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TeV-scale scalar leptoquarks motivated by B anomalies improve Yukawa unification in SO(10) GUT

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It is common practice to explain deviations between data and Standard-Model predictions by postulating new particles at the TeV scale ad-hoc. This approach becomes much more convincing, if one successfully embeds the postulated particles into a UV completion which addresses other conceptual or phenomenological shortcomings of the SM. We present a study of an SO(10) grand unified theory which contains scalar leptoquark fields employed to explain the "flavour anomalies" in $b \to s$ and $b \to c$ decays. We find that the additional degrees of freedom improve the renormalization group evolution of the SM parameters and may explain some of the observed fermion masses. In particular, the light leptoquarks modify the renormalization-group evolution of the Yukawa couplings such that successful bottom-tau unification becomes possible in a minimal SO(10) GUT with only a 126-plet coupling to fermions.

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1 Introduction

Grand unification theories (GUTs) [1, 2] offer an appealing framework for new physics beyond the Standard Model (SM). One of their key successes is the explanation of the quantization of hypercharge in units of the weak isospin, which implies the quantization of electric charge in a way that neutron and neutrinos are electrically neutral. No fundamental principles within the SM can forbid deviations from this pattern [3, 4], but it has been tested to extremely high precision; for instance, the neutron charge is constrained to be smaller than about $10^{-21}\ e$ [5]. This puzzle reflects the arbitrariness in constructing anomaly-free representations of the SM gauge group $G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$, while, by contrast, the quantum numbers of SM fermions do not appear random in reality, but directly point to an extended symmetry: Each generation, with a right-handed neutrino, fits neatly into the spinor representation 16_F of SO(10). 'Unifiability' is a rare feature among all possible anomaly-free assignments of $G_{\rm SM}$ representation [6], and strongly suggests that the fundamental gauge group of nature is a single semi-simple group $G_{\rm GUT}$, such as SU(5) or SO(10) containing $G_{\rm SM}$ as a subgroup. Moreover, various fits to gauge coupling unification for SO(10) suggest that the GUT breaking scale $M_{\rm GUT}$ lies around 10^{16} GeV [7–10], in principle making the GUT idea testable by future proton decay experiments [11, 12]. GUTs predict that gauge coupling unify. With the measured values of the SM gauge couplings one finds that this feature holds qualitatively, as the couplings converge to each other at high energies, but fails quantitatively [13, 14]. Quantitative gauge coupling unification can be achieved in multiple ways by "populating the desert" between the electroweak scale and $M_{\rm GUT}$. For example, in supersymmetric GUTs the superpartners of the SM particles with $\mathcal{O}(1-100\ \text{TeV})$ masses make gauge unification possible, but one can also employ mass splittings among the members of the large Higgs multiplets to modify the renormalization group (RG) evolution of the couplings near $M_{\rm GUT}$.

This paper addresses the unification of Yukawa couplings. This topic has two key elements, the number of Higgs multiplets coupling to fermions and the light degrees of freedom invoked to change the RG evolution of the Yukawa couplings between the electroweak scale and $M_{\rm GUT}$. Unfortunately, the most economical SU(5) or SO(10) Yukawa sectors fail in reproducing the observed fermion masses and mixing angles. It is instructive to study the wrong prediction of the $b-\tau$ mass relation. New particles are required to achieve realistic fermion masses and mixings — either heavier than $M_{\rm GUT}$ (entering as effective operators [15–17]) or lighter (such as additional scalars containing Higgs doublets [18–21] or vector-like fermions [22, 23]). The predictions of the *most minimal* GUTs are replaced by fits within *minimal realistic* models which contain additional free parameters; see Refs. [10, 20, 24–31] for examples of non-supersymmetric GUTs. Here with *most minimal* we mean GUTs in which only one Higgs multiplet couples to fermions, in the case of SO(10) the corresponding representations can be 10_H, 120_H, or 126_H. As a common feature of the proposed minimal realistic models, many robust and discriminative predictions of the *most minimal* GUT are lost.

However, it would be premature to claim that *most minimal* GUTs are unrealistic: The loophole is the assumption of a particle desert between the electroweak and GUT scales, usually deduced from naturalness criteria [32]. The impact of the deviation of the desert picture on Yukawa unification is best studied in supersymmetric GUTs, in which the infrared (IR) theory is the Minimal Supersymmetric Standard Model (MSSM). The threshold correction to the matching relation between SM and MSSM Yukawa couplings can be enhanced when $\tan\beta$, the ratio of the vacuum expectation values (VEVs) of the two MSSM Higgs doublets, is large [33–38]. This fact changes the predictions of minimal GUT, and together with successful gauge coupling unification, has been viewed as indirect support for low-scale supersymmetry. However, the SUSY GUT is far from minimal. It requires many more physical particles than the non-SUSY one, and the huge number of physical degrees of freedom brings a new puzzle that perturbative expansion could fail [39, 40]. Recently, the authors of Ref. [41, 42] proposed a new idea, that the wrong $b-\tau$ mass relation in minimal SU(5) can be resolved by introducing a large mass hierarchy among the particles within the same scalar multiplet. This hierarchy requires that some particles lie far below the $M_{\rm GUT}$, and suggests the desert picture together with the naturalness criterion should be reconsidered. The wrong fermion mass pattern does not necessarily falsify the minimal GUT; rather, it implies that SM alone cannot serve as a viable IR theory. Some scalar particles from the GUT sector need to be included in the light spectrum. In some cases, such particles also lead to successful gauge coupling unification; see, for instance, Ref. [16, 17, 43, 44].

From a different perspective, doubts on the particle desert picture are nurtured by experimental data on flavor-changing B meson decays. For more than a decade several observables related to $b \to s\mu^+\mu^-$ or $b \to c\tau\nu$ decays have been found to deviate form their SM predictions. The current status of the "flavor anomalies" is as follows: $b \to c\tau\nu$ is probed through the ratios of branching ratios $R(D^{(*)}) = \mathrm{BR}(B \to D^{(*)}\tau\nu)/\mathrm{BR}(B \to D^{(*)}\ell\nu)$ ($\ell = e, \mu$) and polarisation data with very robust theory predictions, because the only non-perturbative quantity involved is a ratio of form factors multiplying a term suppressed by the mass ratio m_τ^2/m_B^2. An analysis exploiting experimental information on form factor shapes finds the combined $b \to c\tau\nu$ data deviating from the SM predictions by 4.4σ [45]. A recent paper calculating form factors from first principles finds compatible results with slightly larger theory uncertainties. Moreover, if one tried to change the form factor ratio in $R(D^*)$ to a level that the data are reproduced, predictions of measured polarisation data in $B \to D^*\ell\nu$ decays with light leptons $\ell = e, \mu$ would instead severely deviate from their SM predictions [46]. BaBar, Belle, Belle II and LHCb contribute to the $b \to c\tau\nu$ anomaly with mutually consistent measurements [47–54] (combined in Ref. [55]). The $b \to s\mu^+\mu^-$ anomaly is supported by measurements of various branching ratios and angular distributions of b-flavored hadrons by LHCb [56–72] and, more recently, also by CMS [73]. Moreover, all data are compatible with effects of equal size in $b \to se^+e^-$,i.e. lepton-flavor universality in the first two generations [71, 74]. The combination of all data prefers beyond-SM (BSM) scenarios with a significance above 5σ [75] if the SM prediction of [76] is used. The latter has been challenged by several alternative calculational approaches [77–82] and while more conservative estimates of hadronic uncertainties reduce the significance of BSM physics, there is no convincing way to bring the data into good agreement with the SM predictions. Statistically, it is unlikely that all these anomalies will disappear in the future [83], and their BSM explanation requires particles not far above the TeV scale. Specifically, leptoquarks (LQs) with masses between 1 TeV and 50 TeV are well-suited to remedy the flavor anomalies without harming predictions of observables which are in agreement with their SM predictions [84–102].

The state-of-the art is to postulate the required light LQs ad-hoc, which remains unsatisfactory until these particles are embedded into a meaningful theory addressing fundamental puzzles of the SM. Thus it is a natural idea to analyze whether the (multi-)TeV scale leptoquarks are beneficial to GUTs, as we do in this paper. Although LQs can arise naturally in many partial unification frameworks, such as Pati-Salam (PS) theories [103], their masses and interactions with fermions remain puzzling. If the LQs are vector bosons, their masses originate from spontaneous gauge symmetry breaking and are their mass is protected by gauge symmetry. However, if PS unification is realized at the multi-TeV scale, the LQ-fermion coupling structure needs to respect an approximate $U(2)^n$ flavor symmetry to evade the bounds from processes involving light flavors, such as $K_L \to \mu e$ [104–109]. Additional vector-like Fermions and/or extended gauge groups are typically needed to achieve TeV-scale partial unification while preserving $U(2)^n$ at low energies [110–114]. On the other hand, *scalar* LQs can naturally preserve the chiral $U(2)^n$ symmetry, but their masses are are unprotected and suffer from the same fine-tuning problem as the SM-Higgs boson [115]. This conceptual puzzle becomes an explicit problem in a (partial) unification framework whose scale is much higher than a TeV. In our view, the LQ explanation for B anomalies could become more convincing if LQs originate from a *most minimal* GUT. Their light masses and specific coupling structures could emerge as a consistency requirement for successful Yukawa unification, so that the benefits of successful unification and explanation of flavor anomalies outweigh the nuisance of additional unprotected scalar masses.

In this work, we indeed demonstrate that the light scalar particles correcting the $b-\tau$ mass relation could be the same TeV-scale LQs responsible for the B anomalies. We study a minimal SO(10) model, whose Yukawa sector includes merely one scalar multiplet, 126_H. The SM Higgs doublet and the TeV-scale LQs all live within this 126_H representation. This minimal set-up then contains merely one Yukawa coupling matrix, and seemingly cannot reproduce the observed fermion masses and mixing patterns. Yet, the TeV-scale LQs break the desert picture and modify the renormalization group (RG) equations. At the end, we find the masses of the top quark, bottom quark, and τ lepton to emerge correctly at low energy, although the theory contains just two free parameters —the Yukawa coupling at the GUT scale, $y_t(M_{\rm GUT})$, and the Higgs VEV ratio $\tan\beta$. The LQ-fermion couplings emerge as infrared (IR) fixed points (see Ref. [101] for a model-independent study of fixed-points), and the correlations among couplings of different LQ types potentially make the explanation of the B anomalies much less ad-hoc. Although a single scalar multiplet in the Yukawa sector cannot provide flavor mixing angles, we find that in the presence of the TeV-scale LQs, flavor conservation becomes an unstable solution of the RG equations. Flavor mixing can thus serve as an emergent phenomenon when zooming out to larger distance scales. In this work, we do not attempt a global fit to all data, as our result appears to be fairly model-independent and does not rely on the specific choice of the LQ types. A complete explanation of the B anomalies, particularly the new flavor mixing angles, typically requires further modifications and refinements to the light scalar spectrum of 126_H. Rather than searching for an existence proof, our main goal is to analyze a simple scenario that captures the essential physics and to demonstrate that TeV-scale LQs offer a promising path towards a consistent minimal GUT. Our result makes the LQ explanation to B anomalies more convincing: although fine-tuning scalar LQ masses is still needed, it now arises as a consistency requirement imposed by unification.

The paper is structured as follows. In Section 2, we revisit the minimal ways to construct the SO(10) Yukawa sector. We explain why 126_H is preferred and discuss its shortcomings. Next, we discuss the LQ spectrum in 126_H, specify the the LQ-fermion interactions, and summarize the effective operators relevant for $b \to c, s$ transitions. In the following subsection, we present an overview on how these LQs can address the B anomalies and examine what additional constrains are imposed by SO(10). In Section 3, we analyze how TeV-scale LQs can improve the RG evolution for the Yukawa couplings between light scalars and third-generation fermions. We show how the $b - \tau$ mass relationship is improved and how the LQ-fermion couplings exhibit fixed point behaviors. Since addressing B anomalies requires large flavor mixing angles that are unphysical in SM, we then propose a possible solution to this newly arising problem. In Section 4, we summarize our main findings and outline the further work needed to strengthen our idea. To demonstrate the model-independence of our results, we show in Appendix A that $b - \tau$ unification can also be achieved with different light LQ spectra.

2 Minimal SO(10) GUT

2.1 The landscape

We start by revisiting the SO(10) theories with the simplest Yukawa sector [116]. As introduced, the spinor representation of SO(10) contains exactly one generation of SM fermions plus a right-handed neutrino:

$$16_F \;=\; (Q_L, u_R^c, d_R^c) + (\ell_L, \nu_R^c, e_R^c). \tag{2.1}$$

As a chiral theory, Fermions in 16_F get masses via coupling to the scalars developing non-zero VEVs. An approach is to add a vector 10_H:

$$-\mathcal{L}_{Y_{10}} \;=\; Y_{10} 10_H \overline{16_F} 16_F^c, \quad 10_H = \Gamma_i \phi_i. \tag{2.2}$$

Here, Γ_i $(i = 1, ..10)$ are the Gamma matrices in the SO(10) space. The GUT-scale prediction is robust but a bit boring: degenerate masses for top quark, bottom quark, charged τ lepton, and Dirac-type τ neutrino. This is clearly inconsistent with the low energy data. Alternatively, one can replace the vector 10_H with a rank-three tensor 120_H:

$$-\mathcal{L}_{Y_{120}} \;=\; Y_{120} 120_H \overline{16_F} 16_F^c, \quad 120_H = \Gamma_i \Gamma_j \Gamma_k \phi_{[ijk]}. \tag{2.3}$$

However, the ten-dimensional Dirac algebra tells that Y_{120} is anti-symmetric in flavor space. Y_{120} vanishes in the single generation limit. In case of three generations, Y_{120} cannot generate heavy masses for the third-generation fermions if the first two generations remain light. This situation is even worse than the one in 10_H.

For a long time, people believed that the mentioned failure comes from minimal model construction. In our opinion, this trouble somehow stems from the structures of $10_H, 120_H$, which lack complexity. The last remaining choice, the rank-five (self-dual) tensor 126_H, improves the situation much. The Yukawa sector now reads:

$$-\mathcal{L}_{Y_{126}} = Y_{126} 126_H \overline{16_F} 16_F^c, \quad 126_H = \Gamma_i \Gamma_j \Gamma_k \Gamma_l \Gamma_m \phi_{[ijklm]}. \tag{2.4}$$

Firstly, 126_H is tailor-made for tiny neutrino masses. 126_H contains a vacuum singlet under $G_{\rm SM}$ and can give the right-handed neutrino ν_R a high-scale mass term. Upon integrating out ν_R, the dim-5 Weinberg operator [117] can generate a tiny mass for the active neutrino living in ℓ_L [118, 119]. In addition, 126_H contains two Higgs doublets with opposite $SU(2)_R$ isospin. As a result, one of the Higgses only couples to $\overline{Q_L} u_R, \overline{L_L} \nu_R$ and the other one only couples to $\overline{Q_L} b_R, \overline{Q_L} e_R$. A hierarchical ratio m_t/m_b is now possible, as long as the ratio $\tan\beta$ of the two Higgs doublets VEVs is large. Furthermore, Y_{126} is symmetric and can be assigned third-generation specific, which gives the desired hierarchy structure among the charged fermion masses of the three generations. Indeed, Eq (2.4) cannot account for quark mixing and is unlikely to yield the precise quantities for the first- and second-generation fermion masses. However, we do not think these shortcomings require extending the Yukawa sector immediately. If one neglects Yukawa couplings much smaller than unity, the light-flavor structure contains merely vanishing or unphysical observables. Large neutrino mixing is not problematic either, because the Yukawa couplings enter the left-handed neutrino mass matrix M_{ν_L} non-linearly. The relation $M_{\nu_L} \propto Y_{126}$ could break down even with higher-order corrections. Therefore, the $\mathcal{O}(1)$ predictions of 126_H are approximately correct. In the following, we refer to the theory whose Yukawa sector can be well-approximated by Eq (2.4) as 'minimal SO(10) GUT'.

However, the $b-\tau$ mass relation remains an issue: 126_H predicts $m_\tau = 3m_b$ at $M_{\rm GUT}$ [116], but the experimental measurements, combined with the SM RG equations, indicate $m_\tau = 1.67 m_b$ at $M_{\rm GUT}$ [120, 121]. This discrepancy cannot be ignored. Therefore, we have to conclude that the low-energy theory of minimal SO(10) contains all SM fields with couplings close to —but still not reasonably agreeing with— the measured quantities. The landscape of this minimal theory is *a SM-like theory*, while *the SM* itself lies in the swampland, as illustrated in Figure 1. Can *the SM* be included inside the landscape without modifying the deep-UV physics? Our conjecture is: if some scalar particles living in 126_H, such as LQs, are fine-tuned to be light, they can change the RG equation and reshape the boundary of the minimal SO(10) landscape, which may include *the SM*.

2.2 Leptoquarks in 126_H

To discuss the LQ spectrum, we decompose the spectrum of 126_H with the Pati-Salam type subgroup $SU(4)_c \times SU(2)_L \times SU(2)_R$ by [122]:

$$126_H = (6_c, 1_L, 1_R) + (10_c, 3_L, 1_R) + (\overline{10_c}, 1_L, 3_R) + (15_c, 2_L, 2_R). \tag{2.5}$$

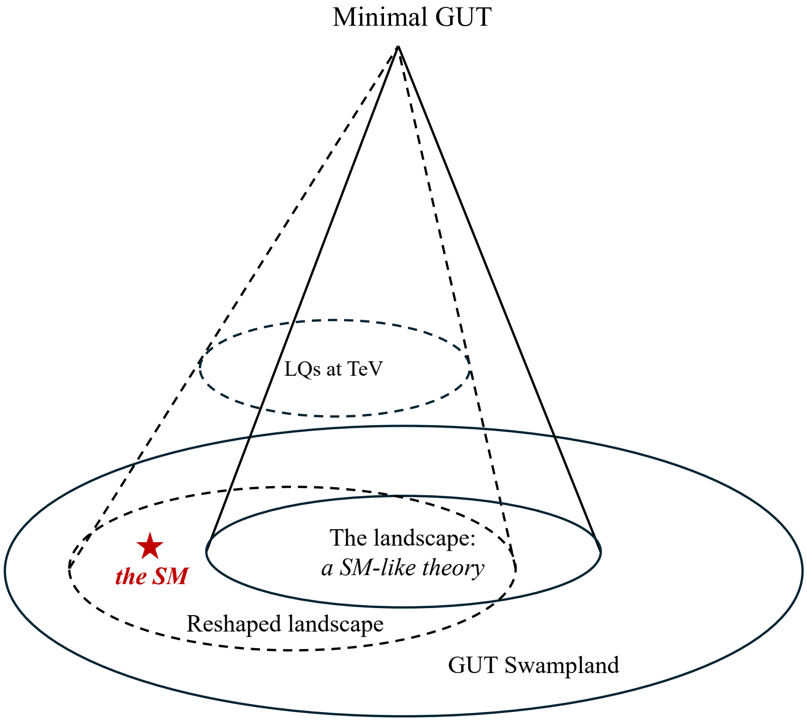

Figure 1. The small solid ellipse represents the landscape of minimal SO(10), which is *a SM-like theory*. The red star marks *the SM* itself. It is not consistent with the minimal theory and therefore lies in the GUT swampland (shown with the large solid ellipse). The dashed ellipse illustrates the reshaped landscape induced by TeV-scale LQs, within which *the SM* is now included.

The $SU(4)_c$ group takes the anomaly-free $U(1)_{B-L}$ charge as the 'fourth color', implying that leptons and quarks can convert into each other. Here, the scalar LQs are identified as the color triplet mediators:

$$\begin{aligned}
(6_c, 1_L, 1_R) &\supset S_1(3,1,-1/3) + S_1'(\overline{3},1,1/3), \\
(10_c, 3_L, 1_R) &\supset S_3(3,3,-1/3), \\
(\overline{10}_c, 1_L, 3_R) &\supset \bar{S}_1(\overline{3},1,-2/3) + S_1''(\overline{3},1,1/3) + \widetilde{S}_1(\overline{3},1,4/3), \\
(15_c, 2_L, 2_R) &\supset R_2(\overline{3},2,-7/6) + \widetilde{R}_2(\overline{3},2,-1/6) + R_2'(3,2,7/6) + \widetilde{R}_2'(3,2,1/6).
\end{aligned} \tag{2.6}$$

We follow the LQ notation of Ref. [85] up to charge conjugation, with the numbers in the parentheses indicating their representations under $G_{\rm SM}$. $\bar{S}_1$, S_1'', and $\widetilde{S}_1$ form an $SU(2)_R$ triplet, while S_1' is an $SU(2)_R$ singlet. They carry symmetric and antisymmetric $SU(2)_R$ indices, respectively, and can be written in a compact form as:

$$\widehat{S}_3 = \begin{pmatrix} \overline{S}_1 & S_1''/\sqrt{2} \\ S_1''/\sqrt{2} & \widetilde{S}_1 \end{pmatrix} \qquad \widehat{S}_1 = \begin{pmatrix} 0 & S_1'/\sqrt{2} \\ -S_1'/\sqrt{2} & 0 \end{pmatrix} \tag{2.7}$$

Both $\widehat{S}_3$ and $\widehat{S}_1$ have $Q_{B-L} = 1/3$. The pairs $(R_2, \widetilde{R}_2)$ and $(R_2', \widetilde{R}_2')$ are two distinct $SU(2)_R$ doublets, with $Q_{B-L} = 2/3$ and $Q_{B-L} = -2/3$ respectively. Their Yukawa interactions with fermions are given by:

$$
\begin{aligned}
-\mathcal{L}_Y^{\rm LQ} \;=\;& Y_3^{LL}\overline{Q_L}S_3L_L^c + Y_3^{RR}\overline{Q_R^c}\widehat{S}_3L_R + Y_1^{LL}\overline{Q_L}S_1L_L^c + Y_1^{RR}\overline{Q_R^c}\widehat{S}_1L_R \\
&+ Y_2^{LR}\overline{Q_L}(R_2', \widetilde{R}_2')L_R + Y_2^{RL}\overline{Q_R^c}(R_2, \widetilde{R}_2)L_L^c + \text{h.c.}
\end{aligned}
\tag{2.8}
$$

Here,^{c} denotes the standard charge conjugation operator, $Q_R = (u_R, d_R)$, $L_R = (\ell_R, \nu_R)$ are $SU(2)_R$ doublets, and flavor indices are implicit. Eq (2.8) shows R-type LQs couple to quarks and leptons with opposite chiralities, while the S-type LQs — S_3, S_1 for left-handed fields and $\widehat{S}_3, \widehat{S}_1$ for right-handed fields— are chirality-specific.

The Yukawa couplings $Y_{1,2,3}$ are 3×3 matrices in flavor space. At the GUT scale, they are aligned with the quark mass matrices and therefore inherit an approximate $U(2)$ structure:

$$
Y_{1,2,3} \;\propto\; Y_{126} \;\sim\; \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & \epsilon' & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \text{at GUT scale.} \tag{2.9}
$$

Here, $\epsilon \ll \epsilon' \ll 1$, following the known hierarchy of quark masses. This structure, if well preserved at low energies, yields the approximate $U(2)$ symmetry needed to suppress the processes such as $K_L \to \mu e$. In this unification framework, the $U(2)$ LQ-fermions coupling structures are no longer ad-hoc, but emerge naturally since they have the same origin as quark masses. In the following discussion, we focus on third-generation specific LQ couplings and include the second generation only when they are needed for the flavor-violating transitions.

In the energy regime far below 1 TeV, LQs in Eq (2.8) can be safely integrated out, and then the scalar-type effective operators with the form $(\overline{q}\ell)(\overline{\ell'}q')$ are generated at tree-level. We apply a Fierz transformation to convert these operators into the standard basis, as summarized in Table 1. These operators follow the general discussion shown in Ref. [85] and can also be directly inferred from the chirality structure of the LQ-fermion couplings. It is worth noting that the left-right symmetry is not manifest in the table, because ν_R and the top quark are not included. Moreover, the scalar- and tensor-type operators require sizable $R_2 - (R_2')^c$ and $S_1 - (S_1')^c$ (or $S_1 - (S_1'')^c$) mixing. Without such mixings, a single LQ in the generic basis cannot couple simultaneously to both Q_L and Q_R, and thus leads to merely the vector-type operators.

2.3 Addressing the B anomalies

TeV scale LQs are related to the long-standing anomalies observed in semi-leptonic B decays. The S_3 LQ contributes to all the $b \to c, s$ transitions with SM-like operators. If S_3 is the unique LQ responsible for the $R(D^{(*)})$ anomaly, C_L^ν, the coefficient of $(\overline{s}_L\gamma^\mu b_L)(\overline{\nu}_L\gamma^\mu\nu_L)$ is fixed by $SU(2)_L$ invariance. The predicted value for C_L^ν is too large to be compatible with the current bounds on $b \to s\overline{\nu}\nu$ transitions. A simple way out is to combine S_3 with S_1, whose contributions to C_L^ν can partly cancel [90, 95] the one of the former LQ. Assuming a cancellation of about 60% in C_L^ν, and including the operators induced by S_1, $R(D^{(*)})$ can be consistently explained at 1σ level [123]. The constraints on C_L^ν can be relaxed since the recent Belle-II data with the inclusive tagging method indicates $\text{Br}(B \to K\overline{\nu}\nu)$ exceeds

	S_3	S_1	$\widetilde{S}_1$
$b \to c\tau\nu$	$(\bar{c}_L\gamma^\mu b_L)(\bar{\tau}_L\gamma^\mu\nu_L)$	$(\bar{c}_R\sigma^{\mu\nu}b_L)(\bar{\tau}_R\sigma_{\mu\nu}\nu_L)$ $(\bar{c}_L\gamma^\mu b_L)(\bar{\tau}_L\gamma^\mu\nu_L)$ $(\overline{c_R}b_L)(\bar{\tau}_R\nu_L)$	–
$b \to s\tau\tau$	$(\bar{s}_L\gamma^\mu b_L)(\bar{\tau}_L\gamma^\mu\tau_L)$	–	$(\bar{s}_R\gamma^\mu b_R)(\bar{\tau}_R\gamma^\mu\tau_R)$
$b \to s\nu\nu$	$(\bar{s}_L\gamma^\mu b_L)(\bar{\nu}_L\gamma^\mu\nu_L)$	$(\bar{s}_L\gamma^\mu b_L)(\bar{\nu}_L\gamma^\mu\nu_L)$	–
	R_2	$\widetilde{R}_2$	
$b \to c\tau\nu$	$(\bar{c}_R\sigma^{\mu\nu}b_L)(\bar{\tau}_R\sigma_{\mu\nu}\nu_L)$ $(\overline{c_R}b_L)(\bar{\tau}_R\nu_L)$	–	
$b \to s\tau\tau$	$(\bar{s}_L\gamma^\mu b_L)(\bar{\tau}_R\gamma^\mu\tau_R)$	$(\bar{s}_R\gamma^\mu b_R)(\bar{\tau}_L\gamma^\mu\tau_L)$	
$b \to s\nu\nu$	–	$(\bar{s}_R\gamma^\mu b_R)(\bar{\nu}_L\gamma^\mu\nu_L)$	

Table 1. Effective operators induced by LQ exchange, expressed in the standard basis. We assume maximal mixing among the LQs with same quantum numbers under $G_{\rm SM}$. Top quarks and right-handed neutrinos are omitted. Except for charm and strange quarks, we do not include other second- and first-generation fermions either.

the SM prediction by a factor of 5.4. Yet $\text{Br}(B \to K^*\bar{\nu}\nu)$ still imposes a tight bound and necessitates the right-handed operator $(\bar{s}_R\gamma^\mu b_R)(\bar{\nu}_L\gamma^\mu\nu_L)$ generated by $\widetilde{R}_2$. If its coefficient C_R^ν takes a proper value, it can suppress the $B \to K^*$ amplitude while allowing a sizable $B \to K$ rate [124]. $SU(2)_L$ invariance also implies $b \to s\bar{\tau}\tau$ transitions. The current limit on $\text{Br}(B \to K\bar{\tau}\tau)$ is weak due the experimental difficulty in identifying τ leptons. Interestingly, if closing the τ loop and attaching it to an off-shell photon, the penguin diagram can induce $b \to s\ell\ell$ ($\ell = e, \mu$) via lepton flavor universal operators [125]. The coefficient $C_9^U \sim -1$ can account for the $B \to K\ell\ell$ anomalies without violating the bound from $R(K^{(*)})$ [126]. Although $B_s - \bar{B}_s$ mixing constraints disfavor the best-fit value for C_9^U, moderate cancellation by additional operators containing right-handed quarks can relax this tension [75, 127]. Such operators can be generated by $\widetilde{S}_1$ or $\widetilde{R}_2$.

The R_2 LQ can also explain the $R(D^{(*)})$ anomaly with the $(\bar{c}_R\sigma^{\nu\nu}b_L)(\bar{\tau}_R\sigma_{\nu\nu}\nu_L)$ and $(\overline{c_R}b_L)(\bar{\tau}_R\nu_L)$ operators, and it does not lead to the $b \to s\bar{\nu}\nu$ transition. This solution requires an $\mathcal{O}(1)$ $R_2 - (R_2')^c$ mixing angle, which brings an inhomogeneous term to the RG equation for the τ lepton mass [101]:

$$16\pi^2 \frac{d}{d\ln\mu} m_\tau \;=\; \ldots - 6 m_t (Y_2^{LR})_{33} (Y_2^{RL})_{33}. \tag{2.10}$$

The $(3,3)$ elements of Y_2^{LR} and Y_2^{RL} are both $\mathcal{O}(1)$ and therefore can lead to an unsuppressed additive correction to the τ lepton mass. This situation is somehow similar to the SUSY threshold correction to the bottom quark mass in the large $\tan\beta$ regime [33–35]. Here, the physical correction to m_τ is further enhanced by a logarithmic factor $\log(M_{\rm GUT}/m_{R_2}) \sim 30$. The predicted m_τ is then too large and cannot be consistent with the observed value in a minimal theory. Moreover, R_2 carries a large hyper-charge $7/6$ and significantly accelerates the running of g_1, the $U(1)_Y$ coupling. With the RG equations of SM, g_1 already increases rapidly and meets the other gauge couplings at around 10^{13} GeV [13, 14], a scale too low to satisfy proton decay constrains. Light R_2 would worsen this problem [128, 129], making gauge coupling unification even more challenging.

The S_1 LQ is constrained to be as heavy as the GUT scale, because it always couples to a pair of quarks in the minimal GUT framework [130] and induces proton decay. On the other hand, the diquark couplings of $S_3, \overline{S}_1, \widetilde{S}_1$ are absent at tree level. This absence is not accidental but a consequence of the $U(1)_{\rm PQ}$ symmetry contained in Eq (2.4).

$$16_F \to 16_F e^{i\theta}, \quad 126_H \to 126_H e^{2i\theta}. \tag{2.11}$$

As a phase rotation of complex fields, this symmetry also emerges in the gauge sector. Taking S_3 as an example, although the SM gauge symmetry allows it to couple to both $(\overline{Q}_L L_L^c)$ and $(\overline{Q_L^c} Q_L)$, the minimal Yukawa sector does not contain the $\overline{Q_L^c} S_3 Q_L$ term because it is not invariant under $U(1)_{\rm PQ}$. To suppress these diquark couplings also at the loop level, the PQ symmetry should also be well preserved in the scalar potential. The $\eta_2(126_H)^4$ and $\gamma_2(45_H)^2(126_H)^2$ terms [131] explicitly break the PQ symmetry. To ensure that the proton decay amplitude is suppressed by $M_{\rm GUT}^2$, the magnitudes of γ_2 and η_2 can be much larger than about $\mathcal{O}\left(\frac{m_S}{M_{\rm GUT}}\right)^2$, where the m_S denotes lightest mass among $S_3, \overline{S}_1$ and $\widetilde{S}_1$[3]. If $m_S \sim$ TeV, γ_2 and η_2 are constrained extremely small. By itself, this does not directly introduce a new hierarchy puzzle, since $\gamma_2 = \eta_2 = 0$ enhances the PQ symmetry. However, the $H_u - H_d$ mixing term is proportional to $\gamma_2 M_{\rm GUT}^2$. To achieve a sizable mixing angle β, the mass of H_d cannot be far larger than $\sqrt{\gamma_2} M_{\rm GUT} \lesssim \mathcal{O}(m_{\rm S})$. This requires additional fine-tuning, while it enriches the low-energy spectrum with an extra Higgs doublet. Its charged component can contribute to $b \to c\tau\nu$ [132–135] and further relax the tension between explaining $R(D^{(*)})$ and avoiding $B \to K\overline{\nu}\nu$ constrains.

To conclude, 126_H contains six types of LQs: $S_3, S_1, \overline{S}_1, \widetilde{S}_1$ and $R_2, \widetilde{R}_2$. Among them, S_1 is excluded from being light due to the proton decay constrains. R_2 is disfavored by both Yukawa and gauge coupling unification. $\overline{S}_1$ only couples to ν_R. On the other hand, $S_3, \widetilde{S}_1$ and $\widetilde{R}_2$ can address the B anomalies, and a consistent explanation typically requires a combination of two or all of them.

[3] Since the hierarchy could be large, we omit the possible loop factors, which can relax the bound by $(16\pi^2)^n$, where n is the loop order.

3 Improved RG evolution

3.1 $b-\tau$ masses

To quantify the modified $b-\tau$ mass relation, we first specify the relevant energy scales. Proton decay constraints and hints from gauge coupling unification imply that the breaking scale of SO(10) generally lies around 10^{16} GeV [7–10]. The intermediate scale can be defined by the heaviest right-handed neutrino mass m_{ν_R}, satisfying the relation:

$$m_{\nu_L} = \frac{m_{\nu_D}^2}{m_{\nu_R}} = \frac{(3m_t)^2}{m_{\nu_R}} < 0.064 \text{ eV [136] (0.45 eV [137])}. \tag{3.1}$$

Considering the cosmological constraint from the combination of DESI and CMB [136], m_{ν_R} is larger than around 10^{15} GeV. The limit from KATRIN [137] is weaker by a factor of about 7 but it is model-independent. In either case, the intermediate scale m_{ν_R} remains high[4] and fairly close to 10^{16} GeV. The low energy spectrum contains SM particles and the TeV-scale scalar particles originating from 126_H, including an additional Higgs doublet as well as $S_3, \widetilde{S}_1, \widetilde{R}_2$. We consider a simpler scenario without $\widetilde{S}_1$ as well, although it may not achieve a global fit as good as the full three-LQ case. For completeness, we also analyze the $S_3, \widetilde{S}_1$ scenario to highlight model-independence. Since the precise mass values are not the focus of this work, we simply assume a common mass of 1 TeV for all light LQs and the Higgs doublet, and set the masses of all other BSM particles to $M_{\text{GUT}} \equiv 10^{16}$ GeV. This two-scale approximation is valid at the leading-log order. A more specified mass distribution cannot introduce large hierarchies which would significantly change the RG evolution.

The Yukawa couplings relevant for the running are defined by:

$$\begin{aligned} -\mathcal{L}_Y &= y_t \overline{Q_L^3} t_R H_u + y_b \overline{Q_L^3} b_R H_d + y_\tau \overline{L_L^3} \tau_R H_d \\ &\quad + y_1 \overline{b_R^c} \tau_R \widetilde{S}_1 + y_2 \overline{b_R^c} L_L^{3c} \widetilde{R}_2 + y_3 \overline{Q_L^{3c}} L_L^3 S_3 + \text{h.c.} \end{aligned} \tag{3.2}$$

Here, $Q_L^3 = (t_L, b_L)$ and $L_L^3 = (\tau_L, \nu_{\tau L})$ are third-generation specific, as we do not include light flavors at this stage. All these Yukawa couplings can be chosen real. At M_{GUT}, they are related by the group structure of 126_H:

$$y_b = y_t, \quad y_\tau = -3y_b, \quad y_1 = y_2 = y_3 = 2\sqrt{3}y_t, \quad \text{at GUT scale.} \tag{3.3}$$

These relations serve as the initial condition of RG evolution from IR to UV. The new factor $2\sqrt{3}$ can be interpreted as a generalized Clebsch-Gordan coefficient, and we take this number from Ref. [130]. This factor enhances the LQ Yukawa coupling over the top quark one by more than a factor of three, yielding a large impact on the running.

[4]This only holds with the non-minimal Yukawa sector.

The explicit RG equations are form Refs. [101, 138, 139]. We reduce them by keeping only the third-generation related couplings:

$$\begin{aligned}
16\pi^2 \frac{d}{d\ln\mu} y_t &= y_t \left(-\frac{17g_1^2}{12} - \frac{9g_2^2}{4} - 8g_3^2 + \frac{9y_t^2}{2} + \frac{y_b^2}{2} + \frac{3y_3^2}{2} \right), \\
16\pi^2 \frac{d}{d\ln\mu} y_b &= y_b \left(-\frac{5g_1^2}{12} - \frac{9g_2^2}{4} - 8g_3^2 + \frac{y_t^2}{2} + \frac{9y_b^2}{2} + y_\tau^2 + \frac{y_1^2}{2} + y_2^2 + \frac{3y_3^2}{2} \right), \\
16\pi^2 \frac{d}{d\ln\mu} y_\tau &= y_\tau \left(-\frac{15g_1^2}{4} - \frac{9g_2^2}{4} + \frac{5y_\tau^2}{2} + 3y_b^2 + \frac{3y_1^2}{2} + \frac{3y_2^2}{2} + \frac{9y_3^2}{2} \right), \\
16\pi^2 \frac{d}{d\ln\mu} y_1 &= y_1 \left(-2g_1^2 - 4g_3^2 + y_b^2 + \frac{y_\tau^2}{2} + 3y_1^2 + y_2^2 \right), \\
16\pi^2 \frac{d}{d\ln\mu} y_2 &= y_2 \left(-\frac{13g_1^2}{20} - \frac{9g_2^2}{4} - 4g_3^2 + y_b^2 + \frac{y_\tau^2}{2} + \frac{y_1^2}{2} + \frac{7y_2^2}{2} + \frac{9y_3^2}{2} \right), \\
16\pi^2 \frac{d}{d\ln\mu} y_3 &= y_3 \left(-\frac{g_1^2}{2} - \frac{9g_2^2}{2} - 4g_3^2 + \frac{y_t^2}{2} + \frac{y_b^2}{2} + \frac{y_\tau^2}{2} + \frac{3y_2^2}{2} + 8y_3^2 \right).
\end{aligned} \tag{3.4}$$

Here, no inhomogeneous terms arise because the $U(1)_{\rm PQ}$ symmetry ensures each LQ only couples to a single type of fermion bilinear. This feature further simplifies the RG equation compared to the general case [101]. The beta function for every coupling is always proportional to itself. Moreover, the running receives negative contributions from the gauge interactions and positive contributions from Yukawa couplings.

The IR observables are the third-generation charged fermions' masses at 1 TeV, defined by:

$$m_t = \frac{1}{\sqrt{2}} y_t v \sin\beta, \quad m_b = \frac{1}{\sqrt{2}} y_b v \cos\beta, \quad m_\tau = \frac{1}{\sqrt{2}} y_\tau v \cos\beta, \quad v \equiv 246 \text{ GeV}. \tag{3.5}$$

Their specific values at 1 TeV can be found in Ref. [140]

$$m_t = 151.1 \pm 1.6 \text{ GeV}, \quad m_b = 2.414 \pm 0.024 \text{ GeV}, \quad m_\tau = 1.7780 \pm 0.0014 \text{ GeV}. \tag{3.6}$$

As a result, there are merely two free input parameters in the system: the GUT-scale value of y_t and the Higgs VEV ratio $\cot\beta$. They are constrained by m_t and m_b. The τ lepton mass then becomes a prediction and thus must reasonably agree with the experimental value.

We illustrate how light LQs from 126_H change the mass running in Figure 2. We assume exact Yukawa unification at $M_{\rm GUT}$ (Eq (3.3) holds exactly) and fix the free inputs $y_t(M_{\rm GUT}) = 0.56,\ \tan\beta = 42$. The solid lines indicate how the $b-$, $\tau-$, $t-$ masses evolve from 10^{16} GeV down to 10^2 GeV. For 1 TeV $< \mu < 10^{16}$ GeV, we use the running equation shown in Eq (3.4). Below 1 TeV, we directly interpolate the SM running data included in the SMDR package. [120]. The resulting $b-$, $\tau-$, $t-$masses at 1 TeV can agree reasonably well with the values extracted from the SM. While the matching at 1 TeV would be perfect if the solid lines were continuous, the gaps visible in the plots are not problematic. At leading-log level, TeV-scale LQs can reduce the $b-\tau$ mass tension from an $\mathcal{O}(1)$ discrepancy to a suppressed $\mathcal{O}(\epsilon)$ mismatch. For comparison, we also show the evolution without TeV-scale

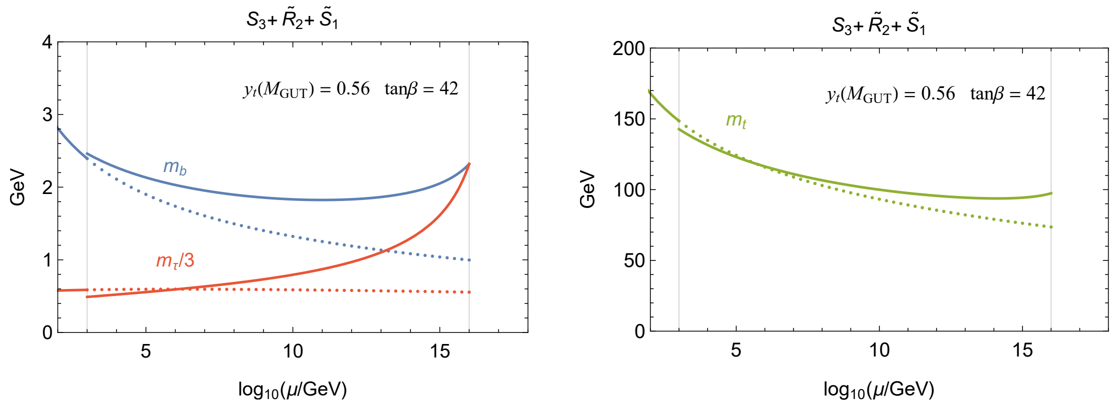

Figure 2. RG evolution of the third-generation charged fermions masses from 10^2 to 10^{16} GeV. The solid (dotted) lines indicate the scenario with (without) LQs at TeV scale. We choose $y_b = y_t = -\frac{1}{3}y_\tau = \frac{1}{2\sqrt{3}}y_1 = \frac{1}{2\sqrt{3}}y_2 = \frac{1}{2\sqrt{3}}y_3 = 0.56$ at GUT scale, and $\tan\beta = 42$. The gray vertical lines indicate $M_{\rm GUT}$ and the light LQ threshold.

LQs, by plotting the SM running parameters from Ref. [120] with dots for $\mu > 1$ TeV. The red and blue dotted curves still diverge significantly at 10^{16} GeV, indicating the relation $m_b = m_\tau/3$ cannot be reached if the theory below $M_{\rm GUT}$ is SM alone.

We also explore scenarios with only $S_3 + \widetilde{R}_2$ and only $S_3 + \widetilde{S}_1$. By choosing different values of $y_t(M_{\rm GUT})$ and $\tan\beta$, both scenarios yield successful Yukawa unification at 10^{16} GeV and reasonably good matching relationships at 1 TeV. Related plots are shown in Appendix A. This universal feature suggests that the improved $b-\tau$ unification is not an accidental outcome of a particular LQ choice, but rather a model-independent effect from colored scalar fields. We understand the underlying reason as follows. If the IR theory is simply the SM, $(m_\tau/3)$ is too small and remains nearly a constant. m_b decreases as the energy scale μ goes up, mainly due to QCD effects, but not fast enough to meet $(m_\tau/3)$ at $M_{\rm GUT}$. As shown in Eq (3.4), the additional homogeneous terms in the β-function of y_τ are always positive and rapidly drive y_τ towards a Landau-pole at UV. Although a similar behavior is also seen for y_b, its growth is slower because y_τ receives an extra enhancement by a factor of $N_c = 3$.

3.2 LQ-fermion couplings

The LQ-fermion couplings y_1, y_2, y_3 are also outcomes from the RG evolution. They are correlated at $M_{\rm GUT}$ by Eq (3.3) and evolve together with y_t, y_b, y_τ according to Eq (3.4). Since $y_t(M_{\rm GUT})$ is fixed by the measured value of m_t, y_1, y_2 and y_3 in the range 1 TeV $<$ $\mu < M_{\rm GUT}$ become predictions of the minimal GUT, as shown in the left panel of Figure 3. A fixed-point behavior [141, 142] emerges: all LQ-fermion couplings drop rapidly near $M_{\rm GUT}$ and remain nearly constant for $\mu \lesssim 10^5 \sim 10^{10}$ GeV. This behavior is similar to what recently found in Ref. [101], that the LQ-fermions couplings approach IR fixed-points of $0.5 \sim 1.0$. Interestingly, the LQ-type considered here is different, suggesting that this feature is a rather general consequence from RG evolution.

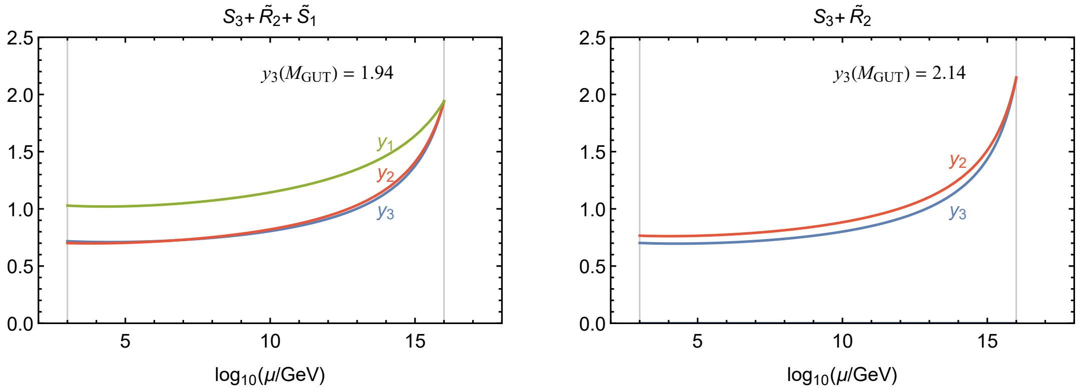

Figure 3. RG evolution of the LQ-fermion coupling y_1, y_2 and y_3 (green, red, and blue, respectively) from 10^2 to 10^{16} GeV. The right panel shows the scenario without $\widetilde{S}_1$ in the light spectrum, with $y_t(M_{\rm GUT}) = 0.62$. All other parameters are as in Figure 2.

Eq (3.4) contains no inhomogeneous terms, so the IR fixed points can be solved analytically. Neglecting g_1, g_2 for simplicity and assuming g_3 varies sufficiently slowly, we find the fixed point solution:

$$y_1 = 0.88 g_3, \quad y_2 = 0.55 g_3, \quad y_3 = 0.51 g_3, \quad \text{at deep IR.} \tag{3.7}$$

As a result, LQ-fermion couplings are numerically close to g_3 and y_t at TeV-scale. Together with Ref. [101], this supports the scalar LQ explanation of the B anomalies, because $\mathcal{O}(1)$ couplings are typically required to account for the data. Moreover, we find y_2 and y_3 almost coincide at all scales. Although this is an accidental result, it can be understood by setting $y_2 = y_3$ and neglecting g_1, g_2, y_t, y_b in Eq (3.4):

$$\begin{aligned} 16\pi^2 \frac{d\ln y_2}{d\ln\mu} &= \left(-4g_3^2 + \frac{y_\tau^2}{2} + \frac{19y_3^2}{2}\left(1 + \frac{y_1^2 - 3y_3^2}{19y_3^2}\right)\right), \\ 16\pi^2 \frac{d\ln y_3}{d\ln\mu} &= \left(-4g_3^2 + \frac{y_\tau^2}{2} + \frac{19y_3^2}{2}\right). \end{aligned} \tag{3.8}$$

This implies $\frac{d\ln y_2}{d\ln\mu} \approx \frac{d\ln y_3}{d\ln\mu}$ so that the ratio $\frac{y_2}{y_3}$ remains nearly a constant under the evolution. A similar result is observed when $\widetilde{S}_1$ is not in the light spectrum ($y_1 = 0$), as shown in the right panel of Figure 3. We consider the unification prediction $y_2 = y_3$ as a critical ingredient for a consistent explanation of the B anomalies. As explained in subsection 2.3, the S_3 and $\widetilde{R}_2$ contributions to $B \to K^* \overline{\nu}\nu$ have to moderately cancel each other. Although the cancellation also requires near degenerate masses and closed $b_L - s_L$ and $b_R - s_R$ mixing angles, the minimal GUT prediction $y_2 = y_3$ makes it much less ad-hoc.

3.3 Emerging flavor mixing angles

A new tension arises. The $b \to c, s$ transition generally requires sizable non-diagonal Yukawa couplings. However, Eq (2.4) contains only one Yukawa matrix Y_{126}, and one can always choose a diagonal basis. Consequently, no flavor mixing can arise at any scale. Where does the necessary flavor mixing come from? One may construct extended models that introduce

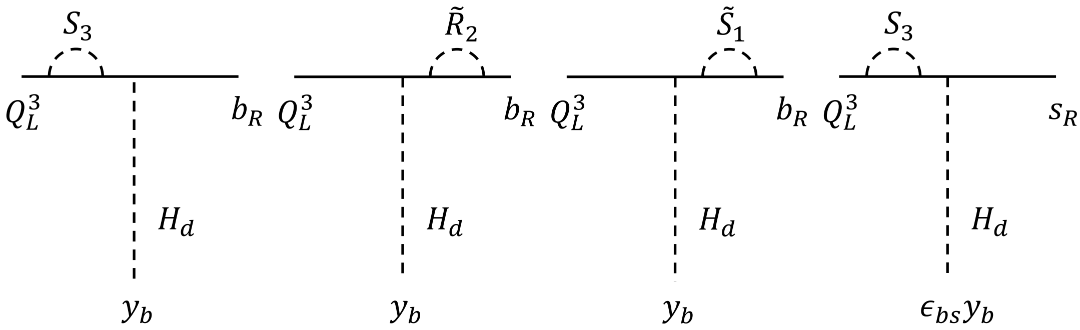

Figure 4. Feynman diagrams illustrating the LQ contributions to the running of y_b and $(\epsilon^{bs}y_b)$. The self-energy correction to Q_L^3, arising from S_3, is universal to both y_b and $(\epsilon^{bs}y_b)$. $\widetilde{S}_1$ and $\widetilde{R}_2$ couple to b_R only so only contribute to y_b. s_R receives no self-energy corrections.

sizable off-diagonal LQ Yukawa couplings while leaving the other sectors unaffected or only mildly modified. Such an approach, nevertheless, is ad hoc and sacrifices the elegance of minimality. We reflect on the robustness of the prediction and realize that although the absolute flavor conserving solution remains stable under the RG equations following the SM [143–145], it becomes unstable when light LQs are introduced. If the flavor-conserving structure at $M_{\rm GUT}$ is not exact, the RG flow from UV to IR can amplify deviations and generate sizable flavor violating interactions at low energies. In this sense, flavor mixing may not mainly originate from explicit next-to-minimal model building, but rather manifest as an emergent phenomenon. The UV theory remains elegantly simple, while complexity arise dynamically through evolving towards IR. This approach is widely applied to address the large neutrino mixing angles [146–150].

Strictly speaking, the flavor violating interactions cannot be exactly zero at $M_{\rm GUT}$. Unlike gauge coupling unification, the diagonal flavor structure does not directly come from the SO(10) symmetry, but rather from the assumption of minimality. Potential corrections may arise due to additional scalars that couple to the fermions via loops [151–153], or high-dimensional operators [15] generated by for instance gravity or by vector-like Fermions. Without loosing generality, we do not specify the underlying source but add the following flavor-violating interactions to Eq (3.2):

$$\mathcal{L}_Y^{\epsilon} \;=\; \epsilon^{ct} y_t \overline{Q_L^2} t_R H_u + \epsilon^{sb} y_b \overline{Q_L^2} b_R H_d + \epsilon^{bs} y_b \overline{Q_L^3} s_R H_d + \epsilon_1^{bs} y_1 \overline{s_R^c} \tau_R \widetilde{S}_1 + \text{h.c.} \tag{3.9}$$

Here, $Q_L^2 = (c_L, s_L)$ donates the second-generation left-handed quark doublet. The flavor basis are chosen such that the S_3 and $\widetilde{R}_2$ couplings are aligned to Q_L^3 and b_R at $M_{\rm GUT}$, respectively. The $t_R - c_R$ rotation remains unphysical, so the $\overline{Q_L^3} c_R H_u$ term is not included. Furthermore, mixings involving leptons or first-generation fermions are omitted, as they are irrelevant to the processes under consideration. $\mathcal{L}_Y^{\epsilon}$ is defined at $\Lambda \gtrsim M_{\rm GUT}$, where Λ is the scale that the new dynamics breaking the minimal GUT prediction arises.

Below $M_{\rm GUT}$, the RG equations for the flavor violating coupling $(\epsilon^{bs}y_b)$ is given by:

$$16\pi^2 \frac{d}{d\ln\mu}(\epsilon^{bs}y_b) \;=\epsilon^{bs}y_b \left(-\frac{5g_1^2}{12} - \frac{9g_2^2}{4} - 8g_3^2 + \frac{y_t^2}{2} + \frac{9y_b^2}{2} + y_\tau^2 + \frac{3y_3^2}{2}\right). \tag{3.10}$$

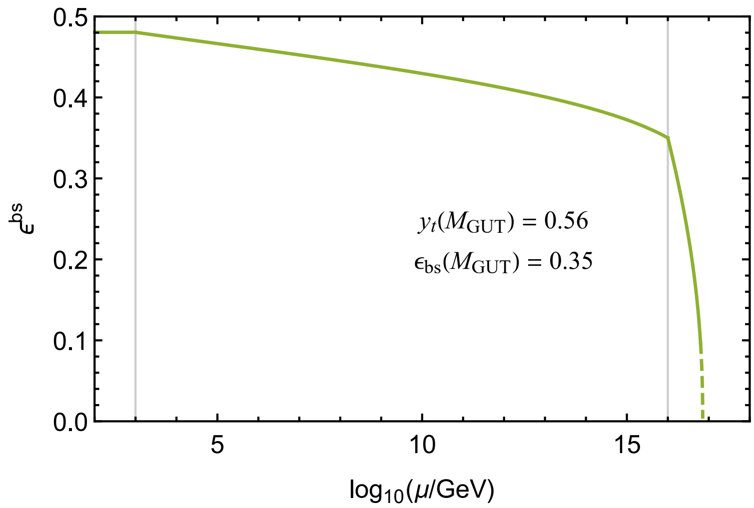

Figure 5. RG evolution of ϵ^{bs} with $y_t(M_{\text{GUT}}) = 0.58$. The dashed line lies in the region that the Yukawa couplings become non-perturbative and may not reflect the physical reality.

Here, we set $\epsilon_1^{bs} = 0$ for simplicity, and neglect higher order flavor violating terms. Comparing with y_b running in Eq (3.4), the $\left(\frac{y_1^2}{2} + y_2^2\right)$ term is absent in Eq (3.10). This is because the LQ couplings are third-generation specific and do not contribute to the self-energy corrections for s_R, as shown in Figure 4. While evolving from UV to IR, $(\epsilon^{bs} y_b)$ decrease more slowly than y_b, and the mixing parameters ϵ^{bs} effectively increase. This behavior is further supported by the negative terms arising in its explicit running equation:

$$16\pi^2 \frac{d}{d\ln\mu} \epsilon^{bs} = -\epsilon^{bs} \left(\frac{y_1^2}{2} + y_2^2\right). \tag{3.11}$$

If ϵ^{bs} is zero at a given scale, it remains exactly zero for all μ since the right-hand side of Eq (3.11) vanishes. However, any slight deviation ϵ^{bs} from zero would be driven towards sizable values as μ decrease. As a consequence, the $s_R - b_R$ mixing is enhanced at IR relative to their UV quantities by a factor of $\left(\frac{M_{\text{GUT}}}{\text{TeV}}\right)^{\frac{y_1^2 + 2y_2^2}{32\pi^2}}$. This leads to flavor misalignment between the Higgs and LQ sector. Upon choosing the physical basis in which the H_d Yukawa coupling is diagonal, the enhanced flavor mixings are then transferred to the $\widetilde{S}_1$ and $\widetilde{R}_2$ couplings.[5]

We illustrate our idea in Figure 5. Keeping $y_t(M_{\text{GUT}}) = 0.58$ as initial condition, the solid line shows the evolution of ϵ^{bs}. Flavor conserving is clearly not a stable solution to the equations, because ϵ^{bs} increase as μ decrease. The shortage is that the evolving below M_{GUT} is too slow: achieving $\epsilon^{bs} \approx 0.5$ at TeV typically requires its GUT scale value be at least 0.35, which does not fit as a perturbation. However, we find ϵ^{bs} changes sufficiently fast

[5] There are additional contributions to the flavor misalignment. The flavor violating LQ interaction itself, defined as zero in Eq (3.9), also receives radiative correction. The beta-function contains an inhomogeneous term proportional to $\epsilon^{bs} y_b^2$. It is much smaller than the $\epsilon^{bs}\left(\frac{y_1^2}{2} + y_2^2\right)$ contribution, because Eq (3.3) tells that y_b^2 is 12 times smaller than y_1^2 or y_2^2 at M_{GUT}. For simplicity, we neglect this sub-leading effect.

above $M_{\rm GUT}$. One reason is that the $(6_c, 1_L, 1_R)$, $(\overline{10_c}, 1_L, 3_R)$, and $(15_c, 2_L, 2_R)$ multiples contained in 126_H all contribute to the b_R self-energy above the threshold. We include their effects according to the matching condition and RG equations from Ref [154–156]. Another reason is the absolute Yukawa coupling strength becomes large. Using the standard one-loop running equations for gauge coupling g_{10} and Yukawa coupling Y_{126} [157, 158], we find Y_{126} increases rapidly and approaches the non-perturbative regime when $\mu \gtrsim 10^{17}$ GeV.[6] When ϵ^{bs} lies in the non-perturbative region, we show it with dashed lines, indicating it is only for illustration and may not reflect the physical reality. Therefore, if tiny ϵ^{bs} originate from physics at scale $\Lambda \gtrsim 10 M_{\rm GUT}$, the running effects yield a sizable value at IR.

Compared with invoking unknown dynamics above GUT scale, a more convincing approach is to add more terms to Eq (3.11) and change the evolution below $M_{\rm GUT}$. This can be achieved by requiring more light scalar components in 126_H, such as di-quarks [159–161], who also speed up the running for all Yukawa couplings. As a result, ϵ^{bs} increases more rapidly when from UV to IR and allows a much lower value at GUT scale. Interestingly, the di-quarks are also relevant for anomalies. They induce effective four-quark interactions that may account for the observed CP asymmetry in $D_0 \to \pi^+\pi^-$ and $D_0 \to K^+K^-$ decays [162–165]. We leave a detailed analysis on this direction for future work.

The behavior of left-handed down-type quark mixing ϵ^{sb} is similar. However, there is an additional requirement to preserve small $V_{cb} \sim V_{ts} \sim 0.04$: $b_L - s_L$ mixing should be well aligned with $t_L - c_L$ mixing. This further requires $\epsilon^{sb} = \epsilon^{ct}$ to good precision at $M_{\rm GUT}$, which can be protected by the $SU(2)_R$ symmetry that connects $t_R H_u$ to $b_R H_d$. At low energies, the unbroken $SU(2)_L$ gauge symmetry ensures the evolution for ϵ^{sb} and ϵ^{ct} remains identical. We think it is worth offering a further comment here. Explaining B anomalies requires misaligned $b - s$ mixing for H_d and LQ Yukawa couplings. In the SM, however, neither $b_L - s_L$ nor $b_R - s_R$ mixing is physical; the only observable is the difference between $b_L - s_L$ and $t_L - c_L$ mixing. In other words, the SM quark mixing pattern and beyond SM flavor transition could have intrinsically different origins, well-separated by symmetries.

[6] Large Y_{126} induces a sizable g_{10} through two-loop effects, while a sizable g_{10} in turn slows down the running of Y_{126} or can even drive it to decrease. If the scalar self-couplings are excluded, asymptotic freedom can be restored in deep UV [158].

4 Conclusion and outlook

In this article, we demonstrate that the TeV-scale LQs addressing the B anomalies can also resolve the wrong $b-\tau$ mass relation in GUTs with simple Yukawa sectors. Among the six types of LQs contained in 126_H, $\widetilde{S}_1, \widetilde{R}_2, S_3$ can account for the $b \to c, s$ anomalies while remaining compatible with grand unification. Fixing exact Yukawa unification at 10^{16} GeV and $m_{\widetilde{S}_1} = m_{\widetilde{R}_2} = m_{S_3} = 1$ TeV, the running masses m_t, m_b, m_τ can match well with low-scale observations. This is a predictive result because the theory contains only two arbitrary inputs at leading-log level: $\tan\beta$ and $y_t(M_{\rm GUT})$. We also show that scenarios without $\widetilde{S}_1$ or $\widetilde{R}_2$ can still lead to successful third-generation Yukawa unification. This implies that the successful prediction of mass ratios is not tied to a specific choice of LQ types. Although our minimal SO(10) theory predicts no flavor mixing, we demonstrate that the flavor conservation solution becomes unstable once the TeV-scale LQs are introduced. Nevertheless, flavor mixing angles large enough to explain the anomalies emerge only when the 126_H spectrum is further modified.

Although the central goal of this work is to illustrate $b-\tau$ unification through a simplified setup, identifying an explicit example that yields an improved global fit for B anomalies would certainly strengthen our conjecture. We note, however, that such a fit introduces additional free inputs. In particular, $m_{\widetilde{S}_1}, m_{\widetilde{R}_2}, m_{S_3}$ all become relevant parameters and can no longer be simply fixed at 1 TeV, as they directly suppress the bottom quark transition amplitudes. Another new relevant set of parameters concerns to which degree the Yukawa sector at the GUT scale deviates from exact Yukawa unification, since the RG evolution of the mixing angles shows a sensitive dependence on initial conditions.

At the end we remark that the physical implication of this work goes beyond unified theories. The original Froggatt-Nielsen paper [166] introduced two distinct ideas to address the SM fermion mass pattern: (a) RG running effects and (b) UV model building. The idea (a) is further developed using the framework of fixed points [141, 142], suggesting that the IR, rather than the UV, gauge structure determines the SM flavor parameters. However, the RG equations derived from SM alone do not lead to a simple UV flavor structure, which brings more efforts to idea (b), UV model building. Thanks to the recent progress at the high-intensity frontier and in lattice QCD calculations, particularly the successful extraction of $R(D^{(*)})$, we now have data-driven motivations to extend the SM into a larger IR theory with LQs at the TeV-scale. The situation has changed compared to 20 years ago and the RG approach to the flavor puzzle deserves new attention. Although light LQs bring a new hierarchy problem, we argue that the RG approach is somehow a more promising path than UV model-building, because the emergence of complexity from simple structures at small distance scales is a universal phenomenon and not limited to particle physics [167]. Well-known examples can be found in condensed matter [168] or other complex systems [169, 170].

Acknowledgments

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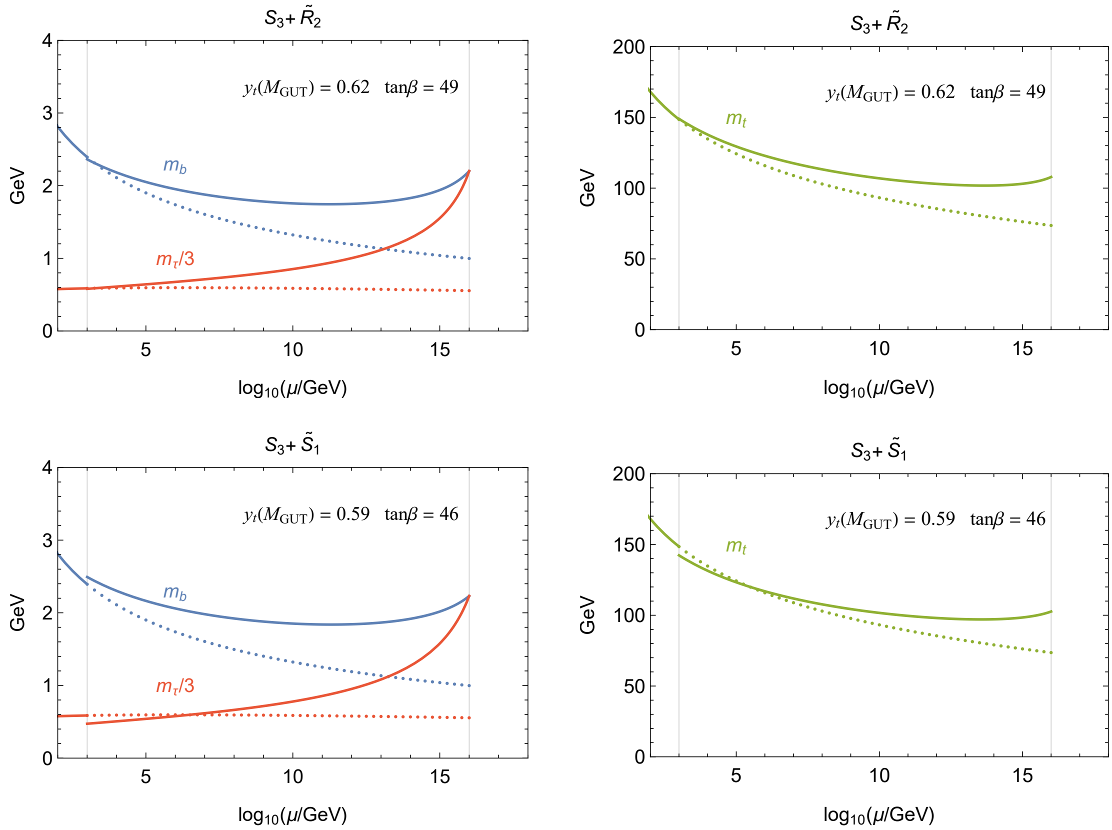

Figure 6. RG evolution in two alternative scenarios to demostrate model independence. Top: $S_3 + \widetilde{R}_2$ with $y_t(M_{\rm GUT}) = 0.62$, $\tan\beta = 49$. Bottom: $S_3 + \widetilde{S}_1$ with $y_t(M_{\rm GUT}) = 0.59$, $\tan\beta = 46$. All others setting are same as in Figure 2.

A Model independence

We show scenarios with $S_3 + \widetilde{R}_2$ and $S_3 + \widetilde{S}_1$ in Figure 6 to demonstrate that our results do not depend on the specific choice of LQ type but are rather universal. We take the same technical setup as the three-LQ scenario, but take $y_t(\mu_{\rm GUT}) = 0.62$, $\tan\beta = 49$, $y_1 \equiv 0$ for the $S_3 + \widetilde{R}_2$ case, and $y_t(\mu_{\rm GUT}) = 0.59$, $\tan\beta = 46$, $y_2 \equiv 0$ for the $S_3 + \widetilde{S}_1$ case. Interestingly, the $S_3+\widetilde{R}_2$ scenario, which may serve as the minimal LQ model to address the B-anomaly, yields a better TeV-scale match than the three-LQ result. In contrast, $S_3 + \widetilde{S}_1$ scenario might be unable to fully account for the $R(D^{(*)})$ anomaly without violating the $B \to K^*\overline{\nu}\nu$ constraint, its matching result is also slightly worse. Despite this, all scenarios provide consistent indication that TeV-scale LQs make the $b-\tau$ mass relationship no longer a major concern for minimal SO(10).

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