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Two- and three-loop QCD corrections to the width difference in the $B_s - \bar{B}_s$ system

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We report on a series of two- and three-loop calculations that were necessary to improve the theory prediction for the width difference $\Delta\Gamma_s$ in the $B_s - \bar{B}_s$ mixing. For the first time we are able to predict this important flavor observable at NNLO accuracy, which is necessary to approach the current experimental precision. Our new Standard-Model prediction obtained from the average of calculations done in the $\overline{\text{MS}}$ and potential subtracted (PS) schemes reads $\Delta\Gamma_s = (0.076 \pm 0.017)\ \text{ps}^{-1}$.

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1. Introduction

The successful start of the LHC Run 3 promises a rich harvest of new results that will further stimulate searches for beyond the Standard Model (BSM) phenomena. In fact, the existing tensions in the flavor sector and their growing statistical significance readily suggest that the long-awaited new physics might be just around the corner. One of the main tasks of the theory is to ensure that possible deviations between the existing predictions and experimental measurements can be indeed attributed to new physics effects rather than uncalculated or neglected corrections. At higher orders in perturbation theory such calculations may become highly nontrivial, requiring the expertise and know-how of the multiloop community. Nevertheless, such laborious investigations are in no way less important for the field than new advances in the model-building sector.

Oscillations of neutral B mesons provide a prominent source of flavor observables, for which higher order corrections are absolutely necessary to approach the existing experimental precision. Experimental measurements of the $B_s - \bar{B}_s$ system give rise to three observables known as the width difference between mass eigenstates, $\Delta\Gamma_s$, their mass difference ΔM_s and the flavor-specific CP asymmetry a^s_{fs}. These quantities are related to Γ_{12}, M_{12} and the physical CP-violating phase ϕ_{12} as follows

$$\Delta M = M_H - M_L \approx 2|M_{12}|, \quad \Delta\Gamma = \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}|\cos(\phi_{12}), \quad a_{\text{fs}} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi_{12}, \tag{1}$$

where

$$M_{12} = |M_{12}|e^{i\phi_M}, \quad \Gamma_{12} = |\Gamma_{12}|e^{i\phi_\Gamma}, \quad \cos(\phi_\Gamma - \phi_M) = -\cos(-\pi + \phi_\Gamma - \phi_M) \equiv -\cos(\phi_{12}) \tag{2}$$

and the perturbative parts of Γ_{12} and M_{12} can be obtained from suitable matching calculations.

In this proceeding we would like to focus on $\Delta\Gamma_s$ which can be related to the dispersive part of Feynman diagrams describing the process $b\bar{s} \to \bar{b}s$ and measured by looking at the lifetimes in different decay modes. Combining [1] the measurements done by ATLAS [2], CMS [3], LHCb [4], CDF [5], DØ [6] experiments one arrives at

$$\Delta\Gamma^{\text{exp}} = (0.085 \pm 0.005)\ \text{ps}^{-1} \tag{3}$$

To put this number into perspective, let us also provide the most precise theoretical prediction [7–12] as of 2020, which comprises the most relevant NLO contributions as well as the fermionic n_f-piece of the dominant NNLO corrections

$$\Delta\Gamma_s = (0.077 \pm 0.015_{\text{pert.}} \pm 0.002_{B,\tilde{B}_S} \pm 0.017_{\Lambda_{\text{QCD}}/m_b}) \times \text{ps}^{-1}\ \text{(pole)} \tag{4}$$

$$\Delta\Gamma_s = (0.088 \pm 0.011_{\text{pert.}} \pm 0.002_{B,\tilde{B}_S} \pm 0.014_{\Lambda_{\text{QCD}}/m_b}) \times \text{ps}^{-1}\ (\overline{\text{MS}}). \tag{5}$$

Owing to the fact that $b\bar{s} \to \bar{b}s$ is a loop-induced flavor changing neutral current process in the SM, NLO and NNLO in this context mean two- and three-loop accuracy respectively. By scrutinizing the above numbers one can readily identify two main sources for the large theoretical uncertainties. These are the missing QCD correction at leading power in the $1/m_b$ expansion (denoted as pert.) as well as uncalculated terms that are subleading in $1/m_b$.

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In our work we addressed the former type of uncertainties by explicitly calculating all the missing pieces at two and three loops [13–15] and finally presenting a new theory prediction for $\Delta\Gamma_s$ at NNLO accuracy [15]. In the following we would like to describe selected aspects of the relevant calculations in more details.

2. Calculation

In order to calculate $\Delta\Gamma_s$ (or a_{fs}^s) one needs to perform a matching calculation between two effective theories. In this setup the high-energy theory describing $|\Delta B| = 1$ transitions is obtained from the SM by integrating out degrees of freedom heavier than the b quark mass m_b. For historical reasons, this effective theory can be formulated using two different operator bases [16, 17], and we choose to work in the so-called Chetyrkin-Misiak-Münz (CMM) [17] basis, as it is better suited for automated higher-order calculations and avoids possible issues with γ^5 [1]. The $|\Delta B| = 1$ effective Hamiltonian is given by

$$\mathcal{H}_{\mathrm{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \Bigg[-\lambda_t^s \Big(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8 \Big) - \lambda_u^s \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Bigg] + \mathrm{h.c.}, \tag{6}$$

with CKM matrix elements V_{ij}, $\lambda_a^s = V_{as}^* V_{ab}$ and the Fermi constant G_F. Explicit definitions of the current-current operators Q_{1-2}, four-quark penguin operators Q_{3-6} and the chromomagnetic penguin operator Q_8 can be found in Ref. [17]. The subscripts cu, uc and u signify that one or both of the c quarks in the operator definition should be replaced with u quarks (cf. e. g. Ref. [18]). The Wilson coefficients C_i are known at three-loop order [19–21]. Apart from the physical operators Q_i the effective Hamiltonian also comprises evanescent operators $E[Q_i]$ [22, 23] that capture ambiguities of purely 4-dimensional algebraic relations (e. g. Chisholm or Fierz identities) in d-dimensions. They become relevant at NLO and beyond, and, more importantly, they mix with the physical operators under renormalization.

Using optical theorem we can relate imaginary[2] parts of the amplitudes calculated on the $|\Delta B| = 1$ side of the matching to Γ_{12}, which can be conveniently written as [10]

$$\Gamma_{12} = -(\lambda_c^s)^2 \Gamma_{12}^{cc} - 2\lambda_c^s \lambda_u^s \Gamma_{12}^{uc} - (\lambda_u^s)^2 \Gamma_{12}^{uu}\,, \tag{7}$$

However, to this aim we also need to consider contributions (including loop corrections) from the low-energy theory. This so-called $|\Delta B| = 2$ effective theory emerges from the Heavy Quark Expansion (HQE) [24–33] of the real part of a time-ordered bilocal matrix element induced by two insertions of $|\Delta B| = 1$ effective Hamiltonians. We find

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[H^{ab}(z) \langle B_s | Q | \bar{B}_s \rangle + \widetilde{H}_S^{ab}(z) \langle B_s | \widetilde{Q}_S | \bar{B}_s \rangle \right] + O(\Lambda_{\mathrm{QCD}}/m_b), \tag{8}$$

[1] In our calculation we do not encounter any chiral traces, and so we employ the naive dimensional regularization (NDR) scheme with anticommuting γ^5 in d-dimensions.

[2] In the literature one often writes "absorptive" and "dispersive" instead of real and imaginary to stress the fact that the CKM matrix element multiplying the corresponding amplitudes are complex quantities. In this sense, when talking about real and imaginary parts we explicitly mean parts of the relevant loop integrals.

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where M_{B_s} denotes the B_s meson mass and $z \equiv m_c^2/m_b^2$. The physical operators are defined as

$$Q = \bar{s}_i \gamma^\mu (1-\gamma^5) b_i \, \bar{s}_j \gamma_\mu (1-\gamma^5) b_j \,, \qquad \widetilde{Q}_S = \bar{s}_i (1+\gamma^5) b_j \, \bar{s}_j (1+\gamma^5) b_i, \tag{9}$$

with i, j being quark color indices in the fundamental representation. At intermediate stages of the calculation and especially when doing comparisons to the existing literature results one also needs to consider the operators

$$\widetilde{Q} = \bar{s}_i \gamma^\mu (1-\gamma^5) b_j \, \bar{s}_j \gamma_\mu (1-\gamma^5) b_i \,, \qquad Q_S = \bar{s}_i (1+\gamma^5) b_i \, \bar{s}_j (1+\gamma^5) b_j. \tag{10}$$

In practice, we work in the basis of three operators being Q, $\widetilde{Q}_S$ and the $1/m_b$-suppressed operator R_0 [7, 10] defined as

$$R_0 = Q_S + \alpha_1 \widetilde{Q}_S + \frac{1}{2}\alpha_2 Q. \tag{11}$$

For the renormalization of the $|\Delta B| = 2$ operators one needs the three operators Q, $\widetilde{Q}_S$ and R_0. Then, $\alpha_{1,2}$ are perturbative coefficients required to ensure that the $\overline{\text{MS}}$-renormalized matrix element $\langle R_0 \rangle$ is indeed $1/m_b$ suppressed. If, furthermore, dimensional regularization is used for IR regularization, subtleties occur, which can lead to incorrect matching coefficients [14]. Definitions of $|\Delta B| = 2$ evanescent operators can be found in Ref. [13]. The hadronic matrix elements $\langle B_s|Q|\bar{B}_s\rangle$ and $\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle$ can be calculated on the lattice [34, 35] or using QCD/HQET sum rules [36–43].

In order to improve the theory prediction for $\Delta\Gamma_s$ we need to calculate the Wilson coefficients $H^{ab}(z)$ and $\widetilde{H}_S^{ab}(z)$ by taking into account different operator insertions on the $|\Delta B| = 1$ side of the matching together with the relevant QCD corrections. To simplify the calculation it is convenient to make use of the good convergence of the Wilson coefficients in z. Therefore, in this work, we expand in z and restrict ourselves to the accuracy of $O(z)$.

We carry out the matching by taking the b quark momenta on-shell (i. e. $p_b^2 = m_b^2$) and neglecting the s quark mass while setting $p_s = 0$. Dimensional regularization is employed both for UV and IR divergences, using the procedure described in [8]. At each order in α_s in the matching we need to calculate one more loop on the $|\Delta B| = 1$ side compared to the $|\Delta B| = 2$ side. This is because LO in the former theory is given by one-loop diagrams, while in the latter the leading contribution is a tree-level one.

At two loops we consider all possible combinations of two $|\Delta B| = 1$ operator insertions (e. g. $Q_{1,2} \times Q_{1,2}$, $Q_{1,2} \times Q_{3-6}$, $Q_{3-6} \times Q_8$ etc.). At three loops we calculate only the insertions of two current-current operators $Q_{1,2}$ (including their u, uc and cu varieties). It is worth noting that for many of the above operator insertions z-exact results proportional to the number of flavors n_f are available in the literature. The reason for this is that such fermionic pieces are simpler to compute as compared to the nonfermionic contributions. A summary comparing our results with what was previously known is given in Table 1.

3. Technical details

The technical side of our matching calculation is handled by a well tested setup employing QGRAF [45] (diagram generation), q2e/exp [46, 47] or tapir [48, 49] (insertion of Feynman rules and topology identification), and the in-house FORM-based [50] calc-code (amplitude evaluation).

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Contribution	Literature result	This work
$Q_{1,2} \times Q_{3-6}$	two loops, z-exact, n_f-part only [11]	two loops, $O(z)$, full
$Q_{1,2} \times Q_8$	two loops, z-exact, n_f-part only [11]	two loops, $O(z)$, full
$Q_{3-6} \times Q_{3-6}$	one loop, z-exact, full [44]	two loops, $O(z)$, full
$Q_{3-6} \times Q_8$	one loop, z-exact, n_f-part only [11]	two loops, $O(z)$, full
$Q_8 \times Q_8$	one loop, z-exact, n_f-part only [11]	two loops, $O(z)$, full
$Q_{1,2} \times Q_{1,2}$	three loops, $O(\sqrt{z})$, n_f-part only [12]	three loops, $O(z)$, full

Table 1: Overview of the existing and new results required for the NNLO theory prediction of $\Delta\Gamma_s$ that were considered in this work. With "n_f-part only" we signify that the corresponding literature result provides only fermionic contributions, while "full" means that both fermionic and nonfermionic pieces are included.

The numerators of multi-loop integrals are handled using two independent approaches, where we either make use of suitable Dirac and color projectors (cf. appendix of Ref. [13]) or employ explicit tensor-reduction formulas obtained with the aid of FeynCalc [51–54] and Fermat [55]. In order to reduce the occurring loop integrals by means of IBP techniques we use FIRE [56] and LiteRed [57].

At two loops the naive Taylor expansion in z is equivalent to the proper asymptotic expansion [58, 59] up to $O(z)$. At three loops this is true for most diagrams, except for those containing a closed charm loop. Since those fermionic contributions are already known, we can take them from Ref. [11, 12] while still employing naive expansion for the rest of the diagrams. Therefore, the master integrals appearing in the final results are single scale propagator-type on-shell integrals, which can always be cut without touching a massive line. This is because integrals with continuous massive lines have no imaginary parts and hence cannot contribute to Γ_{12}. All integrals up to two loops are either already known [60] or can be trivially computed using standard techniques [61]. The three-loop master integrals turn out to be more challenging in the sense that only few of them can be readily found in the literature [12]. In total, we have four massless three-loop integrals that can be taken over from Mincer [62] and 23 genuine masters with internal massive lines.

The new master integrals can be calculated using HyperInt [63], where we first employ FeynCalc to derive the Feynman parametrization of each integral and check that it is projective. Then, the integrand is handed over to HyperInt, regularized using the built-in analytic regularization routines [63–65] and directly integrated in the Feynman parameters. This way we obtain intermediate results in terms of Goncharov Polylogarithms (GPLs) [66] containing 6th root of unity. Using HyperLogProcedures [67] and ultimately also PolyLogTools [68] the imaginary parts of these integrals can be significantly simplified and expressed in terms of familiar constants such as $\pi, \ln(2), \zeta_2, \zeta_3, \zeta_4, \mathrm{Cl}_2(\pi/3), \sqrt{3}, \mathrm{Li}_4(1/2)$ and $\ln\left((1+\sqrt{5})/2\right)$. Notice that the golden ratio stems only from the first integral in Fig. 1 as well as its variety with a dot on one of the massless lines. The appearance of this constant in the final result for $\Delta\Gamma_s$ has already been observed in Ref. [12].

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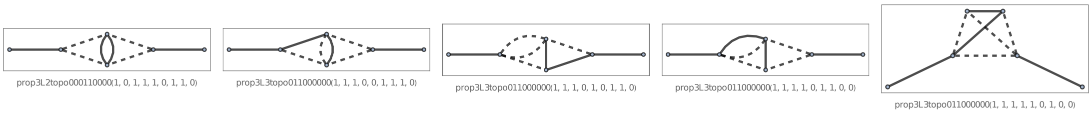

Figure 1: Some of the nonfactorizing on-shell three-loop master integrals occurring in our matching calculation. A solid line denotes a massive propagator with m_b, while a dashed line corresponds to a massless propagator.

4. Results

Having obtained all the necessary ingredients to predict $\Delta\Gamma_s$ with NNLO accuracy, we finally were able to provide the long-awaited numbers in Ref. [15]. Let us start by explaining the scheme dependence of the matching coefficients that enters through the heavy quark masses[3]. In the matching we employ the on-shell scheme, i. e. $z = (m_c^{\text{pole}}/m_b^{\text{pole}})^2$. These results can be readily converted to the $\overline{\text{MS}}$ scheme by introducing $\bar{z} = (m_c(\mu_c)/m_b(\mu_b))^2$. Here we set $\mu_c = \mu_b = m_b$ since this choice leads to a cancellation of potentially large $z \log z$-terms [69]. Apart from $H^{ab}(\bar{z})$ and $\tilde{H}_S^{ab}(\bar{z})$ that now contain only $\overline{\text{MS}}$ quantities, we still have the m_b^2 prefactor in Eq. (8), where different scheme choices are possible. Apart from leaving the prefactor untouched ("pole" scheme) we may also convert it to the $\overline{\text{MS}}$ or e. g. to the potential subtracted (PS) [70] scheme. We refer to Refs. [14, 15] for further information on the choice of the input parameters.

In view of the existing tensions between inclusive and exclusive $|V_{cb}|$ determinations, it is useful to consider the ratio $\Delta\Gamma_s/\Delta M_s$, which does not depend on the CKM parameters V_{ts} and V_{tb}, while the dependence on the hadronic bag parameters cancels to a large extent. In this sense the ratio is a theoretically much cleaner observable than $\Delta\Gamma_s$ alone. Here our predictions read

$$
\begin{aligned}
\frac{\Delta\Gamma_s}{\Delta M_s} &= (3.79^{+0.53}_{-0.58}{}_{\substack{\text{LP}\\\text{scale}}}\,{}^{+0.09}_{-0.19}{}_{\substack{\text{NLP}\\\text{scale}}} \pm 0.11_{B\tilde{B}_S} \pm 0.78_{1/m_b} \pm 0.05_{\text{input}}) \times 10^{-3} \quad (\text{pole}), \\
\frac{\Delta\Gamma_s}{\Delta M_s} &= (4.33^{+0.23}_{-0.44}{}_{\substack{\text{LP}\\\text{scale}}}\,{}^{+0.09}_{-0.19}{}_{\substack{\text{NLP}\\\text{scale}}} \pm 0.12_{B\tilde{B}_S} \pm 0.78_{1/m_b} \pm 0.05_{\text{input}}) \times 10^{-3} \quad (\overline{\text{MS}}), \\
\frac{\Delta\Gamma_s}{\Delta M_s} &= (4.20^{+0.36}_{-0.39}{}_{\substack{\text{LP}\\\text{scale}}}\,{}^{+0.09}_{-0.19}{}_{\substack{\text{NLP}\\\text{scale}}} \pm 0.12_{B\tilde{B}_S} \pm 0.78_{1/m_b} \pm 0.05_{\text{input}}) \times 10^{-3} \quad (\text{PS}),
\end{aligned}
\tag{12}
$$

where "LP scale" and "NLP scale" refer to the variations of the renormalization scale μ at leading and subleading powers in the $1/m_b$ expansion. The subscripts "$1/m_b$", "$B\tilde{B}_S$" and "input" denote the uncertainties in the matrix elements entering the $1/m_b$-corrections, leading-power bag parameters and the remaining input parameters respectively. As one can see, the former type of uncertainties now dominate the error budget, while the scale uncertainty at leading power is under much better control than before. This can be also inferred from Fig. 2. Evidently, both $\overline{\text{MS}}$- and PS-scheme results feature an improved μ-dependence at NNLO as compared to NLO, while at $\mu \approx 9$ GeV the NNLO correction vanishes altogether. Another interesting observation is the obvious failure of the pole scheme at NNLO in the sense that the predicted value is much smaller than that of the other

[3]The Wilson coefficients of all operators are always renormalized in the $\overline{\text{MS}}$ scheme. The same is true also for the coupling constants, gauge parameter ξ and the quark field renormalization constant Z_2.

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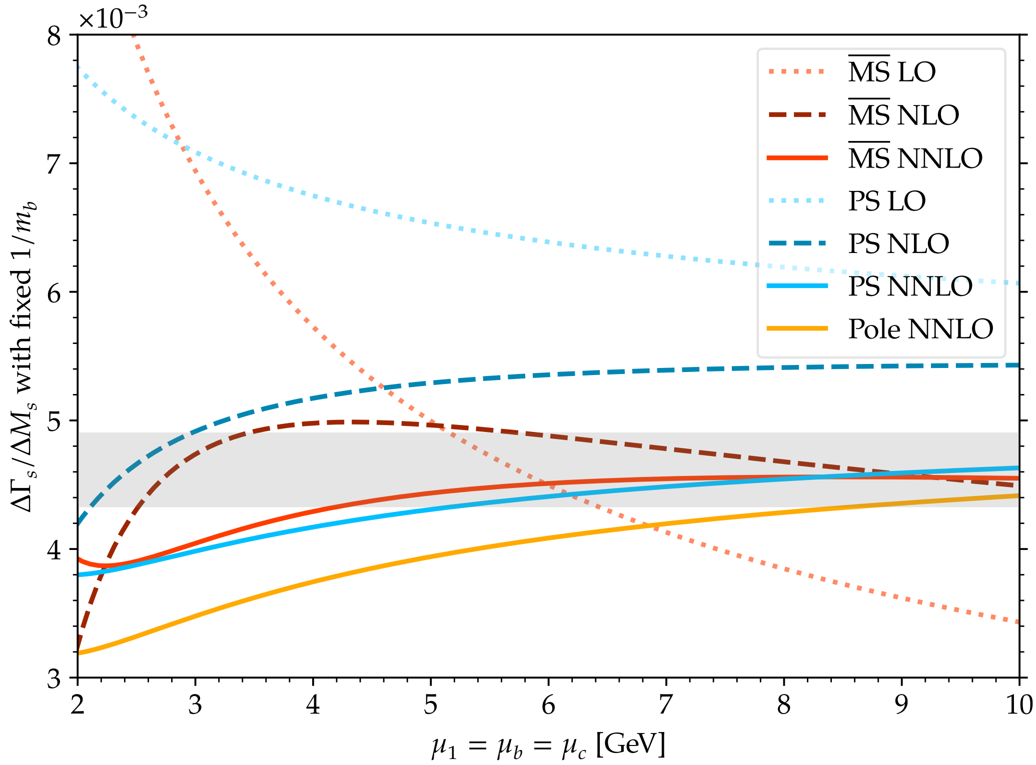

Figure 2: Dependence of the ratio $\Delta\Gamma_s/\Delta M_s$ on the renormalization scale at LO, NLO and NNLO for the $\overline{\text{MS}}$ and PS schemes from [15]. To illustrate the impact of our work, we keep the μ-dependence of the subleading power terms fixed. The gray band denotes the experimental value.

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schemes. Technically, this behavior can be linked to large perturbative corrections present in the two-loop conversion formula between $\overline{\text{MS}}$ and the pole bottom quark mass. Therefore, we conclude that this scheme is poorly suited for predicting $\Delta\Gamma_s$ at NNLO.

Finally, we can use the experimentally measured value for ΔM_s [71]

$$\Delta M_s^{\text{exp}} = 17.7656 \pm 0.0057 \text{ ps}^{-1}, \tag{13}$$

to extract $\Delta\Gamma_s$ from our prediction for the ratio. Our final prediction obtained from taking the average of the $\overline{\text{MS}}$ and PS results, adding the uncertainties in quadrature and symmetrizing the scale dependence reads

$$\Delta\Gamma_s = (0.076 \pm 0.017) \text{ ps}^{-1}. \tag{14}$$

Comparing this number to Eq. (3) we see that the theory uncertainty is now only three times bigger than the experimental one. Furthermore, as already explained above, the perturbative QCD corrections at leading power are not anymore the main obstacle on the way to a more precise determination of $\Delta\Gamma_s$.

5. Summary

Using matching coefficients calculated in a series of related publications [13–15] we were finally able to achieve the NNLO accuracy in our theory prediction for $\Delta\Gamma_s$. Our results in $\overline{\text{MS}}$ and PS schemes are in good agreement with the experimental measurements and feature significantly reduced uncertainties as compared to the previous predictions. More importantly, the main source for the remaining theoretical errors now lies in the $1/m_b$-contributions to $\Delta\Gamma_s$, where missing perturbative corrections and poorly known hadronic matrix elements severely impact the achievable theory precision.

All two-loop matching coefficients were made available in a computer-readable format [13, 14], while a further publication providing analytic three-loop matching coefficients and related building blocks (e. g. two-loop $|\Delta B| = 2$ operator renormalization matrix) is currently in preparation. While our current results are accurate at $O(z)$, the calculation of higher orders in the z-expansion at two and three loops is already in progress.

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