

## On the impact of mixed QCD-electroweak corrections to the Drell-Yan process in the high invariant mass region

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We summarise the calculation of mixed QCD-electroweak corrections to the neutral-current-mediated production of a pair of massless leptons performed in Ref. [1]. The study is focused on the high invariant mass region. We find mixed corrections to be  $O(-1\%)$  for values of the dilepton invariant mass around 200 GeV. For invariant masses larger than 1 TeV, we observe these corrections to be  $O(-3\%)$ , and well reproduced by the product of next-to-leading order QCD and electroweak corrections.

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## 1. Motivation, technicalities and phenomenological results

The Drell-Yan process offers an interesting opportunity to test the Standard Model (SM) and possibly reveal New Physics beyond it. Indeed, dilepton production at high invariant masses is sensitive to New Physics effects, which can be modelled using Standard Model Effective Field Theory (SMEFT) [2]. Improvements on constraining the Wilson coefficients of the relevant SMEFT operators are expected from LHC measurements, as argued in Refs. [3], and require percent-level predictions within the SM. Such precision, in turn, justifies our interest in electroweak (EW) corrections, which are enhanced at large invariant masses because of the so-called Sudakov logarithms [4]. In this proceeding we report on the recent calculation of mixed QCD×EW corrections to the neutral-current-mediated production of a pair of massless leptons [1]. Qualitatively, our results are in agreement with the analysis of Ref. [5], although a direct comparison is not possible because of different the set up.

Before presenting our results, we notice that mixed QCD×EW corrections have already been studied for *resonant* production of  $Z$  and  $W$  bosons [6, 7]. Despite the similarities between the resonant and the off-shell calculation, several complications arise when considering the full  $q\bar{q}' \rightarrow \ell_1\bar{\ell}_2$  process. This is so because interactions between initial and final state become important in the high invariant mass region, while they are negligible for resonant production [8].

Double-virtual corrections to  $q\bar{q}' \rightarrow \ell_1\bar{\ell}_2$  are obtained starting from the two-loop amplitudes presented in Refs. [9]. The one-loop QCD amplitudes for the process  $q\bar{q} \rightarrow \ell^-\ell^+ + \gamma$  are obtained from the QCD amplitudes for the  $q\bar{q} \rightarrow Z + j$  process [11], and implemented as in MCFM [12]. The one-loop electroweak corrections to the processes  $q\bar{q} \rightarrow \ell^-\ell^+$  and  $q\bar{q} \rightarrow \ell^-\ell^+ + g$  are computed using OPENLOOPS2 [13]. We used a variant of the nested soft-collinear subtraction scheme [14] to handle soft and collinear singularities arising from real radiation (see Ref. [15] for a recent review on the topic). In doing so, we benefit from previous studies on resonant vector boson production [6].

We consider proton-proton collisions at 13.6 TeV center-of-mass energy. The results reported below are computed using the NNPDF31\_nnlo\_as\_0118\_luxqed PDFs, available through the LHAPDF library [16]. We use the  $G_\mu$  input scheme for the EW parameters and complex-mass scheme [10]. Further details on the set up can be found in Ref. [1]. We recombine photons and leptons into dressed leptons, and impose cuts on their invariant mass, transverse momenta and rapidities following Refs. [17]. In particular, we set  $m_{\ell\ell} > 200 \text{ GeV}$ ,  $p_{T,\ell^\pm} > 30 \text{ GeV}$ ,  $\sqrt{p_{T,\ell^-}p_{T,\ell^+}} > 35 \text{ GeV}$ ,  $|y_{\ell^\pm}| < 2.5$ . The central value is computed by setting the renormalization and factorisation scale to be  $\mu_R = \mu_F = \mu = m_{\ell\ell}/2$ , where  $m_{\ell\ell}$  is the invariant mass of the dilepton system. Theoretical uncertainties correspond to the envelop of scale uncertainties and EW input scheme uncertainties (see Ref. [1]). We introduce the notation

$$d\sigma = \sum_{i,j=0} d\sigma^{(i,j)}, \quad \delta\sigma^{(i,j)} = \int d\sigma^{(i,j)} \quad \text{with} \quad \sigma^{(0,0)} \equiv \delta\sigma^{(0,0)}, \quad (1)$$

where  $d\sigma^{(i,j)}$  and  $\delta\sigma^{(i,j)}$ ,  $i, j > 0$ , indicate to corrections of order  $\mathcal{O}(\alpha_s^i\alpha^j)$  with respect to the leading-order (LO) cross sections  $d\sigma^{(0,0)}$  and  $\sigma^{(0,0)}$ . We then compare results for the fiducial cross section before and after including the mixed QCD×EW corrections

$$\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} = 1928.3^{+1.8\%}_{-0.15\%} \text{ fb}, \quad (2)$$

$\sigma[\text{fb}]$	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$	$\delta\sigma_{\text{fact.}}^{(1,1)}$	$\sigma_{\text{QCD}\times\text{EW}}$
$\Phi^{(1)}$	1169.8	254.3	-30.98	10.18	-10.74	-6.734	$1392.6^{+0.75\%}_{-0\%}$
$\Phi^{(2)}$	368.29	71.91	-11.891	2.85	-4.05	-2.321	$427.1^{+0.41\%}_{-0.02\%}$
$\Phi^{(3)}$	82.08	14.31	-4.094	0.691	-1.01	-0.7137	$91.98^{+0.22\%}_{-0.14\%}$
$\Phi^{(4)} \times 10$	9.107	1.577	-1.124	0.146	-0.206	-0.1946	$9.500^{+0\%}_{-0.97\%}$

**Table 1:** Results for the fiducial cross sections defined in Eq. (1) and (5) in the invariant mass windows given in Eq. (4). See the text and Ref. [1] for details.

$$\sigma_{\text{QCD}\times\text{EW}} \equiv \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} = 1912.6^{+0.65\%}_{-0.04\%} \text{ fb}. \quad (3)$$

We notice that next-to-leading-order (NLO) QCD corrections,  $\delta\sigma^{(1,0)}$ , impact the LO cross section by about 20%, and the NLO EW corrections  $\sim 3\%$ , compatible with the expectations based on the magnitude of the coupling constants. In contrast, NNLO QCD corrections are smaller than naive power counting predictions due to a strong cancellation between the  $q\bar{q}$  and the  $gq$  channels. Mixed corrections impact the LO cross section by about  $-1\%$ , exceeding by roughly one order of magnitude the power counting estimate  $\alpha\alpha_s \sim 0.1\%$ . In fact, these corrections are *larger* than the NNLO QCD ones. We notice that including mixed corrections reduces the theoretical uncertainties below percent level.

To estimate the impact of universal Sudakov logarithms on EW corrections we consider different invariant mass windows

$$\begin{aligned} \Phi^{(1)} : 200 \text{ GeV} < m_{\ell\ell} < 300 \text{ GeV}, & \quad \Phi^{(2)} : 300 \text{ GeV} < m_{\ell\ell} < 500 \text{ GeV}, \\ \Phi^{(3)} : 500 \text{ GeV} < m_{\ell\ell} < 1.5 \text{ TeV}, & \quad \Phi^{(4)} : 1.5 \text{ TeV} < m_{\ell\ell} < \infty, \end{aligned} \quad (4)$$

and compare the exact result for mixed contributions with the corresponding factorised approximation

$$\delta\sigma_{\text{fact}}^{(1,1)} = \delta_{\text{NLO}}^{(1,0)} \delta_{\text{NLO}}^{(0,1)} \sigma^{(0,0)}, \quad \text{with} \quad \delta_{\text{NLO}}^{(1,0)} = \frac{\delta\sigma^{(1,0)}}{\sigma^{(0,0)}}, \quad \delta_{\text{NLO}}^{(0,1)} = \frac{\delta\sigma^{(0,1)}}{\sigma^{(0,0)}}. \quad (5)$$

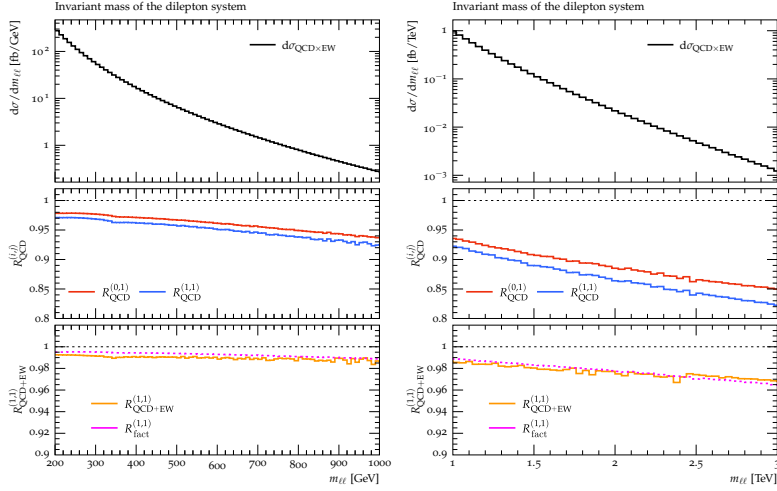
Such approximation should capture the leading Sudakov logarithms that are expected to be the dominant contribution, at least for high values of  $m_{\ell\ell}$ . Indeed, the values in Table 1 confirm this claim for  $m_{\ell\ell} > 1 \text{ TeV}$ . In contrast, the factorised approximation underestimates the mixed corrections for lower invariant masses. We also notice that the inclusion of QCD $\times$ EW corrections reduces the theoretical uncertainties to sub-percent level in all the invariant mass windows.

We now present the effects of the different corrections to the dilepton invariant mass distribution. Our best prediction for the fiducial cross section is defined as

$$d\sigma_{\text{QCD}\times\text{EW}} = d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)} + d\sigma^{(2,0)} + d\sigma^{(1,1)}. \quad (6)$$

We study the relative impact of NLO EW and QCD $\times$ EW corrections on the results computed through NLO QCD

$$R_{\text{QCD}}^{(0,1)} = \frac{d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}}, \quad R_{\text{QCD}}^{(1,1)} = \frac{d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)} + d\sigma^{(1,1)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}}, \quad (7)$$



**Figure 1:** Distribution of the dilepton invariant mass in the range  $200 \text{ GeV} < m_{\ell\ell} < 1 \text{ TeV}$  (left pane) and  $1 \text{ TeV} < m_{\ell\ell} < 3 \text{ TeV}$  (right pane) for  $\sqrt{s} = 13.6 \text{ TeV}$  LHC. The upper panel shows predictions for the cross section (see Eq.(3)). The middle panel shows the ratio of the NLO EW and mixed QCD×EW corrections to the full NLO QCD result (see Eq.(7)). The lower pane shows in orange the ratio  $R_{\text{QCD+EW}}^{(1,1)}$  as defined in Eq.(8), and the corresponding factorised approximation in pink. See the text and Ref. [1] for details.

and introduce the ratio of the two quantities in Eq.(7) as

$$R_{\text{QCD+EW}}^{(1,1)} = R_{\text{QCD}}^{(1,1)} / R_{\text{QCD}}^{(0,1)} = \frac{d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)} + d\sigma^{(1,1)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)}}. \quad (8)$$

We also define the factorised approximation  $R_{\text{fact}}^{(1,1)}$  as the right-hand side of Eq.(8) up to substituting  $d\sigma^{(1,1)}$  with  $d\sigma_{\text{fact}}^{(1,1)}$ . The corresponding distributions are shown in Fig. 1. It follows from the figure that NLO EW corrections grow by a factor of 10 when  $m_{\ell\ell}$  increases from 200 GeV to 3 TeV. Mixed corrections exhibit a similar shape and increase from  $\mathcal{O}(-3\%)$  to  $\mathcal{O}(-18\%)$  in the same invariant mass range. We further note that the ratio  $R_{\text{QCD+EW}}^{(1,1)}$  grows by about a factor of 4 when moving from  $m_{\ell\ell} = 200 \text{ GeV}$  to  $m_{\ell\ell} = 3 \text{ TeV}$ , and is well approximated by  $R_{\text{fact}}^{(1,1)}$  for high values of  $m_{\ell\ell}$ . Interestingly, mixed corrections seem to be enhanced with respect to naive expectations at low values of  $m_{\ell\ell}$  (see also Table 1). We indeed notice these corrections to be only three times smaller than the EW corrections, and we do not expect large Sudakov logarithms at such energy scales. It is unclear if such an enhancement is caused by an artificial numerical effect.

We then consider angular distributions, which are potentially sensitive to the nature of quark-lepton currents. In particular, we focus on the forward-backward asymmetry, that we define as

$$A_{\text{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}, \quad \sigma_F = \int_0^1 d \cos \theta^* \frac{d\sigma(pp \rightarrow \ell^- \ell^+)}{d \cos \theta^*}, \quad \sigma_B = \int_{-1}^0 d \cos \theta^* \frac{d\sigma(pp \rightarrow \ell^- \ell^+)}{d \cos \theta^*}, \quad (9)$$

where  $\theta^*$  is the Collins-Soper angle [18]. After including all the corrections up to NNLO QCD and QCD×EW we find  $A_{\text{FB}} = 0.1580_{-0.07\%}^{+0.15\%}$ . Mixed corrections change this prediction by about 2 per mille which is comparable with the uncertainty on the central value. However, considering different

	$\tilde{A}_{\text{FB}}$	$A_{\text{FB}}$
$\Phi^{(1)}$	$0.1442^{+0.05\%}_{-0.31\%}$	$0.1440^{+0.11\%}_{-0.09\%}$
$\Phi^{(2)}$	$0.1852^{+0.08\%}_{-0.40\%}$	$0.1847^{+0.10\%}_{-0.19\%}$
$\Phi^{(3)}$	$0.2401^{+0.13\%}_{-0.64\%}$	$0.2388^{+0.06\%}_{-0.47\%}$
$\Phi^{(4)}$	$0.3070^{+0.49\%}_{-1.5\%}$	$0.3031^{+0.19\%}_{-1.2\%}$

**Table 2:** Values of the forward-backward asymmetry in the invariant mass windows defined in Eq. (4).  $\tilde{A}_{\text{FB}}$  includes the LO, NLO-QCD, NLO-EW and NNLO-QCD contributions, whereas  $A_{\text{FB}}$  further includes the mixed QCD $\times$ EW correction computed in Ref. [1].

invariant mass windows (see Table 2) we notice that the impact of mixed corrections reaches  $-1.3\%$  at high  $m_{\ell\ell}$ . Such shifts should become observable at the high-luminosity (HL) LHC.

## 2. Conclusions

We reported on the calculation of mixed QCD-electroweak correction to the production of a massless dilepton pair at the LHC. We investigated the high invariant mass region,  $m_{\ell\ell} > 200$  GeV, and found mixed corrections to the fiducial cross section to be about  $-1\%$  of the LO contribution. For invariant masses above 1 TeV, mixed corrections become even larger, and reach  $\mathcal{O}(-3\%)$  at  $m_{\ell\ell} \sim 3$  TeV. Their behaviour is compatible with the growth of Sudakov logarithms, and can be well approximated by the product of NLO QCD and EW contributions. For  $m_{\ell\ell} > 1.5$  TeV this factorised approximation captures more than 90% of the exact result. We stress that the inclusion of mixed corrections reduces the theoretical uncertainties below a percent. We also studied the impact of mixed QCD $\times$ EW corrections on the forward-backward asymmetry and found a percent level effect for dilepton invariant masses above a TeV. We believe these results to be of interest for New Physics searches at the HL-LHC.

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