# Virtual QCD corrections to $g g \rightarrow Z Z$ : top-quark loops from a transverse-momentum expansion 



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#### Abstract

We present the virtual corrections due to the top-quark loops for the process $g g \rightarrow Z Z$ at next-to-leading order in QCD. The associated two-loop box diagrams are computed using a small-transverse-momentum expansion. Our results are then merged with those available in the complementary energy region, obtained via a high-energy expansion, in order to provide an analytic result that is valid in the whole phase space. The results presented allow for an efficient modelling of the signal-background interference as well as the irreducible background in off-shell Higgs production.


[^0]
## 1 Introduction

The quest for the determination of the properties of the Higgs boson, after its discovery at the LHC, has been very rewarding. While the main Higgs production channels have been established and new challenging decay modes are being studied (see e.g. refs. [1, 2]), a large variety of measurements have entered the precision domain, requiring improved predictions for Higgs-related processes within the Standard Model (SM) [3, 4]. A significant role in this program is played by the production of a pair of $Z$ bosons, $p p \rightarrow Z Z$, which has been one of the discovery channels of the Higgs and today is important both as a probe of the electroweak (EW) symmetry breaking and for precision Higgs physics [5, 6].

In the theoretical SM prediction of $p p \rightarrow Z Z$ production two partonic sub-processes have to be considered. The first one is quark-antiquark annihilation, $q \bar{q} \rightarrow Z Z$, which gives the largest contribution to the hadronic cross section. The leading-order (LO) amplitude for this channel is related to purely EW tree-level diagrams [7], and corrections through next-to-next-to-leading order (NNLO) in QCD [8-13] and through next-to-leading order (NLO) in the EW theory [14-16] are available. The second partonic contribution, which is the main topic of this paper, comes from the gluon-initiated channel, $g g \rightarrow Z Z$. The gluon-initiated channel at LO is associated to one-loop diagrams, which contribute as an important $\mathcal{O}\left(\alpha_{s}^{2}\right)$ correction, accounting for about $10 \%$ of the hadronic cross section at NNLO for $\sqrt{s}=13 \mathrm{TeV}$ [17].

The one-loop diagrams for $g g \rightarrow Z Z$ at LO have been computed for the first time in refs. [18, 19]. They feature two topologies: the triangles (see fig. 1(a)) are associated to Higgs production via the sub-process $g g \rightarrow H \rightarrow Z Z$, while the box diagrams (fig. 1(b)) are related to the process of non-resonant (a.k.a. continuum) $Z Z$ production, which constitutes an irreducible background in experimental Higgs searches. Moreover, continuum production plays a relevant role in the indirect determination of the Higgs total decay width, $\Gamma_{H}$ [20, 21]. Indeed, in refs. [22, 23] it has been suggested that upper limits on $\Gamma_{H}$ can be obtained from the investigation of the invariant-mass distribution of the two $Z$ bosons system, $M_{Z Z}$, away from the region where the Higgs is produced on shell ${ }^{1}$ [24]. Under the assumption that no beyond-SM physics spoils the known correlation between on- and off-shell amplitudes [25, 26], the latest experimental measurements could exclude, for the first time, deviations in $\Gamma_{H}$ below $\mathcal{O}(100 \%)$ from its SM prediction [27, 28].

In this situation, a good theoretical control over the destructive interference between the Higgs-mediated and continuum amplitudes is crucial. This motivates the inclusion of NLO QCD corrections to the gluon-initiated channel. The NLO corrections to the Higgsmediated diagrams are known exactly, adapting results for the production of a single Higgs with virtuality $M_{z Z}$ [33-35]. Concerning the continuum term, exact analytic results for the related two-loop box integrals have been obtained in the case of loops of massless quarks [36, 37]. We remark that, at the level of the inclusive $g g \rightarrow Z Z$ cross section, the contribution from light quarks is the dominant one, and furthermore it constitutes more than $50 \%$ of the

[^1]$\mathcal{O}\left(\alpha_{s}^{2}\right)$ corrections to $Z Z$ production [10, 17].
The top-quark contribution to continuum $g g \rightarrow Z Z$ starts to become relevant for invariant masses in the range $M_{Z Z}>2 m_{Z}$ and it is expected to be substantial in the region where $M_{Z Z}$ is larger than twice the mass of the top, $m_{t}$. At present, the top-quark contribution is not known in full analytic form, as the scale associated to the mass of the heavy quark complicates the calculation of the two-loop box integrals. Several approximate analytic evaluations of this contribution have been discussed in the literature. The large-mass expansion (LME) has been used in refs. [38 40] to obtain reliable predictions in the region $2 m_{Z}<M_{Z Z} \lesssim 2 m_{t}$. In ref. [40] an improvement of the LME results by means of conformal mapping and Padé approximants has also been studied. A further improvement of the LME via an expansion around the top threshold has been presented in ref. [41]. Analytic predictions that are reliable for large invariant masses have been obtained using the so-called High-Energy (HE) expansion [42]. Still, the latter approach is expected to break down in the region $M_{Z Z} \lesssim 750 \mathrm{GeV}$, and in ref. 42 Padé approximants were used in order to improve the expansion. Finally, exact results based on numerical approaches have been obtained in refs. [43, 44], showing good agreement with ref. 42 in the ranges of small or large invariant masses. Very recently, the numerical results of ref. 43] have been used to compute the full NLO QCD corrections to $g g \rightarrow Z Z$ [45].

In this paper, we present the computation of the contribution from top-quark loops to the virtual NLO QCD corrections to $g g \rightarrow Z Z$. In the calculation we employ an expansion in the transverse momentum of the $Z, p_{T}$, following refs. [46, 47]. Our first goal is to provide an accurate approximation of the virtual QCD corrections that can be valid in the invariant-mass region that so far has not been covered by any of the analytic approaches discussed above, namely the region $350 \lesssim M_{Z Z} \lesssim 750 \mathrm{GeV}$. Since the $p_{T}$ expansion "contains" the LME result, the region of validity of our approach is actually given by $2 m_{Z}<M_{Z Z} \lesssim 750 \mathrm{GeV}$.

Furthermore, it has been previously shown that an expansion in the forward kinematics [48, 49], like the $p_{T}$ expansion, and the HE expansion can be combined in order to approximate the two-loop box amplitudes with good accuracy over the whole phase space. In the second part of the paper, then, we show that this can be done also for $g g \rightarrow Z Z$, and we discuss the combination of our new results with those of ref. 42].

The paper is organized as follows. In section 2 we set the notation and we comment on the application of the $p_{T}$ expansion to the $g g \rightarrow Z Z$ case. The next section is devoted to a comparison between the known LO amplitude and our approximation. In section 4 we then consider the application of the $p_{T}$ expansion at NLO and and discuss how our $p_{T}$-expanded results can be merged with those derived via the HE expansion. Section 5 contains our NLO results, and we present our conclusions in section 6. The paper is complemented by two appendices. In appendix A we report the explicit expressions for the projectors we employ in the calculation. We present also the relation between our form factors and the ones used in ref. [42]. In appendix B we report the exact results for the NLO triangle and the reducible double-triangle contributions.

## 2 Definitions and the $p_{T}$-expansion method

### 2.1 Definitions

We consider the process $g_{a}^{\mu}\left(p_{1}\right) g_{b}^{\nu}\left(p_{2}\right) \rightarrow Z^{\rho}\left(p_{3}\right) Z^{\sigma}\left(p_{4}\right)$. The amplitude can be defined as

$$
\begin{equation*}
\mathcal{A}=\sqrt{2} m_{Z}^{2} G_{F} \frac{\alpha_{s}\left(\mu_{R}\right)}{\pi} \delta_{a b} \epsilon_{\mu}^{a}\left(p_{1}\right) \epsilon_{\nu}^{b}\left(p_{2}\right) \epsilon_{\rho}^{*}\left(p_{3}\right) \epsilon_{\sigma}^{*}\left(p_{4}\right) \hat{\mathcal{A}}^{\mu \nu \rho \sigma}\left(p_{1}, p_{2}, p_{3}\right), \tag{1}
\end{equation*}
$$

where $G_{F}$ is the Fermi constant, $\alpha_{s}\left(\mu_{R}\right)$ is the strong coupling constant evaluated at a renormalisation scale $\mu_{R}$ and the polarization vectors of the gluons and the $Z$ bosons are $\epsilon_{\mu}^{a}\left(p_{1}\right), \epsilon_{\nu}^{b}\left(p_{2}\right)$ and $\epsilon_{\rho}\left(p_{3}\right), \epsilon_{\sigma}\left(p_{4}\right)$, respectively. The Lorentz structure of the amplitude is encoded in the tensor $\hat{\mathcal{A}}^{\mu \nu \rho \sigma}\left(p_{1}, p_{2}, p_{3}\right)$, whose most general decomposition consists of 138 Lorentz structures. However, by imposing the transversality of the external polarization vectors w.r.t. the relative four-momentum

$$
\begin{equation*}
\epsilon\left(p_{i}\right) \cdot p_{i}=0 \quad i=1, \ldots, 4, \tag{2}
\end{equation*}
$$

and by fixing the gauge of the external gluons with

$$
\begin{equation*}
\epsilon\left(p_{1}\right) \cdot p_{2}=0 \quad \epsilon\left(p_{2}\right) \cdot p_{1}=0, \tag{3}
\end{equation*}
$$

$\hat{\mathcal{A}}^{\mu \nu \rho \sigma}$ can be written as a linear combination of 20 Lorentz structures [36, 42, 50]

$$
\begin{equation*}
\hat{\mathcal{A}}^{\mu \nu \rho \sigma}\left(p_{1}, p_{2}, p_{3}\right)=\sum_{i=1}^{20} S_{i}^{\mu \nu \rho \sigma} f_{i}\left(\hat{s}, \hat{t}, \hat{u}, m_{t}, m_{z}\right) \tag{4}
\end{equation*}
$$

where the scalar form factors $f_{i}$ depend, besides $m_{t}$ and $m_{Z}$, on the partonic Mandelstam variables. Assuming all momenta to be incoming, the latter are defined as

$$
\begin{equation*}
\hat{s}=\left(p_{1}+p_{2}\right)^{2}, \quad \hat{t}=\left(p_{1}+p_{3}\right)^{2}, \quad \hat{u}=\left(p_{2}+p_{3}\right)^{2} \tag{5}
\end{equation*}
$$

and the relation $\hat{s}+\hat{t}+\hat{u}=2 m_{Z}^{2}$ is satisfied. We checked that the Lorentz structures that give a nonzero contribution to the amplitude are

$$
\begin{array}{clll}
S_{1}^{\mu \nu \rho \sigma}=g^{\mu \nu} g^{\rho \sigma} & S_{2}^{\mu \nu \rho \sigma}=g^{\mu \rho} g^{\nu \sigma} & S_{3}^{\mu \nu \rho \sigma}=g^{\mu \sigma} g^{\nu \rho} & S_{4}^{\mu \nu \rho \sigma}=p_{1}^{\rho} p_{3}^{\nu} g^{\mu \sigma} \\
S_{5}^{\mu \nu \rho \sigma}=p_{2}^{\rho} p_{3}^{\nu} g^{\mu \sigma} & S_{6}^{\mu \nu \rho \sigma}=p_{1}^{\rho} p_{3}^{\mu} g^{\nu \sigma} & S_{7}^{\mu \rho \sigma}=p_{2}^{\rho} p_{3}^{\mu} g^{\nu \sigma} & S_{8}^{\mu \nu \rho \sigma}=p_{3}^{\mu} p_{3}^{\nu} g^{\rho \sigma} \\
S_{9}^{\mu \nu \rho \sigma}=p_{1}^{\rho} p_{1}^{\sigma} g^{\mu \nu} & S_{10}^{\mu \nu \rho \sigma}=p_{1}^{\rho} p_{2}^{\sigma} g^{\mu \nu} & S_{11}^{\mu \nu \rho \sigma}=p_{1}^{\sigma} p_{2}^{\rho} g^{\mu \nu} & S_{12}^{\mu \nu \rho \sigma}=p_{2}^{\rho} p_{2}^{\sigma} g^{\mu \nu} \\
S_{13}^{\mu \nu \rho \sigma}=p_{1}^{\sigma} p_{3}^{\nu} g^{\mu \rho} & S_{1 \nu}^{\mu \nu \rho \sigma}=p_{2}^{\sigma} p_{3}^{\mu} g^{\mu \rho} & S_{15}^{\mu \nu \rho \sigma}=p_{1}^{\sigma} p_{3}^{\mu} g^{\nu \rho} & S_{1 \nu}^{\mu \nu \rho \sigma}=p_{2}^{\sigma} p_{3}^{\mu} g^{\nu \rho} \\
S_{17}^{\mu \nu \sigma}=p_{1}^{\rho} p_{1}^{\sigma} p_{3}^{\mu} p_{3}^{\nu} & S_{18}^{\mu \nu \rho \sigma}=p_{1}^{\rho} p_{2}^{\sigma} p_{3}^{\mu} p_{3}^{\nu} & S_{19}^{\mu \nu \rho \sigma}=p_{1}^{\sigma} p_{2}^{\rho} p_{3}^{\mu} p_{3}^{\nu} & S_{20}^{\mu \nu \rho \sigma}=p_{2}^{\rho} p_{2}^{\sigma} p_{3}^{\mu} p_{3}^{\nu}
\end{array}
$$

where we follow the numbering of ref. [42].

(a)

(b)

Figure 1: Representative Feynman diagrams contributing to the $g g \rightarrow Z Z$ amplitude at LO. Only the contribution from top-quark loops is shown.

In order to simplify the evaluation of the cross section, in our work we express the amplitude in terms of a set of orthonormal projectors

$$
\begin{equation*}
\hat{\mathcal{A}}^{\mu \nu \rho \sigma}\left(p_{1}, p_{2}, p_{3}\right)=\sum_{i=1}^{20} \mathcal{P}_{i}^{\mu \nu \rho \sigma} \mathcal{A}_{i}\left(\hat{s}, \hat{t}, \hat{u}, m_{t}, m_{z}\right), \tag{7}
\end{equation*}
$$

where the tensors $\mathcal{P}_{i}^{\mu \nu \rho \sigma}$ are constructed as linear combinations of the Lorentz structures defined in eqs. (6), using a Gram-Schmidt procedure for the orthogonalization. In appendix A we give the explicit expressions for the projectors. Here we point out that, to efficiently perform the $p_{T}$ expansion [47, we choose the projectors to be either symmetric or antisymmetric under the interchange $\left\{\mu \leftrightarrow \nu, p_{1} \leftrightarrow p_{2}\right\}$. This choice also allows to reduce the number of relevant form factors ${ }^{2}$ from 20 to 16 . We present our results in terms of the $\mathcal{A}_{i}$ form factors of eq. (7), while in appendix A we include the relations to obtain the latter as a combination of the $f_{i}$ in eqs. (4).

We consider a perturbative expansion of the form factors in the strong coupling

$$
\begin{equation*}
\mathcal{A}_{i}=\mathcal{A}_{i}^{(0)}+\frac{\alpha_{s}}{\pi} \mathcal{A}_{i}^{(1)}+\mathcal{O}\left(\alpha_{s}^{2}\right) \tag{8}
\end{equation*}
$$

where one- and two-loop diagrams contribute respectively to the LO $\left(\mathcal{A}_{i}^{(0)}\right)$ and $\operatorname{NLO}\left(\mathcal{A}_{i}^{(1)}\right)$. According to the topology of the relevant Feynman diagrams (see fig. 11 , we identify a triangle and a box contribution to the LO form factors

$$
\begin{equation*}
\mathcal{A}_{i}^{(0)}=\mathcal{A}_{i}^{(0, \Delta)}+\mathcal{A}_{i}^{(0, \square)} . \tag{9}
\end{equation*}
$$

The above classification is modified at NLO, where the two-loop triangle and box topologies (see fig. 2) are supplemented with one-particle-reducible double-triangle diagrams as in fig. 2(c). Therefore the NLO form factors are defined as

$$
\begin{equation*}
\mathcal{A}_{i}^{(1)}=\mathcal{A}_{i}^{(1, \triangle)}+\mathcal{A}_{i}^{(1, \square)}+\mathcal{A}_{i}^{(1, \bowtie)} . \tag{10}
\end{equation*}
$$

[^2]

Figure 2: Representative Feynman diagrams contributing to the $g g \rightarrow Z Z$ amplitude at NLO. Loops of bottom quarks are included only in the double-triangle diagrams (c).

The main result of this paper is the evaluation of the $\mathcal{A}_{i}^{(1, \square)}$ using the $p_{T}$ expansion, which is described below. For completeness, we provide also the results for $\mathcal{A}_{i}^{(1, \Delta)}$ and $\mathcal{A}_{i}^{(1, \bowtie)}$. We notice that only for $\mathcal{A}_{i}^{(1, \bowtie)}$ we include the contributions from loops of bottom as well as top quarks, since the whole fermion generation must be considered to remove the axial anomaly.

With the previous definitions, the partonic cross section at LO is expressed as

$$
\begin{equation*}
\hat{\sigma}^{(0)}(\hat{s})=\frac{m_{Z}^{4} G_{F}^{2}}{512 \pi \hat{s}^{2}}\left(\frac{\alpha_{s}\left(\mu_{R}\right)}{\pi}\right)^{2} \int_{\hat{t}^{-}}^{\hat{t}^{+}} d \hat{t} \sum_{i}\left|\mathcal{A}_{i}^{(0)}\right|^{2} \tag{11}
\end{equation*}
$$

with $\hat{t}^{ \pm}=1 / 2\left[-\hat{s}+2 m_{Z}^{2} \pm \sqrt{\hat{s}^{2}-\hat{s} 4 m_{Z}^{2}}\right]$.

### 2.2 The $p_{T}$-expansion method

We now briefly recall the main points of the $p_{T}$ expansion. The details of the method are presented in refs. [46, 47]. The Feynman amplitude is expanded in the transverse momentum at the integrand level via the introduction of the vector $r^{\mu}=p_{1}^{\mu}+p_{3}^{\mu}$, which satisfies the relations

$$
\begin{equation*}
r^{2}=\hat{t} \quad r \cdot p_{1}=t^{\prime} \quad r \cdot p_{2}=-t^{\prime} \tag{12}
\end{equation*}
$$

and can be written as

$$
\begin{equation*}
r^{\mu}=\frac{t^{\prime}}{s^{\prime}}\left(p_{2}-p_{1}\right)^{\mu}+r_{\perp}^{\mu}, \tag{13}
\end{equation*}
$$

where the space-like vector $r_{\perp}^{\mu}$ is such that

$$
\begin{equation*}
r_{\perp} \cdot p_{1}=r_{\perp} \cdot p_{2}=0, \quad r_{\perp}^{2}=-p_{T}^{2}, \tag{14}
\end{equation*}
$$

with

$$
\begin{equation*}
p_{T}^{2}=\frac{\hat{t} \hat{u}-m_{Z}^{4}}{\hat{s}} . \tag{15}
\end{equation*}
$$

In eqs. 12 13) we introduce the primed Mandelstam variables

$$
\begin{equation*}
s^{\prime}=p_{1} \cdot p_{2}=\frac{\hat{s}}{2}, \quad t^{\prime}=p_{1} \cdot p_{3}=\frac{\hat{t}-m_{Z}^{2}}{2}, \quad u^{\prime}=p_{2} \cdot p_{3}=\frac{\hat{u}-m_{Z}^{2}}{2}, \tag{16}
\end{equation*}
$$

such that

$$
\begin{equation*}
s^{\prime}+t^{\prime}+u^{\prime}=0, \quad p_{T}^{2}=\frac{2 t^{\prime} u^{\prime}}{s^{\prime}}-m_{Z}^{2} \tag{17}
\end{equation*}
$$

The above relations allow to rewrite $t^{\prime}$ as

$$
\begin{equation*}
t^{\prime}=-\frac{s^{\prime}}{2}\left\{1 \pm \sqrt{1-2 \frac{p_{T}^{2}+m_{Z}^{2}}{s^{\prime}}}\right\} . \tag{18}
\end{equation*}
$$

We are interested in the expansion of the amplitude in the forward limit, corresponding to $t^{\prime} \sim 0$, i.e the minus-sign case in eq. (18). As discussed in refs. [46, 47], the forward expansion is sufficient to obtain the correct result for the cross section, if the (anti-)symmetry of the form factors with respect to the exchange $t^{\prime} \leftrightarrow u^{\prime}$ is ensured. The region of validity of the expansion is given by the condition $p_{T}^{2} /\left(4 m_{t}^{2}\right) \lesssim 1$.

The $p_{T}$ expansion of the relevant Feynman diagrams returns a result in terms of the ratios of small vs large quantities, $x / y$, where $x \in\left\{p_{T}^{2}, m_{z}^{2}\right\}$ and $y \in\left\{\hat{s}, m_{t}^{2}\right\}$. We notice that, in the $p_{T}$ expansion, the transverse momentum and $m_{Z}$ are treated on the same footing, i.e. a term $\mathcal{O}\left(p_{T}^{2} / m_{Z}^{2}\right)$ is assumed to be $\mathcal{O}(1)$, not to be $\mathcal{O}\left(p_{T}^{2}\right)$. Therefore the expansion can be also considered in terms of the quantities $m_{Z}^{2}$ and of the combination $p_{T}^{2}+m_{Z}^{2}$ that enters in $t^{\prime}$ (see eq. 18) ).

After the scalar form factors $\mathcal{A}_{i}$ are expanded in the small parameters above, and after the scalar integrals are decomposed along a basis of master integrals (MI) using Integration-by-Parts (IBP) identities, the form factors can be written as the following series

$$
\begin{equation*}
\mathcal{A}_{i}=\mathcal{N}\left(p_{T}^{2}, m_{Z}^{2}\right) \sum_{N=0}^{\infty} \sum_{i+j=N} c_{i j}\left(p_{T}^{2}\right)^{i}\left(m_{Z}^{2}\right)^{j}, \tag{19}
\end{equation*}
$$

where the $c_{i j}$ coefficients are linear combinations of the MIs resulting from the IBP reduction, which in turn depend only on $\hat{s}$ and $m_{t}^{2}$, while $\mathcal{N}\left(p_{T}^{2}, m_{Z}^{2}\right)$ is an overall normalization factor which may depend on $p_{T}^{2}$ and $m_{Z}^{2}$.


Figure 3: Absolute value of the form factor $\mathcal{A}_{9}^{(0, \square)}$ for moderate (a) and high (b) partonic centre-of-mass energies as a function of the transverse momentum. The exact result and the results obtained at various orders in the $p_{T}$ expansion are shown.

## 3 Validation of the $p_{T}$ expansion at LO

In this section we assess the level of accuracy of the $p_{T}$ expansion in reproducing the known exact result at LO. We used FeynArts [51] to generate the relevant amplitudes, which we expanded in the limit of small $p_{T}$ using private code. The latter relies on several functions implemented in FeynCalc [52, 53]. The expanded amplitude has been decomposed along a basis of MIs with LiteRed [54, 55]. The MIs can be expressed in terms of the $B_{0}$ and $C_{0}$ Passarino-Veltman functions [56] of argument $m_{t}^{2}$ and $\hat{s}$ or $-\hat{s}$. We also used FeynCalc to recompute the exact LO result for the amplitude and found it in agreement ${ }^{3}$ with the one provided in ref. [42. Since at NLO only the box contributions will be computed in an approximate way via the $p_{T}$ expansion, in the following we focus on the results of the box form factors, $\mathcal{A}_{i}^{(0, \square)}$. In particular, we consider the absolute value of the form factor $\mathcal{A}_{9}^{(0, \square)}$ which provides the largest contribution to the cross section. We notice that the $\mathcal{P}_{9}^{\mu \nu \rho \sigma}$ projector, as well as others, exhibits a $\mathcal{O}\left(p_{T}^{-2}\right)$ term (see eq. A.9p) that can question the fact that the corresponding form factor, $\mathcal{A}_{9}^{(0, \square)}$, must be regular in the limit $p_{T} \rightarrow 0$. However, the $\mathcal{O}\left(p_{T}^{-2}\right)$ terms that appear in the projectors are always multiplied by combination of the

[^3]| $M_{Z Z}(\mathrm{GeV})$ | $\hat{\sigma}^{(0)}(\mathrm{fb})$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathcal{O}\left(p_{T}^{0}\right)$ | $\mathcal{O}\left(p_{T}^{2}\right)$ | $\mathcal{O}\left(p_{T}^{4}\right)$ | Exact |
| 250 | 0.12486 | 0.12418 | 0.12412 | 0.12412 |
| 345 | 0.26619 | 0.26460 | 0.26448 | 0.26447 |
| 475 | 0.44887 | 0.44387 | 0.44368 | 0.44367 |
| 659 | 0.43135 | 0.38051 | 0.38644 | 0.38509 |
| 808 | 0.44108 | 0.30622 | 0.34712 | 0.32024 |
| 1255 | 0.70815 | 1.0303 | 0.66139 | 0.1883 |

Table 1: The partonic LO cross section as a function of $M_{Z Z}$. The exact result and the ones obtained at various orders in the $p_{T}$ expansion are shown.
$S_{i}^{\mu \nu \rho \sigma}(i=17,18,19,20)$ Lorentz structures that, once the gauge condition eq. $(3)$ is enforced, exhibit an $r_{\perp}^{\mu} r_{\perp}^{\nu}$ dependence. This dependence ensures that the form factors are regular in the limit $p_{T} \rightarrow 0$.

In fig. 3 we plot the exact result for the $\mathcal{A}_{9}^{(0, \square)}$ form factor, compared to various $p_{T^{-}}$ expanded results for two values of the partonic centre-of-mass energy. The expanded-overexact ratio in the bottom parts of the figure shows that the convergence of the expansion is rather fast and that already the $\mathcal{O}\left(p_{T}^{2}\right)$ result can reproduce the exact one with an accuracy below the percent level. The spikes that are visible in the expanded-over-exact ratio are due to the fact that the form factor is zero near $p_{T} \sim 100 \mathrm{GeV}$, and the ratio is not numerically stable. From a comparison between figs. 3 (a) and 3 (b) we observe that, independently on the value of $\sqrt{\hat{s}}$ the expansion is convergent only for values $p_{T} \lesssim 300 \mathrm{GeV}$. This is consistent with the limit $p_{T}^{2} \lesssim 4 m_{t}^{2}$ that is assumed in the expansion.

In table 1 we present the values of the LO partonic cross section for $g g \rightarrow Z Z$ as a function of the invariant mass of the two $Z$ bosons system. The exact result is compared to various orders in the $p_{T}$ expansion. The triangle contribution (see fig. 11(a)) is evaluated always exactly. The value of the strong coupling is fixed at $\alpha_{s}=0.118$, while we use as other parameters $G_{F}=1.1663786 \cdot 10^{-5} \mathrm{GeV}^{-2}, m_{H}=125.1 \mathrm{GeV}, m_{Z}=91.1876 \mathrm{GeV}, m_{t}=$ 173.21 GeV . The table shows that for $M_{Z Z} \lesssim 500 \mathrm{GeV}$ already the $\mathcal{O}\left(p_{T}^{2}\right)$ term is in agreement with the exact result at the level of less than 1 per mille. Increasing $M_{Z Z}$ up to 650 GeV the agreement between the exact and the $p_{T}$-expanded result starts to deteriorate reaching a $\sim 1$ per cent difference. However, the inclusion of the $\mathcal{O}\left(p_{T}^{4}\right)$ term brings back the difference to a few per mille level. For higher values of $M_{Z Z}$ the most important contributions to the cross section come from regions in the phase space where the $p_{T}$ expansion is not valid and the agreement between the exact and the $p_{T}$-expanded results is poor.

## 4 Virtual corrections at NLO

Having showed that the $p_{T}$ expansion can provide accurate results for the LO contribution, we move now to discuss its application to the NLO QCD corrections, see fig. 2 for representative Feynman diagrams. In fact, we are going to use the $p_{T}$ expansion only for the evaluation of the two-loop box diagrams. For the two-loop one-particle-irreducible triangles we have adapted the full analytic results of ref. [34], while the reducible double triangles have been computed exactly using FeynCalc and checked with the results of ref. 40. With reference to eq. (10), we list the results for the form factors $\mathcal{A}_{i}^{(1, \Delta)}$ and $\mathcal{A}_{i}^{(1, \bowtie)}$ in appendix B .

### 4.1 Box diagrams

The evaluation of the two-loop box diagrams was carried out as follows. We generated the diagrams with FeynArts and performed all the manipulations of the amplitude within Mathematica and FeynCalc. The amplitude was contracted with the 18 projectors in appendix A and each form factor expanded up to $\mathcal{O}\left(p_{T}^{4}\right)$. This led to the identification of a set of 9 families of scalar integrals. These families were analyzed using LiteRed and reduced to a basis of 52 known MIs [33, 34, $57+60$, which we found to be the same as those encoutered in $H H$ and $Z H$ production. Among the 52 MI , fifty can be expressed in terms of (generalised) harmonic polylogarithms while two are elliptic integrals. The evaluation of the (generalised) harmonic polylogarithms was done using the code handyG [61, while the elliptic integrals were evaluated using the routines of ref. 62]. The top quark mass was renormalised in the on-shell scheme and the infra-red (IR) poles were subtracted as in refs. 63, 64].

As a first check of our computation, we compared our $p_{T}$-expanded result with the LME one presented in ref. [42]. It should be noticed that $p_{T}$-expanded result actually "contains" the LME one. The LME differs from the expansion in $p_{T}$ by the fact that $\hat{s}$ is treated as a small parameter with respect to $m_{t}^{2}$, and not on the same footing as in the latter case. This implies that when the $p_{T}$-expanded result is further expanded in terms of the $\hat{s} / m_{t}^{2}$ ratio the LME result has to be recovered. This way, we were able to reproduce, at the analytic level, the LME result of ref. [42] up to the order of our calculation.

As a second check of our computation we compare, in table 2, the contribution of the twoloop box diagrams at $\mathcal{O}\left(p_{T}^{4}\right)$ with the results of ref. [43], which rely on the numerical evaluation of the scalar integrals. The comparison is done at the level of the helicity amplitudes $\int^{4}$ for the fixed phase-space point presented in ref. [43], defined as $\hat{s}=142 / 17 m_{t}^{2}$ and $\hat{t}=-125 / 22 m_{t}^{2} \sim$ $-5.7 m_{t}^{2}$. We remark that the IR subtraction scheme employed in ref. 43] ( $q_{T}$ subtraction) differs from ours, so that to reproduce the form factors of ref. [43], $\mathcal{A}_{i}^{(1) q_{T}}$, a shift in our form factors has to be introduced as follows:

$$
\begin{equation*}
\mathcal{A}_{i}^{(1) q_{T}}=\mathcal{A}_{i}^{(1)}+\frac{C_{A}}{4} \pi^{2} . \tag{20}
\end{equation*}
$$

[^4]| $\lambda_{1} \lambda_{2} \lambda_{3} \lambda_{4}$ | Numerical | $\mathcal{O}\left(p_{T}^{4}\right)$ |
| :---: | :---: | :---: |
| ++++ | $3.15549(8)+\mathrm{i} 0.47235(8)$ | $3.16038+\mathrm{i} 0.46980$ |
| +++- | $0.15950(7)+\mathrm{i} 0.14052(8)$ | $0.16305+\mathrm{i} 0.13885$ |
| +-+- | $-0.38609(7)+\mathrm{i} 0.10539(7)$ | $-0.38617+\mathrm{i} 0.11085$ |
| -+++ | $-0.46990(8)+\mathrm{i} 0.40207(8)$ | $-0.46956+\mathrm{i} 0.40506$ |
| +++0 | $1.1248(2)-\mathrm{i} 0.0805(2)$ | $1.1256-\mathrm{i} 0.0811$ |
| +-+0 | $-1.4803(2)+\mathrm{i} 0.4940(2)$ | $-1.4799+\mathrm{i} 0.4977$ |
| ++00 | $17.2585(6)+\mathrm{i} 29.5669(6)$ | $17.2602+\mathrm{i} 29.5643$ |
| +-00 | $10.2869(5)-\mathrm{i} 1.0571(6)$ | $10.2840-\mathrm{i} 1.0640$ |

Table 2: Comparison of the NLO helicity amplitudes between the exact numerical values and the values given by the $p_{T}$ expansion at $\mathcal{O}\left(p_{T}^{4}\right)$ in the kinematic point provided in ref. 43].

We notice that the point chosen in ref. 43 lies somewhat beyond the region of validity of the $p_{T}$ expansion. Yet, table 2 shows that the relative difference between the $\mathcal{O}\left(p_{T}^{4}\right)$ values and the numerical ones is, in general with the exception of only one helicity amplitude, at the per mille level and often better, as in the case of the amplitudes involving two longitudinallypolarized $Z$ bosons. The latter amplitudes are the dominant ones, as consequence of the Goldstone Boson Equivalence Theorem.

### 4.2 Merging the $p_{T}$ and High-Energy expansions

The results we have presented so far allow to efficiently approximate the two-loop box integrals with full top-mass dependence over a specific region of the phase space, which spans the $Z Z$ production threshold up to moderate partonic centre-of-mass energies (for the LHC). In the high-energy region the $p_{T}$ expansion is still accurate for any values of $\hat{s}$ and $|\hat{t}| \lesssim 4 m_{t}^{2}$ or $|\hat{u}| \lesssim 4 m_{t}^{2}$ but it cannot cover the complementary region $|\hat{t}|,|\hat{u}| \gtrsim 4 m_{t}^{2}$ that becomes allowed by the kinematics.

In ref. [48] we showed that the $p_{T}$ expansion and the HE expansion are accurate in complementary phase-space regions. The HE expansion is an expansion in terms of the ratios $x / y$ where $x \in\left\{m_{t}^{2}, m_{z}^{2}\right\}$ and $y \in\{\hat{s}, \hat{t}, \hat{u}\}$ and it is valid in phase-space regions where both $|\hat{t}| /\left(4 m_{t}^{2}\right) \gtrsim 1$ and $|\hat{u}| /\left(4 m_{t}^{2}\right) \gtrsim 1$. We also showed that if each expansion is extended beyond its border of validity via the use of Padé approximants the two results can be merged into a single prediction that is accurate over the complete phase space.

Results for the evaluation of the two-loop box contribution in the $g g \rightarrow Z Z$ process via the HE expansion were presented in ref. [42]. Applying the method of ref. [48] we merge our results for the box contribution with those obtained in ref. [42]. Below we recall the main steps of the merging method:

1. We use the formulas in eqs. $(\overline{\mathrm{A} .21)}-(\sqrt{\mathrm{A} .38)}$ to adapt the form factors of ref. 42 to our decomposition of the amplitude.
2. We rewrite the small quantities on which the $p_{T}$ and the HE expansions are based in terms of a scaling parameter $x$.
3. We construct Padé approximants for each form factor, defined as

$$
\begin{equation*}
[m / n](x)=\frac{p_{0}+p_{1} x+\cdots+p_{m} x^{m}}{1+q_{1} x+\ldots q_{n} x^{n}} \tag{21}
\end{equation*}
$$

where $p_{i}, q_{i}$ are expressed as linear combinations of the coefficients of each expansion. We refer to these approximants as the $p_{T}$-Padé and HE-Padé.
4. In evaluating the amplitude, we use the HE-Padé approximant for phase-space points such that $|\hat{t}|>4 m_{t}^{2}$ and $|\hat{u}|>4 m_{t}^{2}$ whereas for all the remaining phase-space points we use the $p_{T}$-Padé approximant.

With our three terms in the $p_{T}$ expansion we constructed a $[1 / 1] p_{T}$-Padé. The several terms in the HE expansion presented in ref. [42] allow to construct different [x/y] HE-Padé. We construct both the [5/5] and [6/6] HE-Padé and compare the results using the different orders. As a result we found that both the [5/5] and [6/6] HE-Padé give very similar results.

## 5 Results

Before applying the merging approach for the form factors at NLO, we verified its reliability testing it against the exact LO contribution. In fig. 4(a) the exact result (solid black line) for the form factor, $\mathcal{A}_{9}^{(0, \square)}$ as a function of $-\hat{t} /\left(4 m_{t}^{2}\right)$ is compared to different approximate evaluations. In the figure the fixed order results $\mathcal{O}\left(p_{T}^{4}\right)$ in the $p_{T}$ expansion (solid blue) and $\mathcal{O}\left(m_{t}^{32}\right)$ in the HE expansion (solid orange) are shown against the Padé-improved versions, [1/1] $p_{T}$-Padé (dashed light blue) and [6/6] HE-Padé (dashed yellow). The bottom panel shows the ratio of the different approximations over the exact result. It is evident that the region $|\hat{t}| \sim 4 m_{t}^{2}$, where both the $p_{T}$ and HE expansions (solid blue and orange) begin to diverge, is well covered by the respective Padé approximants, so that one has a way to accurately reproduce the exact result for any value of $|\hat{t}|$. In fig. 4 (b) we compare different predictions at the level of the LO partonic cross section. One can see that the accuracy of the merged result (dashed green) is at the level of per mille or below, an order of magnitude better than what observed in ref. [48]. We notice that, this remarkable degree of accuracy is not achieved for each individual form factor. However, the impact of larger deviations is suppressed with the integration over $\hat{t}$.

We now present our results at NLO. First, we checked their the validity in two ways:

- We compared the $p_{T}$-Padé and HE-Padé in the vicinity of $|\hat{t}| \sim 4 m_{t}^{2}$, for several values of $\hat{s}$ in the range [800, 2000] GeV , ensuring that the two analytic approximations are indeed complementary and that the matching point is chosen well. In fig. 5 the NLO


Figure 4: (a) Absolute value of a form factor for a fixed $\sqrt{\hat{s}}$ using the $p_{T}$ and the HE expansion and the best Padé approximants. (b) Partonic cross section at LO using the exact result, the $p_{T}$ expansion and the prediction using the merged $p_{T}$ and HE Padé approximants.
form factor $\mathcal{A}_{9}^{(1, \boxed{\square})}$ is shown as a function of $-\hat{t} /\left(4 m_{t}^{2}\right)$ for both fixed-order $p_{T}$ and HE expansions and Padé-improved results. The lower panel of the figure shows the relative difference, $\Delta$, between the $p_{T}$-Padé and the HE-Padé, defined as

$$
\begin{equation*}
\Delta=2\left|\frac{\mid 1 / 1]_{p_{T}}-[6 / 6]_{\mathrm{HE}}}{[1 / 1]_{p_{T}}+[6 / 6]_{\mathrm{HE}}}\right|, \tag{22}
\end{equation*}
$$

where one can see that at $|\hat{t}| \sim 4 m_{t}^{2}$ the two expansion are in good agreement (on the level of 0.5 per mille).

- We compared our merged results with the helicity amplitudes of ref. [43], obtained from a numerical evaluation of the scalar integrals. The results for several phase-space points were provided privately by the authors of ref. [43]. In fig. 6] we plot the results for the amplitude ++00 using different colors depending on the level of agreement $\Delta$ between the numerical and merged prediction. The shaded region in the plots corresponds to the high-energy region, where $|\hat{t}|>4 m_{t}^{2}$ and $|\hat{u}|>4 m_{t}^{2}$. In fig. 6(a) we compare the numerical evaluation with our results using only the $[1 / 1] p_{T}$-Padé. Whereas the agreement is generally better than per mille in the region of validity of the $p_{T}$ expansion, deviations larger than $5 \%$ are visible in the high-energy region for the real part. When we consider the merged prediction, the agreement with the numerical evaluation is improved, and the majority of the phase space points have differences below the per mille level. It must be noted that, as seen in table 2, this level of agreement is not shared among all the


Figure 5: NLO form factor for $\sqrt{\hat{s}}=1200 \mathrm{GeV}$ in various approximations: $p_{T}$ expansion (solid blue), $[1 / 1]$ Padé approximant based on the $p_{T}$ expansion (dashed, light blue), highenergy expansion (solid, pink) and [6/6] HE Padé approximant (dashed, rosa). The lower panel shows the relative difference $\Delta$ (see text).
helicity amplitudes. This is however a consequence of the relations used to convert our results in terms of form factors into helicity amplitudes. Indeed, for some phase-space points, we observed delicate cancellations in the combination of the form factors. At the same time, for these particular phase-space points, the corresponding contributions to the helicity amplitudes are tiny. Therefore, we expect an overall accuarcy at the subpercent level for our results.

In fig. 7 one can see our result for the finite part of the virtual corrections defined as

$$
\begin{equation*}
\Delta \sigma_{\mathrm{virt}}=\int_{\hat{t}^{-}}^{\hat{t}^{+}} d \hat{t} \frac{1}{2} \frac{1}{16 \pi \hat{s}^{2}}\left(\frac{\alpha_{s}\left(\mu_{R}\right)}{\pi}\right) \mathcal{V}_{\mathrm{fin}}(\hat{t}) \tag{23}
\end{equation*}
$$

where the finite part of the virtual correction is

$$
\begin{equation*}
\mathcal{V}_{\mathrm{fin}}=\frac{G_{F}^{2} m_{Z}^{4}}{16}\left(\frac{\alpha_{s}\left(\mu_{R}\right)}{\pi}\right)^{2}\left\{\sum_{i}\left|\mathcal{A}_{i}^{(0)}\right|^{2} \frac{C_{A}}{2}\left(\pi^{2}-\log ^{2}\left(\frac{\mu_{R}^{2}}{\hat{s}}\right)\right)+2 \sum_{i} \operatorname{Re}\left[\mathcal{A}_{i}^{(0)}\left(\mathcal{A}_{i}^{(1)}\right)^{*}\right]\right\} \tag{24}
\end{equation*}
$$

We used $\mu_{R}=M_{Z Z} / 2$ as renormalisation scale. The pink line shows the merged result of a $[1 / 1] p_{T}$-expanded Padé approximant and a [6/6] HE Padé approximant. For comparison we


Figure 6: Relative difference between several phase-space points of ref. 43] and different approximations for the helicity amplitude ++00 . Points in the shaded region are outside the formal limit of validity of the $p_{T}$ expansion. (a) [1/1] $p_{T}$-Padé only; (b) merging of the $p_{T}$-Padé and the HE-Padé.


Figure 7: The partonic virtual corrections as a function of $M_{Z Z}$ merging the $p_{T}$ expansion and the HE (pink solid line) compared to the virtual corrections obtained when using the large mass expanded results for the NLO box and triangles (dashed blue line).
show the large mass expanded results of ref. [42]. Within the range of validity of the large mass expansion the results agree very well. We have checked that using a [5/5] HE Padé approximant would change the result only below the integration error.

## 6 Conclusion

The accurate description of the process $g g \rightarrow Z Z$ is needed in order to improve the extraction of the total width of the Higgs boson, as well as a precision test for the SM. In this paper we have presented the calculation of the top-quark loops in the virtual QCD corrections at NLO. Our main result is the computation of the box diagrams using the $p_{T}$ expansion, which gives accurate results in a phase-space region that so far has not been covered by other analytic approximations.

As a by-product of our calculation, we have used the results obtained in ref. [42] to complement our analytic calculation, showing that a combination of the two approaches can provide an efficient approximation for the cross section, at a level of accuracy that is more than adequate for phenomenological applications. We emphasise that our analytic approach is very flexible and can be included into Monte Carlo programmes, as was already done for the case of Higgs pair production, $g g \rightarrow H H$ [67]. It allows to change easily the input parameters, which
is for instance necessary to evaluate the top-mass renormalisation scheme uncertainty that is expected to be large in the top-mediated contributions. Indeed, it is currently the largest uncertainty for the $g g \rightarrow H H$ [65, 66] and $g g \rightarrow Z H$ [68] processes. In addition, the flexibility of the approach allows for a straightforward application to beyond-the-SM scenarios.

We recall that for a full description of $g g \rightarrow Z Z$ at NLO in QCD also the contribution mediated by loops of light quarks needs to be considered [36, 37, as well as the one from real-emission diagrams [17]. We leave the combination of these effects with our results to future work.

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## A Projectors

We present the explicit expressions of the orthonormal projectors $\mathcal{P}_{i}^{\mu \nu \rho \sigma}$ appearing in eq. (7). Following the notation of section 2 , the antisymmetric projectors under the exchange $\{\mu \leftrightarrow \nu$ , $\left.p_{1} \leftrightarrow p_{2}\right\}$ are

$$
\begin{align*}
\mathcal{P}_{1}^{\mu \nu \rho \sigma} & =\frac{m_{Z}}{\sqrt{2} p_{T}}\left[\frac{1}{p_{T}^{2} s^{\prime}}\left(S_{17}^{\mu \nu \rho \sigma}-S_{20}^{\mu \nu \rho \sigma}\right)\right]  \tag{A.1}\\
\mathcal{P}_{2}^{\mu \nu \rho \sigma} & =\frac{m_{Z}}{\sqrt{2} p_{T}}\left[\frac{1}{p_{T}^{2} s^{\prime}}\left(\frac{t^{\prime}}{u^{\prime}} S_{19}^{\mu \nu \rho \sigma}-\frac{u^{\prime}}{t^{\prime}} S_{18}^{\mu \nu \rho \sigma}\right)\right]  \tag{A.2}\\
\mathcal{P}_{3}^{\mu \nu \rho \sigma} & =\frac{m_{Z}}{\sqrt{2} p_{T}}\left[\frac{S_{5}^{\mu \nu \rho \sigma}}{u^{\prime}}-\frac{S_{6}^{\mu \nu \rho \sigma}}{t^{\prime}}-\frac{1}{p_{T}^{2} s^{\prime}}\left(S_{17}^{\mu \nu \rho \sigma}-S_{20}^{\mu \nu \rho \sigma}+\frac{u^{\prime}}{t^{\prime}} S_{18}^{\mu \nu \rho \sigma}-\frac{t^{\prime}}{u^{\prime}} S_{19}^{\mu \nu \rho \sigma}\right)\right]  \tag{A.3}\\
\mathcal{P}_{4}^{\mu \nu \rho \sigma} & =\frac{m_{Z}}{\sqrt{2} p_{T}}\left[\frac{1}{s^{\prime}}\left(S_{9}^{\mu \nu \rho \sigma}-S_{12}^{\mu \nu \rho \sigma}\right)+\frac{1}{p_{T}^{2} s^{\prime}}\left(S_{17}^{\mu \nu \rho \sigma}-S_{20}^{\mu \nu \rho \sigma}\right)\right]  \tag{A.4}\\
\mathcal{P}_{5}^{\mu \nu \rho \sigma} & =\frac{m_{Z}}{\sqrt{2} p_{T}}\left[\frac{1}{s^{\prime}}\left(\frac{t^{\prime}}{u^{\prime}} S_{11}^{\mu \nu \rho \sigma}-\frac{u^{\prime}}{t^{\prime}} S_{10}^{\mu \nu \rho \sigma}\right)+\frac{1}{p_{T}^{2} s^{\prime}}\left(\frac{t^{\prime}}{u^{\prime}} S_{19}^{\mu \nu \rho \sigma}-\frac{u^{\prime}}{t^{\prime}} S_{18}^{\mu \nu \rho \sigma}\right)\right]  \tag{A.5}\\
\mathcal{P}_{6}^{\mu \nu \rho \sigma} & =\frac{m_{Z}}{\sqrt{2} p_{T}}\left[\frac{S_{13}^{\mu \nu \rho \sigma}}{u^{\prime}}-\frac{S_{16}^{\mu \nu \rho \sigma}}{t^{\prime}}-\frac{1}{p_{T}^{2} s^{\prime}}\left(S_{17}^{\mu \nu \rho \sigma}-S_{20}^{\mu \nu \rho \sigma}+\frac{t^{\prime}}{u^{\prime}} S_{19}^{\mu \nu \rho \sigma}-\frac{u^{\prime}}{t^{\prime}} S_{18}^{\mu \nu \rho \sigma}\right)\right]  \tag{A.6}\\
\mathcal{P}_{7}^{\mu \nu \rho \sigma} & =\frac{m_{Z}^{2}}{\sqrt{2} p_{T}^{2}}\left[\frac{\left(m_{Z}^{2}-p_{T}^{2}\right)}{2 m_{Z}^{2}}\left(\frac{S_{13}^{\mu \nu \rho \sigma}}{u^{\prime}}-\frac{S_{16}^{\mu \nu \rho \sigma}}{t^{\prime}}\right)+\frac{1}{m_{Z}^{2} s^{\prime}}\left(u^{\prime} S_{14}^{\mu \nu \rho \sigma}-t^{\prime} S_{15}^{\mu \nu \rho \sigma}\right)\right.  \tag{A.7}\\
& \left.+\frac{1}{m_{Z}^{2} s^{\prime}}\left(S_{17}^{\mu \nu \rho \sigma}-S_{20}^{\mu \nu \rho \sigma}+\frac{t^{\prime}}{u^{\prime}} S_{19}^{\mu \nu \rho \sigma}-\frac{u^{\prime}}{t^{\prime}} S_{18}^{\mu \nu \rho \sigma}\right)\right] \\
\mathcal{P}_{8}^{\mu \nu \rho \sigma} & =\frac{m_{Z}^{2}}{\sqrt{2} p_{T}^{2}}\left[\frac{1}{m_{Z}^{2} s^{\prime}}\left(u^{\prime} S_{4}^{\mu \nu \rho \sigma}-t^{\prime} S_{7}^{\mu \nu \rho \sigma}\right)+\frac{\left(m_{Z}^{2}-p_{T}^{2}\right)}{2 m_{Z}^{2}}\left(\frac{S_{5}^{\mu \nu \rho \sigma}}{u^{\prime}}-\frac{S_{6}^{\mu \nu \rho \sigma}}{t^{\prime}}\right)\right.  \tag{A.8}\\
& \left.+\frac{1}{m_{Z}^{2} s^{\prime}}\left(S_{17}^{\mu \nu \rho \sigma}-S_{20}^{\mu \nu \rho \sigma}+\frac{u^{\prime}}{t^{\prime}} S_{18}^{\mu \nu \rho \sigma}-\frac{t^{\prime}}{u^{\prime}} S_{19}^{\mu \nu \rho \sigma}\right)\right] .
\end{align*}
$$

The symmetric projectors are

$$
\begin{align*}
& \mathcal{P}_{9}^{\mu \nu \rho \sigma}=\frac{m_{Z}^{2}}{\sqrt{p_{T}^{4}+m_{Z}^{4}}}\left[\frac{1}{p_{T}^{2} s^{\prime}}\left(S_{17}^{\mu \nu \rho \sigma}+S_{20}^{\mu \nu \rho \sigma}\right)\right]  \tag{A.9}\\
& \mathcal{P}_{10}^{\mu \nu \rho \sigma}=\frac{\sqrt{p_{T}^{4}+m_{Z}^{4}}}{2 p_{T}^{2}}\left[\frac{\left(m_{Z}^{4}-p_{T}^{4}\right)}{\left(m_{Z}^{4}+p_{T}^{4}\right) p_{T}^{2} s^{\prime}}\left(S_{17}^{\mu \nu \rho \sigma}+S_{20}^{\mu \nu \rho \sigma}\right)+\frac{1}{p_{T}^{2} s^{\prime}}\left(\frac{u^{\prime}}{t^{\prime}} S_{18}^{\mu \nu \rho \sigma}+\frac{t^{\prime}}{u^{\prime}} S_{19}^{\mu \nu \rho \sigma}\right)\right]  \tag{A.10}\\
& \mathcal{P}_{11}^{\mu \nu \rho \sigma}=\frac{m_{Z}^{2}}{\sqrt{p_{T}^{4}+m_{Z}^{4}}}\left[\frac{1}{s^{\prime}}\left(S_{9}^{\mu \nu \rho \sigma}+S_{12}^{\mu \nu \rho \sigma}\right)+\frac{1}{p_{T}^{2} s^{\prime}}\left(S_{17}^{\mu \nu \rho \sigma}+S_{20}^{\mu \nu \rho \sigma}\right)\right]  \tag{A.11}\\
& \mathcal{P}_{12}^{\mu \nu \rho \sigma}=\frac{\sqrt{p_{T}^{4}+m_{Z}^{4}}}{2 p_{T}^{2}}\left[\frac{\left(m_{Z}^{4}-p_{T}^{4}\right)}{\left(m_{Z}^{4}+p_{T}^{4}\right) s^{\prime}}\left(S_{9}^{\mu \nu \rho \sigma}+S_{12}^{\mu \nu \rho \sigma}\right)+\frac{1}{s^{\prime}}\left(\frac{u^{\prime}}{t^{\prime}} S_{10}^{\mu \nu \rho \sigma}+\frac{t^{\prime}}{u^{\prime}} S_{11}^{\mu \nu \rho \sigma}\right)\right.  \tag{A.12}\\
& \left.+\frac{\left(m_{Z}^{4}-p_{T}^{4}\right)}{\left(m_{Z}^{4}+p_{T}^{4}\right) p_{T}^{2} s^{\prime}}\left(S_{17}^{\mu \nu \rho \sigma}+S_{20}^{\mu \nu \rho \sigma}\right)+\frac{1}{p_{T}^{2} s^{\prime}}\left(\frac{u^{\prime}}{t^{\prime}} S_{18}^{\mu \nu \rho \sigma}+\frac{t^{\prime}}{u^{\prime}} S_{19}^{\mu \nu \rho \sigma}\right)\right] \\
& \mathcal{P}_{13}^{\mu \nu \rho \sigma}=\frac{m_{Z}}{\sqrt{2} p_{T}}\left[\frac{S_{13}^{\mu \nu \rho \sigma}}{u^{\prime}}+\frac{S_{16}^{\mu \nu \rho \sigma}}{t^{\prime}}-\frac{1}{p_{T}^{2} s^{\prime}}\left(S_{17}^{\mu \nu \rho \sigma}+S_{20}^{\mu \nu \rho \sigma}\right)-\frac{1}{p_{T}^{2} s^{\prime}}\left(\frac{u^{\prime}}{t^{\prime}} S_{18}^{\mu \nu \rho \sigma}+\frac{t^{\prime}}{u^{\prime}} S_{19}^{\mu \nu \rho \sigma}\right)\right]  \tag{A.13}\\
& \mathcal{P}_{14}^{\mu \nu \rho \sigma}=\frac{m_{Z}^{2}}{\sqrt{2} p_{T}^{2}}\left[\frac{\left(m_{Z}^{2}-p_{T}^{2}\right)}{2 m_{Z}^{2}}\left(\frac{S_{13}^{\mu \nu \rho \sigma}}{u^{\prime}}+\frac{S_{16}^{\mu \nu \rho \sigma}}{t^{\prime}}\right)+\frac{1}{m_{Z}^{2} s^{\prime}}\left(u^{\prime} S_{14}^{\mu \nu \rho \sigma}+t^{\prime} S_{15}^{\mu \nu \rho \sigma}\right)\right.  \tag{A.14}\\
& \left.-\frac{1}{p_{T}^{2} s^{\prime}}\left(S_{17}^{\mu \nu \rho \sigma}+S_{20}^{\mu \nu \rho \sigma}+\frac{u^{\prime}}{t^{\prime}} S_{18}^{\mu \nu \rho \sigma}+\frac{t^{\prime}}{u^{\prime}} S_{19}^{\mu \nu \rho \sigma}\right)\right] \\
& \mathcal{P}_{15}^{\mu \nu \rho \sigma}=\frac{\sqrt{2} m_{Z}^{3}}{p_{T}\left(p_{T}^{2}+m_{Z}^{2}\right)}\left[\frac{1}{m_{Z}^{2} s^{\prime}}\left(u^{\prime} S_{4}^{\mu \nu \rho \sigma}+t^{\prime} S_{7}^{\mu \nu \rho \sigma}\right)\right.  \tag{A.15}\\
& \left.+\frac{\left(m_{Z}^{2}+p_{T}^{2}\right)}{2 m_{Z}^{2} p_{T}^{2} s^{\prime}}\left(S_{17}^{\mu \nu \rho \sigma}+S_{20}^{\mu \nu \rho \sigma}+\frac{u^{\prime}}{t^{\prime}} S_{18}^{\mu \nu \rho \sigma}+\frac{t^{\prime}}{u^{\prime}} S_{19}^{\mu \nu \rho \sigma}\right)\right] \\
& \mathcal{P}_{16}^{\mu \nu \rho \sigma}=\frac{p_{T}^{2}+m_{Z}^{2}}{2 \sqrt{2} p_{T}^{2}}\left[\frac{2 m_{Z}^{2}}{s^{\prime} p_{T}^{2}\left(p_{T}^{2}+m_{Z}^{2}\right)}\left(S_{17}^{\mu \nu \rho \sigma}+S_{20}^{\mu \nu \rho \sigma}+\frac{u^{\prime}}{t^{\prime}} S_{18}^{\mu \nu \rho \sigma}+\frac{t^{\prime}}{u^{\prime}} S_{19}^{\mu \nu \rho \sigma}\right)\right.  \tag{A.16}\\
& \left.-\frac{2\left(p_{T}^{2}-m_{Z}^{2}\right)}{\left(m_{Z}^{2}+p_{T}^{2}\right)^{2} s^{\prime}}\left(u^{\prime} S_{4}^{\mu \nu \rho \sigma}+t^{\prime} S_{7}^{\mu \nu \rho \sigma}\right)+\frac{S_{5}^{\mu \nu \rho \sigma}}{u^{\prime}}+\frac{S_{6}^{\mu \nu \rho \sigma}}{t^{\prime}}\right] \\
& \mathcal{P}_{17}^{\mu \nu \rho \sigma}=\frac{p_{T}^{2}+2 m_{Z}^{2}}{2 m_{Z}^{2}}\left\{\frac{m_{Z}^{2}}{2 m_{Z}^{2}+p_{T}^{2}}\left(S_{2}^{\mu \nu \rho \sigma}+S_{3}^{\mu \nu \rho \sigma}\right)-\frac{m_{Z}^{2}}{\left(p_{T}^{2}+2 m_{Z}^{2}\right) s^{\prime} p_{T}^{2}}\left(u^{\prime} S_{4}^{\mu \nu \rho \sigma}+t^{\prime} S_{7}^{\mu \nu \rho \sigma}\right)\right.  \tag{A.17}\\
& -\frac{\left(m_{Z}^{2}+p_{T}^{2}\right) m_{Z}^{2}}{p_{T}^{2}\left(2 m_{Z}^{2}+p_{T}^{2}\right)}\left[\frac{S_{5}^{\mu \nu \rho \sigma}}{2 u^{\prime}}+\frac{S_{6}^{\mu \nu \rho \sigma}}{2 t^{\prime}}-\frac{S_{13}^{\mu \nu \rho \sigma}}{2 u^{\prime}}-\frac{S_{16}^{\mu \nu \rho \sigma}}{2 t^{\prime}}+\frac{1}{p_{T}^{2} s^{\prime}}\left(S_{17}^{\mu \nu \rho \sigma}+S_{20}^{\mu \nu \rho \sigma}\right.\right. \\
& \left.\left.\left.+\frac{u^{\prime}}{t^{\prime}} S_{18}^{\mu \nu \rho \sigma}+\frac{t^{\prime}}{u^{\prime}} S_{19}^{\mu \nu \rho \sigma}\right)\right]+\frac{m_{Z}^{218}}{p_{T}^{2} s^{\prime}\left(2 m_{Z}^{2}+p_{T}^{2}\right)}\left(u^{\prime} S_{14}^{\mu \nu \rho \sigma}+t^{\prime} S_{15}^{\mu \nu \rho \sigma}\right)\right\}
\end{align*}
$$

$$
\begin{align*}
\mathcal{P}_{18}^{\mu \nu \rho \sigma} & =\frac{S_{8}^{\mu \nu \rho \sigma}}{p_{T}^{2}}-\frac{\left(m_{Z}^{2}+p_{T}^{2}\right)}{2 p_{T}^{4} s^{\prime}}\left(S_{17}^{\mu \nu \rho \sigma}+S_{20}^{\mu \nu \rho \sigma}\right)+\frac{u^{\prime}\left(-m_{Z}^{4}+p_{T}^{4}-2 p_{T}^{2}\left(s^{\prime}-t^{\prime}+u^{\prime}\right)\right) S_{18}^{\mu \nu \rho \sigma}}{2 p_{T}^{4}\left(m_{Z}^{2}+p_{T}^{2}\right) s^{\prime} t^{\prime}}  \tag{A.18}\\
& -\frac{t^{\prime}\left(m_{Z}^{4}-p_{T}^{4}+2 p_{T}^{2}\left(s^{\prime}+t^{\prime}-u^{\prime}\right)\right) S_{19}^{\mu \nu \rho \sigma}}{2 p_{T}^{4}\left(m_{Z}^{2}+p_{T}^{2}\right) s^{\prime} u^{\prime}} .
\end{align*}
$$

Finally, we include the last two projectors, which have null norm in $D=4$ dimensions and thus do not contribute to the amplitude.

$$
\begin{align*}
\mathcal{P}_{19}^{\mu \nu \rho \sigma} & =S_{1}^{\mu \nu \rho \sigma}-\frac{S_{2}^{\mu \nu \rho \sigma}}{2}-\frac{S_{3}^{\mu \nu \rho \sigma}}{2}+\frac{1}{2 p_{T}^{2} s^{\prime}}\left(u^{\prime} S_{4}^{\mu \nu \rho \sigma}+t^{\prime} S_{7}^{\mu \nu \rho \sigma}\right)+\frac{\left(m_{Z}^{2}+p_{T}^{2}\right)}{4 p_{T}^{2}}\left(\frac{S_{5}^{\mu \nu \rho \sigma}}{u^{\prime}}+\frac{S_{6}^{\mu \nu \rho \sigma}}{t^{\prime}}\right) \\
& +\frac{S_{8}^{\mu \nu \rho \sigma}}{p_{T}^{2}}-\frac{\left(m_{Z}^{2}+p_{T}^{2}\right)}{2 p_{T}^{2} s^{\prime}}\left(S_{9}^{\mu \nu \rho \sigma}+S_{12}^{\mu \nu \rho \sigma}\right)+\frac{u^{\prime}\left(-m_{Z}^{4}+p_{T}^{4}-2 p_{T}^{2}\left(s^{\prime}-t^{\prime}+u^{\prime}\right)\right) S_{10}^{\mu \nu \rho \sigma}}{2 p_{T}^{2}\left(m_{Z}^{2}+p_{T}^{2}\right) s^{\prime} t^{\prime}}  \tag{A.19}\\
& -\frac{t^{\prime}\left(m_{Z}^{4}-p_{T}^{4}+2 p_{T}^{2}\left(s^{\prime}+t^{\prime}-u^{\prime}\right)\right) S_{11}^{\mu \nu \rho \sigma}}{2 p_{T}^{2}\left(m_{Z}^{2}+p_{T}^{2}\right) s^{\prime} u^{\prime}}-\frac{\left(m_{Z}^{2}+p_{T}^{2}\right)}{4 p_{T}^{2}}\left(\frac{S_{13}^{\mu \nu \rho \sigma}}{u^{\prime}}+\frac{S_{16}^{\mu \nu \rho \sigma}}{t^{\prime}}\right) \\
& -\frac{1}{2 p_{T}^{2} s^{\prime}}\left(u^{\prime} S_{14}^{\mu \nu \rho \sigma}+t^{\prime} S_{15}^{\mu \nu \rho \sigma}\right)+\frac{\left(m_{Z}^{2}+p_{T}^{2}-s^{\prime}+t^{\prime}-u^{\prime}\right) u^{\prime} S_{18}^{\mu \nu \rho \sigma}}{p_{T}^{2}\left(m_{Z}^{2}+p_{T}^{2}\right) s^{\prime} t^{\prime}} \\
& +\frac{t^{\prime}\left(m_{Z}^{2}+p_{T}^{2}-s^{\prime}-t^{\prime}+u^{\prime}\right) S_{19}^{\mu \nu \rho \sigma}}{p_{T}^{2}\left(m_{Z}^{2}+p_{T}^{2}\right) s^{\prime} u^{\prime}} \\
\mathcal{P}_{20}^{\mu \nu \rho \sigma} & =\frac{1}{2 m_{Z}^{2}+p_{T}^{2}}\left[S_{3}^{\mu \nu \rho \sigma}-S_{2}^{\mu \nu \rho \sigma}-\frac{1}{p_{T}^{2} s^{\prime}}\left(u^{\prime} S_{4}^{\mu \nu \rho \sigma}-t^{\prime} S_{7}^{\mu \nu \rho \sigma}+u^{\prime} S_{14}^{\mu \nu \rho \sigma}-t^{\prime} S_{15}^{\mu \nu \rho \sigma}\right) \quad\right. \text { (A.20) }  \tag{A.20}\\
& \left.-\frac{\left(m_{Z}^{2}+p_{T}^{2}\right)}{2 p_{T}^{2}}\left(\frac{S_{5}^{\mu \nu \rho \sigma}}{u^{\prime}}-\frac{S_{6}^{\mu \nu \rho \sigma}}{t^{\prime}}+\frac{S_{13}^{\mu \nu \rho \sigma}}{u^{\prime}}-\frac{S_{16}^{\mu \nu \rho \sigma}}{t^{\prime}}\right)\right]
\end{align*}
$$

The form factors $\mathcal{A}_{i}$ associated to the above projectors can be expressed in terms of the form factors $f_{i}$ defined in eq. (2.8) of ref. [42] via the following relations

$$
\begin{align*}
& \mathcal{A}_{1}=0  \tag{A.21}\\
& \mathcal{A}_{2}=\frac{p_{T}^{2}}{m_{Z}^{2}}\left[\frac{2\left(t^{\prime}-u^{\prime}\right)}{\left(m_{Z}^{2}+p_{T}^{2}\right)}\left(p_{T}^{2} f_{8}-f_{1}\right)+s^{\prime} p_{T}^{2}\left(\frac{u^{\prime}}{t^{\prime}} f_{19}-\frac{t^{\prime}}{u^{\prime}} f_{18}\right)\right.  \tag{A.22}\\
& \left.+2 t^{\prime} f_{4}-2 u^{\prime} f_{5}+2 t^{\prime} f_{6}-2 u^{\prime} f_{7}+\frac{s^{\prime} t^{\prime}}{u^{\prime}} f_{10}-\frac{s^{\prime} u^{\prime}}{t^{\prime}} f_{11}\right] \\
& \mathcal{A}_{3}=\frac{p_{T}^{2}}{m_{Z}^{2}}\left[f_{3}-f_{2}+\frac{\left(m_{Z}^{2}-p_{T}^{2}\right) s^{\prime}}{2}\left(\frac{f_{7}}{t^{\prime}}-\frac{f_{4}}{u^{\prime}}\right)+u^{\prime} f_{5}-t^{\prime} f_{6}\right]  \tag{A.23}\\
& \mathcal{A}_{4}=0  \tag{A.24}\\
& \mathcal{A}_{5}=\frac{p_{T}^{2}}{m_{Z}^{2}}\left[\frac{2\left(t^{\prime}-u^{\prime}\right)}{\left(m_{Z}^{2}+p_{T}^{2}\right)} f_{1}+\frac{s^{\prime} u^{\prime}}{t^{\prime}} f_{11}-\frac{s^{\prime} t^{\prime}}{u^{\prime}} f_{10}\right]  \tag{A.25}\\
& \mathcal{A}_{6}=\frac{p_{T}^{2}}{m_{Z}^{2}}\left[f_{3}-f_{2}+t^{\prime} f_{4}-u^{\prime} f_{7}+\frac{\left(m_{Z}^{2}-p_{T}^{2}\right) s^{\prime}}{2}\left(\frac{f_{6}}{u^{\prime}}-\frac{f_{5}}{u^{\prime}}\right)\right]  \tag{A.26}\\
& \mathcal{A}_{7}=\frac{p_{T}^{2}}{m_{Z}^{2}}\left[f_{3}-f_{2}+\frac{\left(3 s^{\prime} m_{Z}^{2}+p_{T}^{2} s^{\prime}-2\left(u^{\prime} m_{Z}^{2}+t^{\prime}\left(m_{Z}^{2}+u^{\prime}\right)\right)\right) p_{T}^{2}}{4 m_{Z}^{2}}\left(\frac{f_{5}}{t^{\prime}}-\frac{f_{6}}{u^{\prime}}\right)\right]  \tag{A.27}\\
& \mathcal{A}_{8}=\frac{p_{T}^{2}}{m_{Z}^{2}}\left[f_{3}-f_{2} \frac{\left(3 s^{\prime} m_{Z}^{2}+p_{T}^{2} s^{\prime}-2\left(u^{\prime} m_{Z}^{2}+t^{\prime}\left(m_{Z}^{2}+u^{\prime}\right)\right)\right) p_{T}^{2}}{4 m_{Z}^{2}}\left(\frac{f_{4}}{u^{\prime}}-\frac{f_{7}}{t^{\prime}}\right)\right]  \tag{A.28}\\
& \mathcal{A}_{9}=\frac{1}{m_{Z}^{4}}\left[-2 p_{T}^{4}\left(t^{\prime} f_{4}+u^{\prime} f_{5}+t^{\prime} f_{6}+u^{\prime} f_{7}\right)-\left(m_{Z}^{2}+p_{T}^{2}\right) p_{T}^{2}\left(f_{2}+f_{3}\right)\right.  \tag{A.29}\\
& -\frac{\left(s^{\prime}\left(m_{Z}^{2}+p_{T}^{2}\right)\left(m_{Z}^{2}-s^{\prime}\right)+2 m_{Z}^{2}\left(t^{\prime 2}+u^{\prime 2}\right)\right)}{s^{\prime}}\left(p_{T}^{2} f_{8}-f_{1}\right)-s^{\prime}\left(m_{Z}^{4}+p_{T}^{4}\right)\left(f_{9}-p_{T}^{2} f_{20}\right) \\
& \left.+\frac{2\left(u^{\prime} m_{Z}^{2}+t^{\prime}\left(m_{Z}^{2}+u^{\prime}\right)\right)}{s^{\prime}}\left(p_{T}^{2}\left(t^{\prime 2} f_{18}+u^{\prime 2} f_{19}\right)-\left(t^{\prime 2} f_{10}+u^{\prime 2} f_{11}\right)\right)\right] \\
& \mathcal{A}_{10}=\frac{2 p_{T}^{2}}{\left(m_{Z}^{4}+p_{T}^{4}\right)}\left[s^{\prime} p_{T}^{2}\left(p_{T}^{2}\left(\frac{t^{\prime}}{u^{\prime}} f_{18}+\frac{u^{\prime}}{t^{\prime}} f_{19}\right)-\left(\frac{t^{\prime}}{u^{\prime}} f_{10}+\frac{u^{\prime}}{t^{\prime}} f_{11}\right)\right)-2 p_{T}^{2}\left(t^{\prime} f_{4}+u^{\prime} f_{5}+t^{\prime} f_{6}+u^{\prime} f_{7}\right)\right.  \tag{A.30}\\
& \left.+\frac{\left(\left(s^{\prime 2}-t^{\prime 2}-u^{\prime 2}\right) m_{Z}^{2}+p_{T}^{2}\left(s^{\prime 2}+t^{\prime 2}+u^{\prime 2}\right)\right)}{\left(m_{Z}^{2}+p_{T}^{2}\right) s^{\prime}}\left(f_{8} p_{T}^{2}-f_{1}\right)-\left(m_{Z}^{2}+p_{T}^{2}\right)\left(f_{2}+f_{3}\right)\right] \\
& \mathcal{A}_{11}=\frac{1}{m_{Z}^{4}}\left[s^{\prime}\left(m_{Z}^{4}+p_{T}^{4}\right) f_{9}+\frac{2\left(u^{\prime} m_{Z}^{2}+t^{\prime}\left(m_{Z}^{2}+u^{\prime}\right)\right)}{s^{\prime}}\left(t^{\prime 2} f_{10}+u^{\prime 2} f_{11}\right)\right.  \tag{A.31}\\
& \left.-\frac{\left(2 u^{\prime 2} m_{Z}^{2}+s^{\prime}\left(m_{Z}^{2}+u^{\prime}\right)\left(s^{\prime} m_{Z}^{2}+2 t^{\prime 2}+p_{T}^{2} s^{\prime}\right)\right)}{s^{\prime}} f_{1}\right]
\end{align*}
$$

$$
\begin{align*}
& \mathcal{A}_{12}=\frac{2 p_{T}^{2}}{\left(m_{Z}^{4}+p_{T}^{4}\right)}\left[\frac{\left(\left(s^{\prime 2}-t^{\prime 2}-u^{\prime 2}\right) m_{Z}^{2}+p_{T}^{2}\left(s^{\prime 2}+t^{\prime 2}+u^{\prime 2}\right)\right)}{\left(m_{Z}^{2}+p_{T}^{2}\right) s^{\prime}} f_{1}+p_{T}^{2} s^{\prime}\left(\frac{t^{\prime}}{u^{\prime}} f_{10}+\frac{u^{\prime}}{t^{\prime}} f_{11}\right)\right]  \tag{A.32}\\
& \mathcal{A}_{13}=\frac{p_{T}^{2}}{m_{Z}^{2}}\left[\frac{\left(m_{Z}^{2}-p_{T}^{2}\right) s^{\prime}}{2}\left(\frac{f_{5}}{t^{\prime}}+\frac{f_{6}}{u^{\prime}}\right)-\left(f_{2}+f_{3}+t^{\prime} f_{4}+u^{\prime} f_{7}\right)\right]  \tag{A.33}\\
& \mathcal{A}_{14}=-\frac{p_{T}^{2}}{m_{Z}^{2}}\left[f_{2}+f_{3}+p_{T}^{2} s^{\prime}\left(\frac{f_{5}}{t^{\prime}}+\frac{f_{6}}{u^{\prime}}\right)\right]  \tag{A.34}\\
& \mathcal{A}_{15}=\frac{p_{T}^{2}}{2 m_{Z}^{4}}\left[\left(m_{Z}^{2}+p_{T}^{2}\right)\left(f_{2}+f_{3}+t^{\prime} f_{4}+u^{\prime} f_{7}\right)-\left(m_{Z}^{2}-p_{T}^{2}\right)\left(u^{\prime} f_{5}+t^{\prime} f_{6}\right)\right]  \tag{A.35}\\
& \mathcal{A}_{16}=\frac{2 p_{T}^{2}}{m_{Z}^{2}+p_{T}^{2}}\left[f_{2}+f_{3}+\frac{2 p_{T}^{2}}{m_{Z}^{2}+p_{T}^{2}}\left(u^{\prime} f_{5}+t^{\prime} f_{6}\right)\right]  \tag{A.36}\\
& \mathcal{A}_{17}=\frac{2 m_{Z}^{2}}{2 m_{Z}^{2}+p_{T}^{2}}\left[\frac{\left(s^{\prime 2}\left(t^{\prime 2}+u^{\prime 2}\right)-\left(t^{\prime 2}-u^{\prime 2}\right)^{2}\right)}{s^{\prime 3}\left(m_{Z}^{2}+p_{T}^{2}\right)} f_{1}+f_{2}+f_{3}\right]  \tag{A.37}\\
& \mathcal{A}_{18}=\frac{\left(m_{Z}^{2}\left(2 s^{\prime 2}-t^{\prime 2}-u^{\prime 2}\right)+s^{\prime}\left(s^{\prime}\left(p_{T}^{2}-s^{\prime}\right)+t^{\prime 2}+u^{\prime 2}\right)\right)}{m_{Z}^{2} s^{\prime}\left(m_{Z}^{2}+p_{T}^{2}\right)}\left(p_{T}^{2} f_{8}-f_{1}\right) \tag{A.38}
\end{align*}
$$

## B Analytical Results

Results for Double-Triangle Diagrams With reference to eq. (10), we present here the exact results for the double-triangle contributions to the $\mathcal{A}_{i}^{(1, \bowtie)}$ with $i=1, \ldots, 18$. We keep the dependence of the final result on the mass of the bottom quark, $m_{b}$. We find

$$
\begin{align*}
& \mathcal{A}_{1}^{(1, \bowtie)}=0  \tag{B.1}\\
& \mathcal{A}_{2}^{(1, \bowtie)}=\frac{p_{T}\left(\left(m_{Z}^{2}+2 t^{\prime}\right) \Delta\left(t^{\prime}\right)^{2}-\left(t^{\prime} \leftrightarrow u^{\prime}\right)\right)}{32 \sqrt{2} m_{Z}\left(m_{Z}^{2}+p_{T}^{2}\right)}  \tag{B.2}\\
& \mathcal{A}_{3}^{(1, \bowtie)}=-\frac{p_{T}^{3}\left(u^{\prime} \Delta\left(t^{\prime}\right)^{2}-\left(t^{\prime} \leftrightarrow u^{\prime}\right)\right)}{16 \sqrt{2} m_{Z}\left(m_{Z}^{2}+p_{T}^{2}\right)^{2}}  \tag{B.3}\\
& \mathcal{A}_{4}^{(1, \bowtie)}=0  \tag{B.4}\\
& \mathcal{A}_{5}^{(1, \bowtie)}=-\frac{p_{T}\left(t^{\prime} \Delta\left(t^{\prime}\right)^{2}-\left(t^{\prime} \leftrightarrow u^{\prime}\right)\right)}{16 \sqrt{2} m_{Z}\left(m_{Z}^{2}+p_{T}^{2}\right)}  \tag{B.5}\\
& \mathcal{A}_{6}^{(1, \bowtie)}=-\frac{p_{T}\left(\left(m_{Z}^{2}+2 t^{\prime}\right) u^{\prime 2} \Delta\left(t^{\prime}\right)^{2}-\left(t^{\prime} \leftrightarrow u^{\prime}\right)\right)}{16 \sqrt{2} m_{Z}\left(m_{Z}^{2}+p_{T}^{2}\right)^{2} s^{\prime}}  \tag{B.6}\\
& \mathcal{A}_{7}^{(1, \bowtie)}=\frac{\left(s^{\prime} p_{T}^{2}+t^{\prime 2}\right) u^{\prime 2} \Delta\left(t^{\prime}\right)^{2}-\left(t^{\prime} \leftrightarrow u^{\prime}\right)}{16 \sqrt{2}\left(m_{Z}^{2}+p_{T}^{2}\right)^{2} s^{\prime 2}}  \tag{B.7}\\
& \mathcal{A}_{8}^{(1, \bowtie)}=\frac{u^{\prime}\left(t^{\prime}\left(m_{Z}^{2}+p_{T}^{2}\right)-2 s^{\prime} p_{T}^{2}\right) \Delta\left(t^{\prime}\right)^{2}-\left(t^{\prime} \leftrightarrow u^{\prime}\right)}{32 \sqrt{2}\left(m_{Z}^{2}+p_{T}^{2}\right)^{2} s^{\prime}}  \tag{B.8}\\
& \mathcal{A}_{9}^{(1, \bowtie)}=\frac{\left(m_{Z}^{2}-p_{T}^{2}\right)\left(\left(m_{Z}^{2}+2 t^{\prime}\right) \Delta\left(t^{\prime}\right)^{2}+\left(t^{\prime} \leftrightarrow u^{\prime}\right)\right)}{64 m_{Z}^{2} \sqrt{m_{Z}^{4}+p_{T}^{4}}}  \tag{B.9}\\
& \mathcal{A}_{10}^{(1, \bowtie)}=-\frac{p_{T}^{2}\left(\left(m_{Z}^{2}+2 t^{\prime}\right) \Delta\left(t^{\prime}\right)^{2}+\left(t^{\prime} \leftrightarrow u^{\prime}\right)\right)}{32\left(m_{Z}^{2}+p_{T}^{2}\right) \sqrt{m_{Z}^{4}+p_{T}^{4}}}  \tag{B.10}\\
& \mathcal{A}_{11}^{(1, \bowtie)}=\frac{u^{\prime}\left(m_{Z}^{2}+2 t^{\prime}\right)\left(2 s^{\prime} m_{Z}^{2}+\left(m_{Z}^{2}+p_{T}^{2}\right)\left(t^{\prime}+2 u^{\prime}\right)\right) \Delta\left(t^{\prime}\right)^{2}+\left(t^{\prime} \leftrightarrow u^{\prime}\right)}{32 m_{Z}^{2}\left(m_{Z}^{2}+p_{T}^{2}\right) \sqrt{m_{Z}^{4}+p_{T}^{4}} s^{\prime}}  \tag{B.11}\\
& \mathcal{A}_{12}^{(1, \bowtie)}=-\frac{\left(s^{\prime} m_{Z}^{4}+4 p_{T}^{2} s^{\prime} m_{Z}^{2}+3 p_{T}^{4} s^{\prime}+4 p_{T}^{2}\left(t^{\prime 2}+u^{\prime 2}\right)\right)\left(\Delta\left(t^{\prime}\right)^{2}+\Delta\left(u^{\prime}\right)^{2}\right)}{64\left(m_{Z}^{2}+p_{T}^{2}\right) \sqrt{m_{Z}^{4}+p_{T}^{4}} s^{\prime}}  \tag{B.12}\\
& \mathcal{A}_{13}^{(1, \bowtie)}=\frac{p_{T}\left(\left(m_{Z}^{2}+2 t^{\prime}\right) u^{\prime 2} \Delta\left(t^{\prime}\right)^{2}+\left(t^{\prime} \leftrightarrow u^{\prime}\right)\right)}{16 \sqrt{2} m_{Z}\left(m_{Z}^{2}+p_{T}^{2}\right)^{2} s^{\prime}} \tag{B.13}
\end{align*}
$$

$$
\begin{align*}
& \mathcal{A}_{14}^{(1, \bowtie)}=-\frac{\left(s^{\prime} p_{T}^{2}+t^{\prime 2}\right) u^{\prime 2} \Delta\left(t^{\prime}\right)^{2}+\left(t^{\prime} \leftrightarrow u^{\prime}\right)}{64 \sqrt{2}\left(m_{Z}^{2}+p_{T}^{2}\right)^{2} s^{\prime 2}}  \tag{B.14}\\
& \mathcal{A}_{15}^{(1, \bowtie)}=-\frac{p_{T}\left(\left(m_{Z}^{2}+2 t^{\prime}\right) u^{\prime 2} \Delta\left(t^{\prime}\right)^{2}+\left(t^{\prime} \leftrightarrow u^{\prime}\right)\right)}{16 \sqrt{2} m_{Z}\left(m_{Z}^{2}+p_{T}^{2}\right)^{2} s^{\prime}}  \tag{B.15}\\
& \mathcal{A}_{16}^{(1, \bowtie)}=\frac{\left(s^{\prime} p_{T}^{2}+t^{\prime 2}\right) u^{\prime 2} \Delta\left(t^{\prime}\right)^{2}+\left(t^{\prime} \leftrightarrow u^{\prime}\right)}{16 \sqrt{2}\left(m_{Z}^{2}+p_{T}^{2}\right)^{2} s^{\prime 2}}  \tag{B.16}\\
& \mathcal{A}_{17}^{(1, \bowtie)}=0  \tag{B.17}\\
& \mathcal{A}_{18}^{(1, \bowtie)}=\frac{1}{64}\left(\left(\frac{s^{\prime} p_{T}^{2}}{t^{\prime 2}}+1\right) \Delta\left(t^{\prime}\right)^{2}+\left(t^{\prime} \leftrightarrow u^{\prime}\right)\right) \tag{B.18}
\end{align*}
$$

where

$$
\begin{equation*}
\Delta(x)=F\left(x, m_{t}\right)-F\left(x, m_{b}\right) \tag{B.19}
\end{equation*}
$$

and

$$
\begin{align*}
F(x, M) & =\frac{m_{Z}^{2}}{x}\left[B_{0}\left(2 x+m_{Z}^{2}, M^{2}, M^{2}\right)-B_{0}\left(m_{Z}^{2}, M^{2}, M^{2}\right)\right]  \tag{B.20}\\
& +4 M^{2} C_{0}\left(0, m_{Z}^{2}, 2 x+m_{Z}^{2}, M^{2}, M^{2}, M^{2}\right)+2
\end{align*}
$$

Results for Triangle Diagrams at NLO With reference to eq. 10), we present here the exact results for the two-loop one-particle-irreducible triangle contributions to the NLO form factors. We obtain

$$
\begin{align*}
& \mathcal{A}_{1}^{(1, \triangle)}=0  \tag{B.21}\\
& \mathcal{A}_{2}^{(1, \triangle)}=\frac{p_{T}^{2}(\hat{t}-\hat{u})}{m_{Z}^{2}\left(p_{T}^{2}+m_{Z}^{2}\right)} \frac{\hat{s}}{\hat{s}-m_{H}^{2}}\left(C_{F} \mathcal{F}_{1 / 2}^{(2 l)}+C_{A} \mathcal{G}_{1 / 2}^{(2 l, C A)}\right)  \tag{B.22}\\
& \mathcal{A}_{3}^{(1, \triangle)}=0  \tag{B.23}\\
& \mathcal{A}_{4}^{(1, \triangle)}=0  \tag{B.24}\\
& \mathcal{A}_{5}^{(1, \triangle)}=-\frac{p_{T}^{2}(\hat{t}-\hat{u})}{m_{Z}^{2}\left(p_{T}^{2}+m_{Z}^{2}\right)} \frac{\hat{s}}{\hat{s}-m_{H}^{2}}\left(C_{F} \mathcal{F}_{1 / 2}^{(2 l)}+C_{A} \mathcal{G}_{1 / 2}^{(2 l, C A)}\right)  \tag{B.25}\\
& \mathcal{A}_{6}^{(1, \triangle)}=0  \tag{B.26}\\
& \mathcal{A}_{7}^{(1, \triangle)}=0  \tag{B.27}\\
& \mathcal{A}_{8}^{(1, \triangle)}=0  \tag{B.28}\\
& \mathcal{A}_{9}^{(1, \triangle)}=\frac{\left(\hat{s}\left(p_{T}^{2}-m_{Z}^{2}\right)+2 m_{Z}^{2}\left(p_{T}^{2}+m_{Z}^{2}\right)\right.}{2 m_{Z}^{4}} \frac{\hat{s}}{\hat{s}-m_{H}^{2}}\left(C_{F} \mathcal{F}_{1 / 2}^{(2 l)}+C_{A} \mathcal{G}_{1 / 2}^{(2 l, C A)}\right)  \tag{B.29}\\
& \mathcal{A}_{10}^{(1, \triangle)}=\frac{2 p_{T}^{2}\left(p_{T}^{2} \hat{s}+m_{Z}^{4}-p_{T}^{4}\right)}{\left(p_{T}^{2}+m_{Z}^{2}\right)\left(p_{T}^{4}+m_{Z}^{4}\right)} \frac{\hat{s}-m_{H}^{2}}{\hat{s}}\left(C_{F} \mathcal{F}_{1 / 2}^{(2 l)}+C_{A} \mathcal{G}_{1 / 2}^{(2 l, C A)}\right)  \tag{B.30}\\
& \mathcal{A}_{11}^{(1, \triangle)}=-\frac{\left(\hat{s}\left(p_{T}^{2}-m_{Z}^{2}\right)+2 m_{Z}^{2}\left(p_{T}^{2}+m_{Z}^{2}\right)\right.}{2 m_{Z}^{4}} \frac{\hat{s}}{\hat{s}-m_{H}^{2}}\left(C_{F} \mathcal{F}_{1 / 2}^{(2 l)}+C_{A} \mathcal{G}_{1 / 2}^{(2 l, C A)}\right)  \tag{B.31}\\
& \mathcal{A}_{12}^{(1, \triangle)}=-\frac{2 p_{T}^{2}\left(p_{T}^{2} \hat{s}+m_{Z}^{4}-p_{T}^{4}\right)}{\left(p_{T}^{2}+m_{Z}^{2}\right)\left(p_{T}^{4}+m_{Z}^{4}\right)} \frac{\hat{s}}{\hat{s}-m_{H}^{2}}\left(C_{F} \mathcal{F}_{1 / 2}^{(2 l)}+C_{A} \mathcal{G}_{1 / 2}^{(2 l, C A)}\right)  \tag{B.32}\\
& \mathcal{A}_{13}^{(1, \triangle)}=0  \tag{B.33}\\
& \mathcal{A}_{14}^{(1, \triangle)}=0  \tag{B.34}\\
& \mathcal{A}_{15}^{(1, \triangle)}=0  \tag{B.35}\\
& \mathcal{A}_{16}^{(1, \triangle)}=0  \tag{B.36}\\
& \mathcal{A}_{17}^{(1, \triangle)}=-\frac{2 m_{Z}^{2}}{p_{T}^{2}+2 m_{Z}^{2}} \frac{\hat{s}-m_{H}^{2}}{\hat{s}}\left(C_{F} \mathcal{F}_{1 / 2}^{(2 l)}+C_{A} \mathcal{G}_{1 / 2}^{(2 l, C A)}\right)  \tag{B.37}\\
& \mathcal{A}_{18}^{(1, \triangle)}=\frac{\hat{s}}{\hat{s}-m_{H}^{2}}\left(C_{F} \mathcal{F}_{1 / 2}^{(2 l)}+C_{A} \mathcal{G}_{1 / 2}^{(2 l, C A)}\right)  \tag{B.38}\\
& \mathcal{S}^{2}
\end{align*}
$$

where the functions $\mathcal{F}_{1 / 2}^{(2 l)}$ and $\mathcal{G}_{1 / 2}^{(2 l, C A)}$ are defined in eqs. (2.11) and (3.8) in ref. [34.

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[^1]:    ${ }^{1}$ Off-shell Higgs production can also provide an excellent probe of new physics, for instance of light-quark Yukawa couplings [29], modified trilinear Higgs self-coupling [30] or Higgs portal models [31, 32]

[^2]:    ${ }^{2}$ In fact, we observed that enforcing the additional Bose symmetry $\left\{\rho \leftrightarrow \sigma, p_{3} \leftrightarrow p_{4}\right\}$ further reduces the relevant form factors to 12 . While this may be used for improving the practical implementation of our results, in this paper we use the 16 form factors.

[^3]:    ${ }^{3}$ The comparison has been done for each form factor, using the conversion formulas listed in eqs. A.21A.38 in appendix A

[^4]:    ${ }^{4}$ The helicities of the polarization vectors are defined as: $\epsilon_{\mu}^{\lambda_{1}}\left(p_{1}\right) \epsilon_{\nu}^{\lambda_{2}}\left(p_{2}\right) \epsilon_{\rho}^{* \lambda_{3}}\left(p_{3}\right) \epsilon_{\sigma}^{* \lambda_{4}}\left(p_{4}\right)$.

