PROCEEDINGS OF SCIENCE

# Towards NNLO QCD corrections to Higgs boson pair production

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Higgs boson pair production plays an important role in the determination of the Higgs boson self coupling. The predictions based on next-to-leading order corrections show a large dependence on the renormalization scheme of the top quark mass, which requires a next-to-next-to-leading order calculation. We discuss the current status and show first results of the three-loop virtual corrections.

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## 1. Introduction

Higgs boson pair production is an active field of research, both on the experimental side but also in the theory community. It is probably the most promising observable to learn more about the Higgs boson self coupling, a crucial element in the Standard Model Lagrange density.

Theory predictions for Higgs boson pair production are quite advanced; due to size limitations not all references can be listed in these proceedings and we concentrate on those which are most relevant for the results presented below. The leading order (LO) corrections were computed long before the discovery of the Higgs boson [1, 2]. About 25 years ago the first next-to-leading order (NLO) result became available, however, only in the limit of infinitely heavy top quark [3]. Exact NLO results have been known since 2016 [4–6]. To obtain these results, to a large extent numerical methods have been used, which have several drawbacks. For example, there is limited flexibility in connection to renormalization scheme changes and, furthermore, the calculations require a fair amount of CPU time for the numerical evaluation of the cross section.

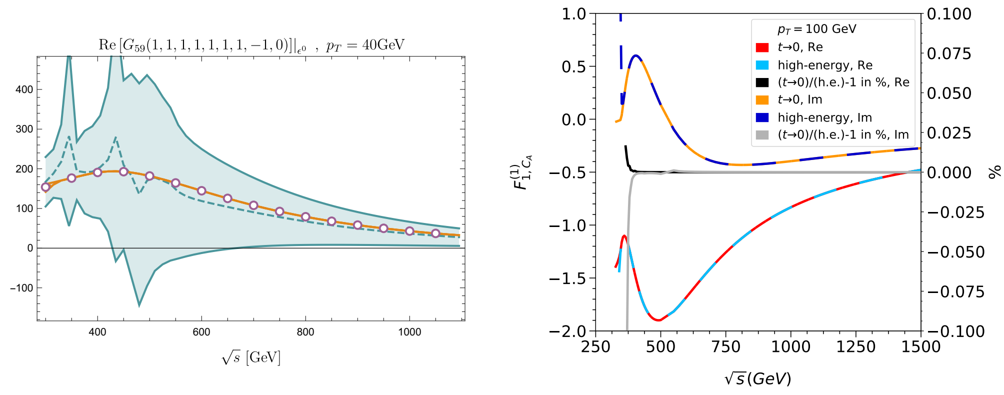
In Section 2 we propose an alternative to the (mostly) numerical approach: analytic expansions. Here the idea is to identify, in a given region of phase space, small parameters and then perform a Taylor or an asymptotic expansion. For Higgs boson pair production first steps in this direction have been undertaken in Ref. [7] where analytic results from [8] have been used in the high-energy region and (expensive) numerical results from [4, 5] are only needed in a restricted phase space. A further development of this idea was realized in Ref. [9] where the expansion around the forward-scattering limit [10] has been combined with the high-energy results [8] abandoning completely the need for numerical evaluations. A different formulation of this idea can be found in Ref. [11], where many more expansion terms have been used in the different limits. This leads to precise results in the whole phase space. The results of Ref. [11] are summarized in Section 2.

In Ref. [12, 13] it has been shown that the NLO prediction for Higgs boson pair production suffers from large uncertainties originating from the renormalization scheme for the top quark mass. In order to tame these uncertainties it is necessary to perform a calculation at next-to-next-to-leading order (NNLO). Until recently NNLO effects were basically only available in the large-$m_t$ limit [14–16]. The most advanced calculation has been performed in Ref. [17] where five expansion terms in $1/m_t$ for the box form factors for $gg \to HH$ have been computed. In the meantime first results for other kinematic regions are available: In Ref. [18] the light-fermion contribution to the form factors has been computed for vanishing transverse momentum of the Higgs bosons and in Ref. [19] results valid in the whole phase space for all reducible contributions have been obtained. The results of Refs. [18] and [19] are summarized in Section 3.

## 2. NLO

In Ref. [11] it was shown that for the NLO QCD corrections to $gg \to HH$ it is possible to cover the whole phase space by combining expansions in the forward scattering kinematics and in the high-energy region.

We have implemented the forward scattering expansion by performing a Taylor expansion of the integrand in the Mandelstam variable $t$ up to order $t^5$ and in the external Higgs boson mass $m_H$ up to quartic order. A similar approach has be used before in Ref. [9]. However, fewer expansion




(a) (b)

**Figure 1:** (a) Comparison of Padé-based approximations constructed from different expansion depths (see text) with numerical results obtained using `FIESTA`, for a non-planar master integral with a numerator. (b) $C_A$ contribution to the two-loop form factor $F^{(1)}_{\mathrm{box1}}$ as a function of $\sqrt{s}$ for $p_T = 100$ GeV.

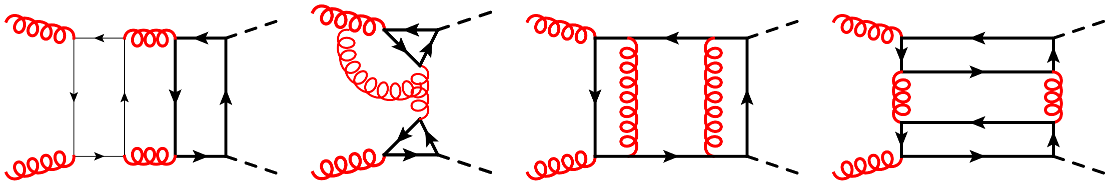
terms were incorporated in the approximation leading to less precise results for the two-loop virtual corrections.

In the high-energy region we have constructed an expansion up to $m_t^{112}$ and up to $m_H^4$. To obtain precise results it is crucial to accompany the deep expansion with a Padé approximation. In our case this leads to precise results, even for quite small values of $p_T$. Furthermore, our approach also provides an estimate of the uncertainty. For details concerning the Padé procedure we refer to Ref. [20].

In Fig. 1 (taken from Ref. [11]) we show results of a non-planar scalar integral (see Fig. 1 of Ref. [11]) as a function of $\sqrt{s}$ for fixed transverse momentum $p_T = 40$ GeV. Let us stress that from the perspective of the high-energy expansion this is a quite small value. Exact results obtained with `FIESTA` [21] are shown as open circles. The greenish line and band correspond to results where an expansion up to $m_t^{32}$ has been used as input. It is interesting to note that the central line is quite close to exact numerical results. Furthermore, the uncertainty estimate is reliable. The orange line and band are obtained using an expansion up to $m_t^{112}$. The band is only visible close to the threshold for top quark pair production. The orange curve reproduces the exact result with impressive accuracy which demonstrates that in our application the Padé approximation is a precision tool.

For the $t$ expansion no Padé improvement is necessary since we observe a fast convergence in the region of phase space where we apply this expansion (see below).

In Fig. 1 we show results for one of the colour structures of the (renormalized and infrared-subtracted) two-loop form factor $F_1$ (see Ref. [11] for a precise definition) for fixed $p_T = 100$ GeV as a function of the partonic center-of-mass energy, $\sqrt{s}$. The red and light blue curves show the real part and the orange and dark blue curves the imaginary part. In both cases we plot the results from the high-energy and the forward scattering approximation. The gray curves show relative differences in percent (see right axis). For the chosen value of $p_T$ one observes an agreement far below the per mille level. Similar results are obtained for a wide range of $p_T$ between 50 GeV $\lesssim p_T \lesssim$ 200 GeV which defines a comfortable overlap region where both approximations are valid. For larger values



(a) (b) (c) (d)

**Figure 2:** Classification of the three-loop virtual corrections to $gg \to HH$.

of $p_T$, i.e. higher energies, the high-energy expansion is supposed to work even better. On the other hand, for smaller $p_T$ we enter deeper into the regime of the forward scattering approximation. Thus, we can cover the whole phase-space based on analytic expansions and without relying on expensive numerical evaluations. The limiting factor of this approach on the final precision is the expansion in $m_H$. Including quartic terms, the corresponding uncertainty can be at the percent level if $p_T$ and $\sqrt{s}$ are small.

## 3. NNLO

The NNLO contribution which is sensitive to the top quark mass scheme used in the NLO prediction are the three-loop virtual corrections. This requires the calculation of triangle and box diagrams with internal and external massive particles. The triangle contributions have been computed in Refs. [22–26] which leaves us with the box diagrams. In Fig. 2 we show a classification of the Feynman diagrams which will be discussed in the following in more detail.

Among the methods used at two loops, in our opinion, the most promising approach for the three-loop calculation is the forward-scattering expansion. Here, one can exploit the fact that the expansion in $t$ is a Taylor expansion.[1] Thus one can expand the integrand before the reduction to master integrals yielding dependence only on the scales $s$ and $m_t^2$. The other methods rely on IBP reductions involving more scales which are currently not feasible.

A straightforward extension of the two-loop calculation of Ref. [11] (cf. Section 2) are the light-fermion contributions, see Fig. 2(a). In Ref. [18] we have performed a proof-of-principle calculation and have computed the form factors for $t = 0$ and $m_H = 0$. At one- and two-loop order this approximation works a the level of 20% or better for $p_T = 100$ GeV. At three-loop order this would be sufficient to reduce the uncertainties from the top quark mass scheme. In Fig. 3 we show the light-fermion contribution of $F_1$ for $t = 0$ and $m_H = 0$.

The findings of Ref. [18] show that the NNLO results can be obtained in the forward scattering approximation with the main bottleneck being the reduction to master integrals. In the meantime we have performed the reduction of all integrals for $t = 0$ and $m_H = 0$ contributing to the class in Fig. 2(c). For the most complicated family the reduction took about 40 days requiring more than 2 terabytes of RAM. In a first step we arrive at more than 30.000 master integrals. For their minimization, which also involves integration-by-parts reductions, we use a refined version of the approach described in Ref. [8]. Here the command `FindRules` implemented in `FIRE` [27] is helpful.

[1] This is not true for the diagram in Fig. 2(d), see below.

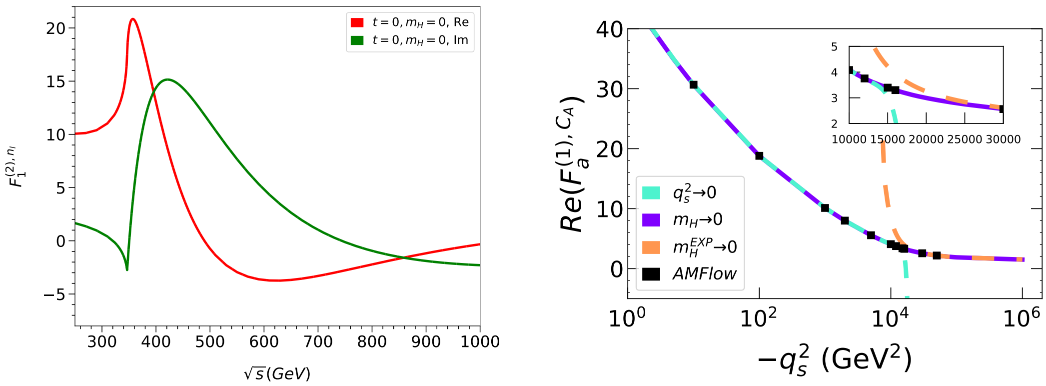



**Figure 3:** Left: Real (red) and imaginary (green) parts of the three-loop light-fermion contributions to the factor $F_1$ as a function of $\sqrt{s}$. Right: Two-loop results to the Higgs-gluon-gluon form factor (see Ref. [19] for a precise definition) with an off-shell gluon with virtuality $q_s^2$.

The approach described above can also be applied to parts of the calculation of the diagrams in Fig. 2(d). This class of diagrams has the additional complication that there are asymptotic contributions beyond the Taylor expansion which need additional investigations.

Results for finite Higgs boson mass which are valid in the whole phase space have been obtained for the class of diagrams in Fig. 2(b) [19]. The calculation factorizes into one- and two-loop form factors with an off-shell external gluon with virtuality $q_s^2$. We have computed this contribution for a finite final-state Higgs boson mass $m_H$ by applying expansions both for small $q_s^2$ and small $m_H$. The right panel in Fig. 3 demonstrates that the expansions (represented by different colours) have sufficient overlap such that the whole physical region can be covered. The contributions in Fig. 2(d) are neither separately finite nor gauge parameter independent. In both cases the contributions from Fig. 2(d) are needed.

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