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Toward a general nested soft-collinear subtraction method for NNLO calculations

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In this proceeding we report on the recent study [1] performed in the context of nested soft-collinear subtraction [2]. The scheme is designed to provide a fully differential and analytic subtraction method for infrared singularities at next-to-next-to-leading order (NNLO) in QCD. At variant with respect to the first formulation of the nested soft-collinear subtraction, Ref. [1] aims at organising the subtraction procedure in a way that allows for a drastic reduction of the complexity related to the proliferation of the subtraction terms, and at dealing with arbitrary final states. In this document we highlight a crucial aspect of the analysis of Ref. [1]: the idea of rearranging NNLO subtraction terms as iterations of few universal structures identified at NLO.

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PoS(EPS-HEP2023)269

1. Introduction and next-to-leading order calculation

Reliable comparisons between theoretical predictions and experimental data require that the former include corrections up to (at least) next-to-next-to-leading order (NNLO) in perturbative QCD. One issue that plagues theoretical calculations beyond leading order (LO) is the rise of infrared (IR) singularities that stem from integrating over loop momenta of virtual partons, and over the phase space of real radiation. The cancellation of these singularities is ensured by well-known theorems. However, the problem of extracting IR singularities while preserving the differential nature of the calculation remains a challenge at NNLO and beyond. While a formula valid for arbitrary production processes at lepton colliders has recently become available [3], such a formula is still elusive for hadronic colliders, although recent work in this direction has been presented in Ref. [4, 5]. Here we report on the study performed in Ref. [1], which provides the first steps towards such a general formulation in the context of the nested soft-collinear subtraction scheme [2]. This scheme has already been successfully applied to compute NNLO QCD and mixed QCD-EW corrections to a variety of processes [6–13], suggesting that it can accommodate multi-particle final states. In the following we will discuss one of the key aspects of the procedure presented in Ref. [1], namely how one can express NNLO counterterms as iterations of universal functions arising at NLO. We will focus on the quark-anti-quark annihilation into an arbitrary number N of gluons, and a generic colorless set of particles X, i.e. $1_a + 2_b \to X + N\, g$, where $a, b \in \{q, \bar{q}\}$.

We begin by expressing the NLO cross section $\mathrm{d}\hat{\sigma}^{\mathrm{NLO}}_{ab}$ in terms of the universal operators that will appear also in the NNLO calculation. The cross section $\mathrm{d}\hat{\sigma}^{\mathrm{NLO}}_{ab}$ receives contribution from a real, a virtual and a PDF collinear renormalisation contribution[1]

$$\mathrm{d}\hat{\sigma}^{\mathrm{NLO}}_{ab} = \mathrm{d}\hat{\sigma}^{\mathrm{V}}_{ab} + \mathrm{d}\hat{\sigma}^{\mathrm{R}}_{ab} + \mathrm{d}\hat{\sigma}^{\mathrm{pdf}}_{ab}\,. \tag{1}$$

The IR singularities in the first term are described by a variation on Catani's operator [14]

$$\bar{I}_1(\epsilon) = \frac{1}{2}\sum_{(ij)}^{N_p} \frac{\mathcal{V}_i^{\mathrm{sing}}(\epsilon)}{\boldsymbol{T}_i^2}\,(\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\, e^{i\pi\lambda_{ij}\epsilon}\, e^{\epsilon L_{ij}}\,, \qquad \mathcal{V}_i^{\mathrm{sing}}(\epsilon) = \frac{\boldsymbol{T}_i^2}{\epsilon^2} + \frac{\gamma_i}{\epsilon}\,, \tag{2}$$

where $\boldsymbol{T}_i$ are color-charge operators, γ_i are the quark and gluon anomalous dimensions, $L_{ij} = \log(\mu^2/s_{ij})$, with $s_{ij} = 2p_i\cdot p_j$, and the parameters λ_{ij} are 1 if i and j are both incoming or outgoing partons and zero otherwise. We then write

$$2s\,\mathrm{d}\hat{\sigma}^{V}_{ab} = [\alpha_s]\big\langle I_{\mathrm{V}}(\epsilon)\cdot F_{\mathrm{LM}}\big\rangle + \big\langle F^{\mathrm{fin}}_{\mathrm{LV}}\big\rangle\,, \qquad I_{\mathrm{V}}(\epsilon) = \bar{I}_1(\epsilon) + \bar{I}_1^{\dagger}(\epsilon)\,, \tag{3}$$

where $[\alpha_s] = \alpha_s(\mu)e^{\epsilon\gamma_{\mathrm{E}}}/(2\pi\,\Gamma(1-\epsilon))$, and F_{LM} is the LO matrix element squared (including all the relevant symmetry factors) describing the scattering $q\bar{q} \to X + Ng$. In Eq. (3) the angular brackets indicate that F_{LM} is integrated over the fiducial final-state phase space. Finally, $F^{\mathrm{fin}}_{\mathrm{LV}}$ is the finite remainder of the one-loop amplitude interfered with the tree-level one.

The second term in Eq. (1) is affected by *implicit* IR singularities, related to the configurations where one parton becomes soft and/or collinear to another parton. These divergences become manifest upon integrating over the unresolved phase space, and partially cancel against those

[1]Throughout this paper we work with UV-renormalized matrix elements, and in $d = 4 - 2\epsilon$ dimensions.

encoded in I_{V}. In order to highlight this interplay, we regulate the real radiation singularities according to the FKS procedure [15]. We first extract the soft singularity and integrate over the corresponding unresolved phase space. We then introduce a partitioning of the angular phase space using functions $\omega^{\mathfrak{m}i}$, after which we extract the collinear singularities and integrate over their phase space. We organise the integrated subtraction terms into universal operators that are as close as possible to I_{V}, and have a specific structure in color space. In particular, we write[2]

$$\begin{aligned} 2s\,\mathrm{d}\hat{\sigma}^{\mathrm{R}}_{ab} &= [\alpha_s]\big\langle \big[I_{\mathrm{S}}(\epsilon) + I_{\mathrm{C}}(\epsilon)\big]\cdot F_{\mathrm{LM}}\big\rangle + \frac{[\alpha_s]}{\epsilon}\Big[\,\big\langle \mathcal{P}^{\mathrm{gen}}_{aa}\otimes F_{\mathrm{LM}}\big\rangle + \big\langle F_{\mathrm{LM}}\otimes \mathcal{P}^{\mathrm{gen}}_{bb}\big\rangle\,\Big] \\ &\quad + \sum_{i=1}^{N_p}\big\langle (\mathbb{1}-S_{\mathfrak{m}})(\mathbb{1}-C_{i\mathfrak{m}})\,\omega^{\mathfrak{m}i}\Delta^{(\mathfrak{m})}F_{\mathrm{LM}}(\mathfrak{m})\big\rangle\,. \end{aligned} \tag{4}$$

In Eq. (4), the term on the last line is fully regulated and manifestly finite. The sum in this line runs over all the resolved partons of the process.[3] We have defined the operators $S_i A = \lim_{E_i\to 0} A$ and $C_{ij}A = \lim_{\eta_{ij}\to 0} A$, with $\eta_{ij} = (1-\cos\theta_{ij})/2$, where θ_{ij} is the angle between the three-momenta of partons i and j. Furthermore, $\Delta^{(\mathfrak{m})}$ is a damping factor that identifies $\mathfrak{m}$ as the only potential unresolved parton, and $F_{\mathrm{LM}}(\mathfrak{m})$ is proportional to squared matrix element for the process $pp \to X{+}N{+}\mathfrak{m}$. The operator I_{S} in the first term in Eq. (4) encodes all the singularities of soft origin. It contains explicit singularities proportional to the color-correlated structures $\boldsymbol{T}_i\cdot\boldsymbol{T}_j$. However, this dependence only arises at $\mathcal{O}(\epsilon^{-1})$, while poles of $\mathcal{O}(\epsilon^{-2})$ can be shown to be proportional to the sum of the Casimir factors $\boldsymbol{T}_i^2$. The second term in Eq. (4) encodes all the singularities of collinear origin that are proportional to Born-like kinematics. The I_{C} operator starts contributing at $\mathcal{O}(\epsilon^{-1})$ and is free of color correlations. It depends on the energies and anomalous dimensions of the external partons. Next, the contribution in square brackets in Eq. (4) completes the divergent collinear content of $\mathrm{d}\hat{\sigma}^{\mathrm{R}}_{ab}$, with the symbol $\otimes$ identifying the convolution of the splitting functions $\mathcal{P}^{\mathrm{gen}}$ and the squared matrix element. This term is free of color correlations and features boosted kinematics. We note that the splitting function $\mathcal{P}^{\mathrm{gen}}$ is equal (up to a sign) to the LO Altarelli-Parisi splitting function at $\mathcal{O}(\epsilon^0)$. By summing $\mathrm{d}\hat{\sigma}^{\mathrm{R}}_{ab}$ and $\mathrm{d}\hat{\sigma}^{V}_{ab}$, the following combination arises

$$\big\langle I_{\mathrm{T}}(\epsilon)\cdot F_{\mathrm{LM}}\big\rangle = \big\langle \big[I_{\mathrm{V}}(\epsilon) + I_{\mathrm{S}}(\epsilon) + I_{\mathrm{C}}(\epsilon)\big]\cdot F_{\mathrm{LM}}\big\rangle = \mathcal{O}(\epsilon^0)\,. \tag{5}$$

The remaining singularities given by the hard-collinear component in squared brackets in Eq. (4) cancel against the PDFs collinear renormalisation contribution. The final expression of the NLO partonic cross section for the process $q\bar{q}\to X + Ng$ reads

$$\begin{aligned} 2s\,\mathrm{d}\hat{\sigma}^{\mathrm{NLO}}_{ab} &= [\alpha_s]\big\langle I^{(0)}_{\mathrm{T}}\cdot F_{\mathrm{LM}}\big\rangle + \langle F^{\mathrm{fin}}_{\mathrm{LV}}\rangle \\ &\quad + [\alpha_s]\Big[\,\big\langle \mathcal{P}^{\mathrm{NLO}}_{aa}\otimes F_{\mathrm{LM}}\big\rangle + \big\langle F_{\mathrm{LM}}\otimes \mathcal{P}^{\mathrm{NLO}}_{bb}\big\rangle\,\Big] + \big\langle \mathcal{O}_{\mathrm{NLO}}\,\Delta^{(\mathfrak{m})}F_{\mathrm{LM}}(\mathfrak{m})\big\rangle\,. \end{aligned} \tag{6}$$

Here $I^{(0)}_{\mathrm{T}}$ is the $\mathcal{O}(\epsilon^0)$ coefficient in the expansion of $I_{\mathrm{T}}(\epsilon)$, $\mathcal{O}_{\mathrm{NLO}} = \sum_{i=1}^{N_p}(\mathbb{1}-S_{\mathfrak{m}})(\mathbb{1}-C_{i\mathfrak{m}})\,\omega^{\mathfrak{m}i}$ is the subtraction operator for the fully-regulated real-emission contribution, and the finite NLO splitting function $\mathcal{P}^{\mathrm{NLO}}$ is given in Eq. (I.3) in Ref. [1].

In the next section we will consider the NNLO contribution and will demonstrate that *iterations* of these operators appear from virtual contributions and integrated subtraction terms. This fact will play an important role in proving the cancellation of poles at NNLO.

[2]More details on the definitions and properties of the operators I_{S} and I_{C} can be found in Appendix A in Ref. [1].
[3]In the case of $q\bar{q}\to X + N$ jets we have $N_p = N + 2$.

2. NNLO calculation and color-correlated components

In this section we discuss the NNLO QCD correction to the $q\bar{q} \to X + Ng$ process. The corresponding cross section, $\mathrm{d}\hat{\sigma}_{ab}^{\mathrm{NNLO}}$, receives contributions from double-virtual, double-real, real-virtual and PDFs collinear renormalization contributions. We then write

$$\mathrm{d}\hat{\sigma}_{ab}^{\mathrm{NNLO}} = \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{VV}} + \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{RV}} + \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{RR}} + \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{pdf}} . \tag{7}$$

As done at NLO, we will be guided by the virtual contributions to identify corresponding IR structures in the real emission terms. We begin by analysing the double-virtual component $\mathrm{d}\hat{\sigma}_{ab}^{\mathrm{VV}}$ in Eq. (7). Following Refs [14, 16, 17], we write

$$\begin{aligned} \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{VV}} = {} & [\alpha_s]^2 \left\langle \left[\frac{1}{2} I_{\mathrm{V}}^2(\epsilon) - \frac{\Gamma(1-\epsilon)}{e^{\epsilon\gamma_{\mathrm{E}}}} \left(\frac{\beta_0}{\epsilon} I_{\mathrm{V}}(\epsilon) - \left(\frac{\beta_0}{\epsilon} + K \right) I_{\mathrm{V}}(2\epsilon) \right) \right] \cdot F_{\mathrm{LM}} \right\rangle \\ & + [\alpha_s]^2 \left\langle \left[-\frac{1}{2} \left[\bar{I}_1(\epsilon), \bar{I}_1^\dagger(\epsilon) \right] + \mathcal{H}_{2,\mathrm{tc}} + \mathcal{H}_{2,\mathrm{tc}}^\dagger + \mathcal{H}_{2,\mathrm{cd}} + \mathcal{H}_{2,\mathrm{cd}}^\dagger \right] \cdot F_{\mathrm{LM}} \right\rangle \\ & + [\alpha_s] \left\langle I_{\mathrm{V}}(\epsilon) \cdot F_{\mathrm{LV}}^{\mathrm{fin}} \right\rangle + \left\langle F_{\mathrm{LV}^2}^{\mathrm{fin}} \right\rangle + \left\langle F_{\mathrm{VV}}^{\mathrm{fin}} \right\rangle . \end{aligned} \tag{8}$$

In Eq. (8), K is a constant, and $F_{\mathrm{LV}^2}^{\mathrm{fin}}$, $F_{\mathrm{VV}}^{\mathrm{fin}}$ are the finite remainders of the NNLO matrix element squared. The two quantities $\mathcal{H}_{2,\mathrm{tc}}^{(\dagger)}$ and $\mathcal{H}_{2,\mathrm{cd}}^{(\dagger)}$ are triple color-correlated and color-diagonal contributions respectively, and were explicitly computed in Refs. [16, 17]. In the expression of $\mathrm{d}\hat{\sigma}_{ab}^{\mathrm{VV}}$ four different color structures appear:

i. *color-uncorrelated* terms, arising for instance from $\mathcal{H}_{2,\mathrm{cd}} + \mathcal{H}_{2,\mathrm{cd}}^\dagger \sim \mathbb{1}$;

ii. *color-correlated* terms, arising from $I_{\mathrm{V}} \sim \boldsymbol{T}_i \cdot \boldsymbol{T}_j$;

iii. *quartic color-correlated* terms, arising from $I_{\mathrm{V}}^2 \sim (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)(\boldsymbol{T}_k \cdot \boldsymbol{T}_l)$;

iv. *triple color-correlated* terms, arising from $[\bar{I}_1, \bar{I}_1^\dagger]$ and $\mathcal{H}_{2,\mathrm{tc}} + \mathcal{H}_{2,\mathrm{tc}}^\dagger \sim f_{abc} T_k^a T_i^b T_j^c$.

Each of the structures listed above has to cancel independently. In these proceedings we summarise the cancellation mechanism for the double color-correlated and the triple color-correlated contributions.

We first focus on terms proportional to $(\boldsymbol{T}_i \cdot \boldsymbol{T}_j)(\boldsymbol{T}_k \cdot \boldsymbol{T}_l)$. They arise from the first term in Eq. (8). Other terms that lead to similar color structures stem from the factorised contribution to the double-soft limit of the double-real matrix element squared. This contribution is indeed proportional to the product of two independent NLO-like soft currents, which, upon integration over the relevant phase space, returns I_{S}^2. Moreover, the soft limit of the real-virtual correction yields the product of a soft current and a color-correlated, one-loop matrix element interfered with the LO one. This results in the product of Catani's operators and I_{S}, and suggests that NNLO double color-correlated contributions can be rearranged in terms of the operator I_{T} found at NLO (see Eq. (5)). Such rearrangement would automatically guarantee that double color-correlated poles vanish. Indeed, as mentioned, I_{T} is of $O(\epsilon^0)$. Since the definition of I_{T} involves also the collinear operator I_{C}, it is crucial to identify it (and its combinations with I_{V} and I_{S}) among the subtraction terms stemming from the double-real and the real-virtual limits. We indeed find the following terms

$$\mathrm{d}\hat{\sigma}_{ab}^{\mathrm{VV}} \supset Y_{\mathrm{VV}} = \frac{1}{2}\left\langle I_{\mathrm{V}}^2 \cdot F_{\mathrm{LM}} \right\rangle, \qquad \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{RR}} \supset Y_{\mathrm{RR}}^{(\mathrm{ss})} = \frac{1}{2}\left\langle I_{\mathrm{S}}^2 \cdot F_{\mathrm{LM}} \right\rangle, \tag{9}$$

$$\mathrm{d}\hat{\sigma}_{ab}^{\mathrm{RR}} \supset Y_{\mathrm{RR}}^{(\mathrm{shc})} = \left\langle I_{\mathrm{S}}\, I_{\mathrm{C}} \cdot F_{\mathrm{LM}} \right\rangle, \qquad \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{RR}} \supset Y_{\mathrm{RR}}^{(\mathrm{cc})} = \frac{1}{2}\left\langle I_{\mathrm{C}}^2 \cdot F_{\mathrm{LM}} \right\rangle, \tag{10}$$

$$\mathrm{d}\hat{\sigma}_{ab}^{\mathrm{RV}} \supset Y_{\mathrm{RV}}^{(\mathrm{s})} = \frac{1}{2}\left\langle \left[I_{\mathrm{V}}\, I_{\mathrm{S}} + I_{\mathrm{S}}\, I_{\mathrm{V}} \right] \cdot F_{\mathrm{LM}} \right\rangle, \qquad \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{RV}} \supset Y_{\mathrm{RV}}^{(\mathrm{shc})} = \left\langle I_{\mathrm{V}}\, I_{\mathrm{C}} \cdot F_{\mathrm{LM}} \right\rangle. \tag{11}$$

Once combined, these objects return

$$Y = \frac{1}{2}\left\langle \left[I_{\mathrm{V}} + I_{\mathrm{S}} + I_{\mathrm{C}} \right]^2 \cdot F_{\mathrm{LM}} \right\rangle \equiv \frac{1}{2}\left\langle I_{\mathrm{T}}^2 \cdot F_{\mathrm{LM}} \right\rangle. \tag{12}$$

Eq. (12) is precisely the square of the structure found at NLO (see Eq. (5)) and collects all the double color-correlated contributions into the operator I_{T}^2, which is finite in ϵ by construction.

In order to extract the $Y_{\mathrm{VV,RV,RR}}$ functions as in Eqs (9)-(11), we have systematically expressed the double-virtual contribution and the soft limit of the real-virtual correction through the I_{V} operator, instead of $\overline{I}_1$ and $\overline{I}_1^{\dagger}$. In doing so, we obtain various commutators of the operators $\overline{I}_1$, $\overline{I}_1^{\dagger}$ and I_{S}. These "remnants" give rise to different color structures, namely to triple color-correlated contributions proportional to $\langle \mathcal{M}_0 |\, f_{abc} T_k^a T_i^b T_j^c \,| \mathcal{M}_0 \rangle$. Such terms have to cancel against other sources of triple-color correlations, such as the operator $\mathcal{H}_{2,\mathrm{tc}} + \mathcal{H}_{2,\mathrm{tc}}^{\dagger}$ in the second line of Eq. (8). Interestingly, explicit triple-color correlations also arise from the non-factorised component of the soft limit of the real-virtual correction. The integration of this term over the unresolved phase space is highly non-trivial (see Appendix H of Ref. [1] for details). We label the resulting integrated counterterm as $I_{\mathrm{tri}}^{\mathrm{RV}}$. Combining all the relevant terms, we find

$$\Sigma_N^{\mathrm{tri}} = [\alpha_s]^2 \left\langle \left(I_{\mathrm{tri}}^{\mathrm{RV}} + I_{\mathrm{tri}}^{(\mathrm{cc})} \right) \cdot F_{\mathrm{LM}} \right\rangle, \tag{13}$$

where $I_{\mathrm{tri}}^{(\mathrm{cc})}$ is defined as

$$I_{\mathrm{tri}}^{(\mathrm{cc})} = -\left[I_+, I_- \right] + \left[2I_+ + I_{\mathrm{S}}, I_- \right] + \mathcal{H}_{2,\mathrm{tc}} + \mathcal{H}_{2,\mathrm{tc}}^{\dagger}, \qquad I_{\pm}(\epsilon) = 1/2\left(\overline{I}_1(\epsilon) \pm \overline{I}_1^{\dagger}(\epsilon) \right). \tag{14}$$

To proceed, we need to compute the commutators of the various I-operators that appear in Eq. (14). We point out that the only non-vanishing contributions come from color-correlated terms, therefore only the irreducible terms proportional to $\boldsymbol{T}_i \cdot \boldsymbol{T}_j$ can play a role. Furthermore, we also exploit a suitable representation of the triple color-correlated operator $\mathcal{H}_{2,\mathrm{tc}}$, which can be found in Ref. [17]. Once all the technical details have been sorted out (see Ref. [1]), we obtain[4]

$$I_{\mathrm{tri}}^{(\mathrm{cc})}(\epsilon) = \frac{\pi}{2} \sum_{(ijk)}^{N_p} f_{abc}\, T_k^a\, T_i^b\, T_j^c \left[\frac{2L_{kj}\,\lambda_{ij}}{\epsilon^2} - \frac{4\phi_{jk}\lambda_{ij}}{\epsilon} + \left(\delta_{ij}^- + \delta_{ji}^- \right)\left(\delta_{kj}^+ + \delta_{jk}^+ - 2\phi_{jk} \right) \right], \tag{15}$$

where $F^{(kij)} = f_{abc}\, T_k^a\, T_i^b\, T_j^c$ and[5]

$$\begin{aligned} \delta_{ij}^+ &= \frac{1}{2}L_{ij}^2 + \frac{\gamma_i}{\boldsymbol{T}_i^2} L_{ij} - \frac{1}{2}\pi^2\lambda_{ij}^2, \qquad \delta_{ij}^- = \frac{\gamma_i}{\boldsymbol{T}_i^2}\lambda_{ij} + L_{ij}\,\lambda_{ij}, \\ \phi_{ij} &= -2\log\left(\frac{2E_{\max}}{\mu} \right)\log(\eta_{ij}) - \frac{1}{2}\log^2(\eta_{ij}) - \mathrm{Li}_2(1-\eta_{ij}). \end{aligned} \tag{16}$$

[4] In the following formula we omit terms of $O(\epsilon)$.

[5] $E_{\max}$ is an upper bound on the soft gluon energy, $E_{\mathrm{m}} \le E_{\max}$. It is an arbitrary quantity that should be larger than the largest energy that a particle in a particular process can have. For additional information, see Ref. [2].

PoS(EPS-HEP2023)269

First, we notice that for a process with only outgoing (or only incoming) partons, we have $\lambda_{ij} = 1$ for all i, j and hence the triple color-correlated poles in Eq. (15) vanish, due to the contraction of the antisymmetric tensor $F^{(kij)}$ with the symmetric functions L_{kj} and ϕ_{jk}. Similarly, the ϵ-poles in $I_{\rm tri}^{\rm RV}$ also vanish if all resolved partons are in the final state. If both incoming and outgoing partons are present, it is convenient to write λ_{ij} in the following way

$$\lambda_{ij} = 1 - \delta_{i1} - \delta_{i2} - \delta_{j1} - \delta_{j2} + 2\delta_{i1}\,\delta_{j2} + 2\delta_{i2}\,\delta_{j1}\,, \tag{17}$$

where 1 and 2 label the initial-state partons. Using this representation we obtain

$$\begin{aligned} I_{\rm tri}^{\rm (cc)} = \sum_{k\neq 1,2}^{N_p} F^{(k12)} \Bigg[-\frac{2\pi}{\epsilon^2} \log\left(\frac{\eta_{2k}}{\eta_{1k}}\right) - \frac{2\pi}{\epsilon} \Bigg(2\log\left(\frac{4E_{\rm max}^2}{\mu^2}\right)\log\left(\frac{\eta_{1k}}{\eta_{2k}}\right) \\ + \log^2\eta_{1k} - \log^2\eta_{2k} + 2{\rm Li}(1-\eta_{1k}) - 2{\rm Li}(1-\eta_{2k})\Bigg)\Bigg] + O(\epsilon^0)\,. \end{aligned} \tag{18}$$

Comparing this result with the expression for $I_{\rm tri}^{\rm RV}$ in Eq. (H.15) of Ref. [1], we find that Eq. (13) is finite, and therefore all the triple color-correlated objects contributing to ${\rm d}\hat{\sigma}_{ab}^{\rm NNLO}$ are of $O(\epsilon^0)$, independently on the number of final state gluons.

We stress that the issue of triple-color correlation is due to the non-abelian nature of QCD: the NNLO correction cannot be reduced to a double copy of the NLO case, as it would be in QED. For the same reason, Eq. (8) also contains color-correlated terms proportional to $\boldsymbol{T}_i \cdot \boldsymbol{T}_j$, as do ${\rm d}\hat{\sigma}_{ab}^{\rm RV}$ and ${\rm d}\hat{\sigma}_{ab}^{\rm RR}$. However, one can show that the single color-correlated pole can be written as a combination of the operators $I_{\rm V}$ and $I_{\rm S}$ only, and it vanishes regardless of the number N of final-state gluons.

Finally, Eq. (8) contains also color-uncorrelated terms that are diagonal in color space, and that can be written as combinations of Casimir operators of external partons. They have to be combined with the hard-collinear contributions contained within ${\rm d}\hat{\sigma}_{ab}^{\rm RV}$, ${\rm d}\hat{\sigma}_{ab}^{\rm RR}$ and ${\rm d}\hat{\sigma}_{ab}^{\rm pdf}$. The computation of these objects is conceptually simple but tedious (see Sec. 5 of Ref. [1]). What is worth underlining, however, is that even for these color-uncorrelated contributions it is possible to show the analytic cancellation of the poles for the production of an arbitrary number of gluons in $q\bar{q}$ annihilation.

3. Conclusions

In this proceeding we have reported on recent developments [1] in the application of the nested soft-collinear subtraction scheme [2] to generic hadron collider process at NNLO in QCD. The key idea consists in identifying recurring universal structures that are iterations of those that arise at NLO, or simple variants of them. Such structures usually combine into finite quantities, that resemble NLO-like cross sections. We organised the calculation of NNLO partonic cross section in such a way that the iterative nature of these finite contributions is fully exposed. This lead to a drastic reduction of the computational complexity. Although we considered a $q\bar{q}$ initial state in this paper, the guiding principles of the calculation can be adapted to accommodate other channels. The main remaining challenge at this point is the combinatorics of quark and gluon collinear limits.

PoS(EPS-HEP2023)269

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