PROCEEDINGS OF SCIENCE

Precision Calculations in B physics

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We discuss recent higher order calculations to properties of B mesons. This includes next-to-next-to-leading order corrections to nonleptonic and next-to-next-to-next-to-leading order corrections to semileptonic B meson decays. The latter is important in connection to the determination of the CKM matrix element V_{cb}. We also discuss next-to-next-to-leading order corrections to the width difference in the neutral B meson system.

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1. Introduction

In this contribution we discuss three physical quantities from B meson physics where higher order corrections are important: Semileptonic and nonleptonic decays in Sections 2 and 3, respectively, and B meson mixing in Section 4.

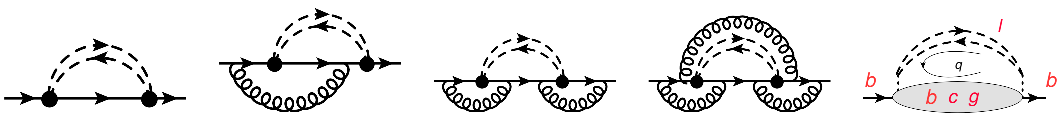
2. Semileptonic B meson decay

Semileptonic B meson decays are important ingredients for the determination of the Cabibbo–Kobayashi–Maskawa matrix elements V_{ub} and V_{cb}. The total decay rate and moments thereof are of particular importance for the inclusive determination of $|V_{cb}|$. It is thus crucial to compute higher order corrections to these quantities in order to move towards a solution of the long-standing tension between the inclusive and exclusive determinations of $|V_{ub}|$ and $|V_{cb}|$.

If Ref. [1] the third-order corrections to the decay rate $b \to c\ell\nu$ have been computed (see also Ref. [2] for partial results). In the following, we briefly outline the main ideas of this computation.

A convenient approach to obtain inclusive quantities is based on the optical theorem. If specified to decay rates one has to compute the imaginary parts of forward-scattering amplitudes of the decaying particle. In our case we have to consider corrections up to five-loop order to the bottom quark two-point function, as can be seen from Fig. 1, where representative diagrams for the leading (LO), next-to-leading (NLO), next-to-next-to-leading (NNLO) and next-to-next-to-next-to-leading order (N^{3}LO) corrections are shown.

To date, the techniques to compute five-loop diagrams which depend on two mass scales (m_c and m_b) are not available. The main idea to nevertheless arrive at precise results is based on expansions in the quantity $\delta = 1 - m_c/m_b$, i.e., around the limit $m_c = m_b$. This approach has been successfully applied at order α_s^2 in Ref. [3]. At order α_s^3 the implementation of this idea consists of the following steps: The starting point are five-loop diagrams, see Fig. 1. One performs the loop-integration involving the charged lepton and the neutrino in $d = 4 - 2\epsilon$ dimensions. This reduces the problem to four loops at the price of introducing a massless propagator with momentum q raised to non-integer power (see Fig. 1(e)). q is an external momentum of the gray blob which represents the remaining three loop integrations over k_i ($i = 1, 2, 3$). At this point, one realizes that all loop-momenta either scale as m_b ("hard") or $\delta \cdot m_b$ ("ultra-soft"). Furthermore, q has to be ultra-soft; otherwise there is no imaginary part. It is possible to determine the dependence of the gray blob on q before actually performing the integrations over k_i. Thus we can integrate over q and remain with three-loop integrals, which can be solved analytically.



(a) (b) (c) (d) (e)

Figure 1: (a) to (d): Representative Feynman diagrams to the semileptonic B meson decay at LO, NLO, NNLO and N^{3}LO. (e): generic diagram.

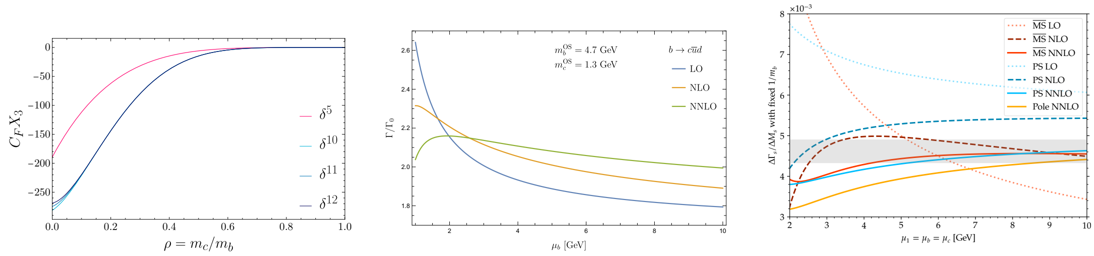


Figure 2: Left: Third-order coefficient of the total semileptonic decay rate of a b quark as a function of m_c/m_b. Middle: The dependence of the rate for $b \to c\bar{u}d$ on the renormalization scale μ_b at LO, NLO and NNLO. Charm and bottom quark masses are renormalized in the on-shell scheme. Right: Renormalization scale dependence of $\Delta\Gamma_s/\Delta M_s$ at LO, NLO and NNLO for the $\overline{\text{MS}}$ and PS scheme. The gray band represents the experimental result.

In Ref. [1] terms up to δ^{12} have been computed. The results are shown in Fig. 2 where the coefficient of order α_s^3 is shown as a function of m_c/m_b. One observes a rapid convergence after including higher order terms in δ which allows for the determination of the third-order coefficient with a precision below the percent level; see Ref. [1] for details.

The results of of Ref. [1] have been used in recent determinations of $|V_{cb}|$ [4–6]. In Ref. [4] it is mentioned that the α_s^3 term leads to a reduction of the uncertainty due to the decay rate by a factor two. Furthermore, it leads to a shift of $|V_{cb}|$ by only 0.6% which demonstrates that the inclusive determination of $|V_{cb}|$ is stable w.r.t. higher order perturbative corrections.

In Ref. [7] the same approach as mentioned above has been applied to obtain results for moments to order α_s^3. Note, however, the no experimental cuts are applied. More recently corrections of order α_s^2 to the leptonic invariant mass spectrum ("q^2 moments") have been computed in Ref. [8]. Recently also the corrections of order α_s^3 to $b \to u\ell\nu$ became available: In Ref. [9] results in the large-N_c limit have been obtained and in Ref. [10] the fermionic contributions have been computed.

3. Nonleptonic B meson decay

Similar to semileptonic B meson decays also the prediction for nonleptonic decays is organized as a double expansion in α_s and Λ_{QCD}/m_b (see, e.g., Ref. [11]). Until recently, the leading term, which describes the partonic decay of the b quark, was only available to NLO [12–14]. As a consequence, the by far dominant contribution to the uncertainty of the theory predictions comes form the variation of the renormalization scale [11]. In Ref. [15] the NNLO correction have been computed[1] for various partonic channels, in particular for $b \to c\bar{u}d$ and $b \to c\bar{c}s$ with one and two massive charm quarks in the final state, respectively.

For the computation the optical theorem has been used, which means that at NNLO the imaginary parts of four-loop diagrams have to be considered. There are contributions with gluon exchanges on the bottom-charm fermion line, which are in analogy to the semileptonic decay. Furthermore, there are QCD corrections in the closed quark-antiquark loop and gluon exchanges between the bottom-charm fermion line and the closed quark-antiquark loop. It is the latter class of

[1]First NNLO results have been obtained in Ref. [16], however, only for massless final-state quarks and without the resummation of $\log(m_W/m_b)$ terms.

diagrams which makes this calculation more involved than in the semileptonic decay. In particular, it is not possible to reduce the number of integrations in a simple way as in the semileptonic case (see Section 2) and thus one has to compute four-loop integrals with two mass scales, m_c and m_b. This requires new techniques, which come with several challenges, see Refs. [15, 17] for details.

One of the challenges is the computation of the Feynman integrals which depend on $\rho = m_c/m_b$. A further challenge of the NLO calculation is the proper choice of operator basis such that we are allowed to apply Fierz relations in $d \neq 4$ dimensions. This concerns in particular the definition of the evanescent operators. More details can be found in Ref. [15].

In Fig. 2 we show the result for the (normalized) decay rate as a function of the renormalization scale μ_b. The blue, orange and green curves represent the LO, NLO and NNLO result. The quark masses are renormalized on-shell. If we estimate the theory uncertainty from the maximum and minimum for $\mu_b \in \{m_b/2, 2m_b\}$ (divided by two), we obtain a reduction from 6.3% at NLO and to 3.5% at NNLO. At the central scale $\mu_b = m_b$ the $O(\alpha_s)$ corrections shift the LO results by 6.5% and the NNLO induce a further shift of less than 3.5%. For a more phenomenological analysis one has to transform the quark masses into short-distance schemes which is planned in future work.

4. *B* meson mixing

The oscillation of a neutral B_q meson (with $q = s, d$) into its anti-particle is induced by the weak interaction. Within the Standard Model such $\Delta B = 2$ transitions are mediated by the exchange of W bosons in box Feynman diagrams. They contain dispersive and absorptive parts which are related to the mass and decay matrices (M^q and Γ^q), which in turn are related to the experimentally accessible quantities as, e.g., $\Delta\Gamma_q/\Delta M_q = -\mathrm{Re}\Gamma_{12}^q/M_{12}^q$. Here, $\Delta\Gamma_q$ and ΔM_q are width and mass differences of the two physical mass eigenstates in the neutral B meson system. In the following we concentrate on $\Delta\Gamma_s$ where precise experimental results exist $\Delta\Gamma_s^{\mathrm{exp}} = (0.082 \pm 0.005)\ \mathrm{ps}^{-1}$ [18].

NLO predictions for $\Delta\Gamma_s$ are available since more than 25 years [19–22]. However, only recently NNLO corrections became available, see [23] and references therein. The theory predictions are based on the construction of two effective theories: In a first step one integrates out the heavy degrees of freedom of the Standard Model and arrives at the so-called $\Delta B = 1$ theory. The decay width is obtained from the absorptive part of a correlator with two $\Delta B = 1$ operator insertions. It is computed by applying the Heavy Quark Expansion. This leads to effective $\Delta B = 2$ operators, which mediate the $B_s - \overline{B}_s$ transition.

At NNLO one has to face several challenges. Some of them are related to the proper choice of the $\Delta B = 2$ operators, in particular their evanescent contributions. Furthermore, there are a number of technical challenges. For example the computation of three-loop Feynman integrals for finite charm and bottom quark masses, traces over products with up to 22 γ matrices or tensor integrals up to rank 11. Their solutions have been discussed at length in a recent publication [24].

In Fig. 2 we show the renormalization scale (μ_1) dependence of the theory prediction of $\Delta\Gamma_s/\Delta M_s$ at LO, NLO and NNLO for two renormalization schemes, $\overline{\mathrm{MS}}$ and PS. One observes that after the inclusion of higher order corrections the curves become flatter. Furthermore, the scheme dependence is significantly reduced and good agreement to the experimental result is observed. The NNLO result is also shown in the pole scheme, which, however, is not adequate to describe $\Delta\Gamma_s/\Delta M_s$.

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