}_2$-symmetric models, we follow Tab. 1 of Ref. [1]. This means that `betaYu2` is the beta function for the up-type quark Yukawa matrix in these models. The SM is defined in analogy to the Type I 2HDM. This implies that $\hat{\lambda}_{22,22} = \hat{\lambda}$ where $\hat{\lambda}$ is the quartic coupling in the SM. Note that $\hat{\lambda}$ differs by a factor of 2 from the definition in Ref. [2].

If an even number of Yukawa matrices are present it is understood that each of them are rescaled by $1/\sqrt{4\pi}$ (see quantities with a hat in [1]). If the number is odd all but one of the matrices are rescaled. Note that the Yukawa matrix beta functions always contains an odd number of Yukawa matrices.

The symbols appearing in our results are listed in Tab. 1

References

[1] F. Herren, L. N. Mihaila and M. Steinhauser, [arXiv:1712.xxxx [hep-ph]].

[2] L. N. Mihaila, J. Salomon and M. Steinhauser, Phys. Rev. D **86** (2012) 096008 doi:10.1103/PhysRevD.86.096008 [arXiv:1208.3357 [hep-ph]].