Lepton Phenomenology of Stueckelberg Portal to Dark Sector

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We propose an extension of the Standard Model (SM) with a $U_{A'}(1)$ gauge invariant Dark Sector connected to the SM via a new portal arising in the framework of dark photon A' mass generation via Stueckelberg mechanism. This mechanism implies the existence of a scalar field σ, which is shift-transformed under this group and resembles an axion-like particle (ALP) widely addressed in the literature in different contexts. The effective dim=5 operators constructed of the covariant derivative of the σ field generate flavor non-diagonal renormalizable couplings of both σ and A' to the SM fermions ψ. Contrary to the conventional kinetic mixing portal, in our scenario flavor diagonal A'-ψ couplings are not proportional to the fermion charges. These features drastically change the phenomenology of dark photon A' relaxing or avoiding some previously established experimental constraints. We focus on the phenomenology of the described scenario of the Stueckelberg portal in the lepton sector and analyze the contribution of the dark sector fields A' and σ to the anomalous magnetic moment of muon $(g-2)_\mu$, Lepton Flavor Violating decays $l_i \to l_k \gamma$ and $\mu - e$ conversion in nuclei. We obtain limits on the model parameters from the existing experimental data on the corresponding observables.

I. INTRODUCTION

The idea of the Dark Sector (DS) of the Universe, existing almost independently of the Standard Model (SM) sector, has attracted growing interest in recent years. Originally, DS was thought to be populated by only one dark species, necessary to make up for the lack of matter in the universe with dark matter (DM). Extensions of DS was, in particular, motivated by the popular scenario of Light sub-GeV DM. It was realized that in this case a dark boson, known as the Dark Photon, would need to be introduced to prevent the Universe from over-closing. An extended DS can have not only cosmological, but also interesting phenomenological consequences. This DS physics beyond the SM can manifest itself in the phenomena observable experimentally (for a status report see, e.g., Ref. [1]).

Presently, there are a number of experiments to search for DS physics and others are planned for the near future. Among them we mention CERN based experiments NA64 [2, 3], NA62 [4], SHiP [5–7], LHCb [8], ATLAS [9], CMS [10] and *BaBar* experiment at SLAC [11], HPS at JLab [12], Belle at KEK [13]. So far no signal of DS or other kind of new physics beyond the SM (BSM) is observed.

An encouraging indication of new physics has recently come from measurements of the anomalous magnetic moment (AMM) of the muon $(g-2)_\mu$. The Fermilab Muon g-2 Collaboration published [14] the observation of 4.2 σ deviation of the $(g-2)_\mu$ from its SM value and stimulated an explosion of the BSM literature. As is known, measurements of $(g-2)_\mu$ are a very sensitive probe of BSM physics. The Fermilab Muon $g-2$ result with such unprecedented precision can severely limit or refute many BSM models.

On the other hand, there is no doubt that the SM is an incomplete theory, requiring some physics beyond its scope to explain a number of problems that cannot be addressed in the SM. Among them, the DM problem is one of the most obvious. As we already mentioned, DM hints at the existence of a DS of the Universe, which not only provides DM particle candidates, but is also populated by other particles involved in interactions governed by some dark symmetries. The DS with possible non-trivial physics could have a phenomenological impact on the SM sector through portals such as the well-known kinetic mixing of dark and normal photons. Other hypothetical DS particles can have access to the SM sector through different portals and contribute to various observables, in particular, to

$(g-2)_\mu$, allowing one to probe the DS.

We should stress that there is much evidence of deviation from SM. Besides $(g-2)$ of muon [15] they are strong CP problem and rare meson decays [16]-[20], flavor non-universality [21, 22], $b-s$ quark anomaly, and others. It motivates theoretical study/constructing of effective Lagrangians beyond Standard Model (BSM) trying to involve new particles/portals, like axion, dark photon, vectors, pseudoscalars, scalars, and axials, etc. [1, 5],[23]-[53].

Here, we propose an extension of the SM by inclusion of DS with a softly-broken $U_{A'}(1)$ symmetry. The corresponding gauge boson A', also known as Dark Photon, acquires a non-zero gauge invariant mass via the Stueckelberg mechanism [54, 55], which implies the existence of a scalar Stueckelberg field σ.

This field opens a new portal from the SM to the Dark sector via the effective dimension-5 operator with the covariant derivative of the σ-field. We call it the Stueckelberg portal. In our setup this portal coexists with the conventional kinetic mixing portal and leads to new phenomenological effects in the SM sector, in particular, flavor violation both in lepton and quark sectors. In the present work we focus on the Lepton Flavor Violation (LFV) and the corresponding experimental observables.

We also introduce one dark fermion, χ, charged under $U(1)_D$, which is a viable Light DM particle candidate. We postulate that DSB plays important role in this model: (1) generate mass of dark gauge boson (DGB) (Dark photon) via Stueckelberg mechanism [54, 55]; (2) generate a mixing of DS with SM fermion including couplings preserving and violating symmetries of SM (e.g., lepton flavor violation (LFV)). Interaction of DGB and DSB with fermions is based on idea of familon (or flavons) [33, 42, 56].

The paper is organized as follows. In Sec. II we describe our theoretical setup. In Sec. III, IV and V we consider application of the Stueckelberg portal to phenomenological aspects of the $g-2$ lepton anomaly, lepton conversion and rare lepton-flavor violating decays $l_i \to l_k \gamma$ which were used to derive limits for couplings occurring in the Stueckelberg portal. In Sec. VI we discuss boundary to couplings from DSB and DGB and a possible contribution to $g-2$ lepton from obtained restrictions for different channels. The Sec. VII is the conclusion. Technical details of our calculations are placed in Appendices.

II. THEORETICAL SETUP

We consider the conventional Dark Sector scenario with the $U_{A'}(1)$ extension of the SM gauge symmetry. We suppose that Dark Sector (DS), blind to the SM interactions, is populated with Dirac fermions χ_i charged under $U_{A'}(1)$. The lightest of which is stable and plays the role of Dark Matter. By definition, all the SM fields are neutral with respect to this group. The gauge boson, A', of the Dark sector $U_{A'}(1)$ group is conventionally called Dark Photon. In the conventional Dark Photon scenario A' acquires its mass $M_{A'}$ from spontaneous breaking of $U_{A'}(1)$ group. In contrast, in our approach its mass is a gauge invariant quantity generated by the Stueckelberg mechanism. The latter requires the introduction of a scalar Stueckelberg field σ. Gauge invariant Lagrangian of the model reads

$$\mathcal{L}_{\text{SM+DS}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{DS}} + \mathcal{L}_{\text{mix}} + \mathcal{L}_{\text{int}}, \tag{1}$$

where $\mathcal{L}_{\text{SM}}$ is the SM Lagrangian. The terms denoted by $\mathcal{L}_{\text{DS}}$ describe Dark Sector according to

$$\mathcal{L}_{\text{DS}} = -\frac{1}{4} A'_{\mu\nu} A'^{\mu\nu} + \frac{1}{2} D_\mu \sigma D^\mu \sigma + \bar{\chi}\left(i\,\slashed{D}_\chi - m_\chi\right)\chi, \tag{2}$$

where, as usual, $A'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu$ and

$$\left(iD_\chi\right)_\mu = i\partial_\mu - g_{A'} A'_\mu. \tag{3}$$

is the conventional covariant derivative. The Stueckelberg covariant derivative is defined as

$$D_\mu \sigma = \partial_\mu \sigma - M_{A'} A'_\mu \tag{4}$$

The $U_{A'}(1)$-symmetry is realized on the Dark Sector fields according to the transformations

$$A'_\mu \ \to \ A'_\mu + \frac{i}{g_{A'}}\,\partial_\mu \Omega_{A'}\,\Omega_{A'}^{-1}, \qquad A'_{\mu\nu} \ \to \ A'_{\mu\nu}, \tag{5}$$

$$\sigma \ \to \ \sigma - \frac{M_{A'}}{g_{A'}}\theta_{A'}, \qquad \partial_\mu \sigma \ \to \ \partial_\mu \sigma + \frac{iM_{A'}}{g_{A'}}\,\partial_\mu \Omega_{A'}\,\Omega_{A'}^{-1}, \qquad D_\mu \sigma \to D_\mu \sigma, \tag{6}$$

$$\chi_D \ \to \ \Omega_{A'}\,\chi_D, \qquad i\,\slashed{D}_{\chi_D}\,\chi_D \to \Omega_{A'}\,i\,\slashed{D}_{\chi_D}\,\chi_D, \tag{7}$$

where

$$\Omega_{A'}(x) = \exp\left[i\theta_{A'}(x)\right]. \tag{8}$$

As seen from from the last relation in (7), the σ is an axion-like field shift-transformed under the $U_{A'}(1)$. In our framework the $U_{A'}(1)$-symmetry is softly broken by the mass M_σ of the σ-field

$$\mathcal{L}_{\text{DS}}^{\text{mass}} = -\frac{1}{2}\,M_\sigma^2\sigma^2\,. \tag{9}$$

We introduce this term in order to avoid massless scalars.

We also introduce the conventional portal to Dark sector via kinetic mixing of the Dark and the SM photons, $A - A'$, according to

$$\mathcal{L}_{\text{mix}} = -\frac{\epsilon_A}{2}\,F_{\mu\nu}A'^{\mu\nu}\,, \tag{10}$$

where ϵ_A is the mixing parameter. This term is necessary in our model in in order to close it under renormalization after we introduce the Stueckelberg portal interactions denoted in Eq. (1) as $\mathcal{L}_{\text{int}}$. These interactions appear at the level effective operators of dim=4 (in case of couplings of Dark photon with SM fermions) and dim=5 (in case of coupling of DSB with SM fermions) and have the form

$$\mathcal{L}_{\text{int}} = \frac{1}{\Lambda}\,D_\mu\sigma\sum_{ij}\left[\bar{Q}^i\chi_\sigma^{ij}\gamma^\mu Q^j + \bar{u}_R^i\chi_{u\sigma}^{ij}\gamma^\mu u_R^j + \bar{d}_R^i\chi_{d\sigma}^{ij}\gamma^\mu d_R^j + \bar{L}^{im}\kappa_{L\sigma}^{ij}\gamma^\mu L^{jm} + \bar{\ell}_R^i\kappa_{R\sigma}^{ij}\gamma^\mu\ell_R^j\right] + \mathcal{L}_{\sigma FF}\,, \tag{11}$$

which is invariant under the $SU_c(3) \times SU_{2L}(2) \times U_Y(1) \times U_{A'}(1)$ gauge group. The fields belong to the following representations of this group $Q(3, 2; 1/3; 0)$, $u_R(3, 1; 4/3; 0)$, $d_R(3, 1; -2/3; 0)$, $L(1, 2; -1; 0)$, $\ell_R(1, 1; -2; 0)$. The parameter Λ is the characteristic scale of this effective operator, defining when it opens up in terms of renormalizabale interactions of an UV completion.

The term $\mathcal{L}_{\sigma FF}$ is defined as

$$\mathcal{L}_{\sigma FF} = \sigma\left[g_{\sigma\gamma\gamma}\,F_{\mu\nu}\tilde{F}^{\mu\nu} + \bar{g}_{\sigma\gamma\gamma}\,F_{\mu\nu}F^{\mu\nu}\right]. \tag{12}$$

where $\tilde{F}^{\mu\nu}$ is the dual tensor. We included this term for the similar reason as the kinetic mixing term (10): they both are generated at loop-level from the interactions of σ with the SM fermions and are needed for closure of the model under renormalization.

In Eq. (11) the parameters χ^{ij} and κ^{ij} form $3 \otimes 3$ hermitian matrices leading to the neutral current flavor violation both in quark and lepton sectors. In the present work we focus only on the lepton sector and make ad hoc assumption $\chi_\sigma = \chi_{u\sigma} = \chi_{d\sigma} = 0$. As it was pointed out in Ref. [31] after spontaneous breaking of electroweak symmetry in SM one should diagonalize the fermions mass matrices by means of unitary transformations involving the matrices V_L^q and V_L^ℓ acting on the left-handed quarks and leptons, respectively, and W_R^U, W_R^D, W_R^ℓ are the transformation matrices acting on right singlets. It will have impact on the couplings of dark scalars with SM fermions. In particular, they will rotate as

$$
\begin{aligned}
\chi_\sigma &\rightarrow X_\sigma = (V_L^q)^\dagger\,\chi_\sigma\,V_L^q\,,\\
\chi_{U\sigma} &\rightarrow X_{U\sigma} = (W_R^U)^\dagger\,\chi_{U\sigma}\,W_R^U\,,\\
\chi_{D\sigma} &\rightarrow X_{D\sigma} = (W_R^D)^\dagger\,\chi_{D\sigma}\,W_R^D\,,\\
\kappa_{L\sigma} &\rightarrow K_{L\sigma} = (V_L^q)^\dagger\,\kappa_{L\sigma}\,V_L^q\,,\\
\kappa_{R\sigma} &\rightarrow K_{R\sigma} = (W_R^D)^\dagger\,\kappa_{R\sigma}\,W_R^U\,.
\end{aligned}
\tag{13}
$$

It is convenient to remove the kinetic mixing term (10) by the field redefinition. As usual, we shift the SM photon field

$$A_\mu \rightarrow A_\mu - \epsilon_A\,A'_\mu \tag{14}$$

and, then, rescale the Dark photon field

$$A'_\mu \rightarrow A'_\mu\,(1 - \epsilon_A^2)^{-1/2}\,. \tag{15}$$

These redefinitions generate flavor diagonal couplings of the SM fermions, ψ, to the dark photon originating from the kinetic mixing term:

$$\mathcal{L}_{\text{mix}}^{\text{A}'-\psi} = e\,\epsilon_A\,A'_\mu\,\bar{\psi}\,\gamma^\mu\,T_Q\,\psi\,, \tag{16}$$

where T_Q is the charge matrix. Now let us examine the first term in (11). It generates interactions of σ and A' fields with the SM fermions. The latter takes form

$$\mathcal{L}^{\text{A}'-\psi} = A'_\mu \sum_{ij} \bar{\psi}_i \gamma^\mu \left(g^V_{ij} + g^A_{ij}\gamma_5 \right) \psi_j\,, \tag{17}$$

where we defined vector g^V and axial-vector g^A dimensionless couplings, which are proportional to the factor $M_{A'}/\Lambda$, which multiplies linear combinations of the dimensionless couplings χ, κ present in Eq. (11). Such mass behavior is crucial for analysis of the A' contribution and setting limits on the couplings taking into account the dependence on the intermediate state mass.

It is clear that the terms (16), originating from the kinetic mixing, are completely absorbed by the redefinition of the flavor diagonal matrix elements of the coupling g^V_{ii}. Thus, in our approach the Lagrangian (17) describes interactions of the Dark Photon A' to the SM fermions with arbitrary dimensionless flavor non-diagonal couplings $g^{V,A}_{ij}$. Let us highlight two principal differences between the conventional kinetic portal and the Stueckelberg portal scenarios of DS. First, in the latter case contrary to the former one the A' couplings to the SM fermions are not proportional to the SM fermion electric charges. Second, these couplings are flavor non-diagonal leading to reach LFV phenomenology. Note that the first point can significantly affect the conclusions of the existing searches of the Dark Photon for the case of the Stueckelberg scenario. In particular, the conventional dark photon from the kinetic portal scenario has been strongly constrained from the data of NA64 experiment at SPS CERN [2, 3].

Finally, let us consider the interaction terms of the σ-field. As follows from (11), we have

$$\mathcal{L}_\sigma = \mathcal{L}_{\sigma FF} + \frac{1}{\Lambda}\partial_\mu\sigma \sum_{ij} \bar{\psi}_i \gamma^\mu \left(v_{ij} + a_{ij}\gamma_5 \right) \psi_j\,. \tag{18}$$

Where v_{ij} and a_{ij} are generic hermitian matrices of couplings. Using equations of motion for fermion fields one can rewrite derivative coupling of scalar field with fermions in the equivalent non-derivative form:

$$\mathcal{L}_{\sigma\bar{\psi}\psi} = i\sigma \sum_{ij} \bar{\psi}_i \left(g^S_{ij} - g^{PS}_{ij}\gamma_5 \right) \psi_j\,, \tag{19}$$

were we introduced dimensionless scalar and pseudoscalar couplings

$$g^S_{ij} = \frac{v_{ij}(m_i - m_j)}{\Lambda}; \qquad g^{PS}_{ij} = \frac{a_{ij}(m_i + m_j)}{\Lambda}\,. \tag{20}$$

Note that matrices v_{ij} and a_{ij} are Hermitian: $v_{ij} = v^*_{ji}$ and $a_{ij} = a^*_{ji}$. From Eq. (19) one can see, that the flavor diagonal scalar couplings of σ to the SM fermions vanish $g^s_{ii}=0$. Note that the similar Lagrangian was obtained in Ref. [56] in the framework of a flavor model with σ in the role of familon. Currently, this part of the effective Lagrangian is considered as the part of low-energy action for axion-like particles (ALPs) [31, 41, 42, 45].

In the subsequent sections we will study contributions of the dark sector fields A' and σ to muon anomalous magnetic moment $(g-2)_\mu$ and LFV decays $l_i \to l_j\gamma$ and $\mu - e$ conversion in nuclei.

III. ANOMALOUS MAGNETIC MOMENT

To derive the boundaries on coupling constants we use existing difference between theoretical prediction based on SM and available actual data on $(g-2)$ of leptons [14, 15, 57, 58]:

$$\begin{aligned}
\Delta a_e &= a_e^{exp} - a_e^{SM} = 8.7 \times 10^{-13}, \\
\Delta a_\mu &= a_\mu^{exp} - a_\mu^{SM} = 251 \times 10^{-11}, \\
\Delta a_\tau &= a_\tau^{exp} - a_\tau^{SM} = 279 \times 10^{-6}\,.
\end{aligned} \tag{21}$$

Discrepancy between theory and experiments for $(g-2)$ in case of τ lepton is huge because the lifetime τ lepton is short and experimental measurements of $(g-2)$ is very difficult.

Study of $(g-2)$ anomaly helps to derive the upper limits for couplings of DGB and DSB with SM fermions. Here DM particles propagate in the loops, see Fig. 1.
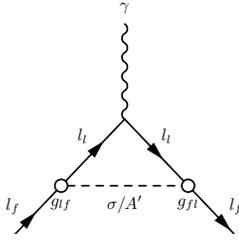
FIG. 1: Feynman diagrams which gives a contribution to AMM of leptons due to the exchange by intermediate DGB or DSB.

A. Vector boson contribution

Contribution massive vector boson to $(g-2)$ with taking into account LFV effect in the covariant gauge has a form:

$$\delta a_f = U_{fl} \frac{g_{lf}^V g_{fl}^V}{4\pi^2} \left(\frac{m_f}{m_l}\right) \int_0^1 dy\, y(1-y) \frac{\frac{m_f}{m_l}(y-1)-1}{\frac{m_A^2}{m_l^2}y + (1-y)\frac{m_f^2}{m_l^2}} \,, \tag{22}$$

where m_l is mass of a fermion which propagates in the loop, m_A is the mass of the dark photon A'. In case when $m_l = m_f$ and in the limits $m_A^2 \to 0$ and $g_{ij}^V \to e$ we reproduce a famous Schwinger term contribution to fermion anomalous magnetic moment.

The upper limits on dimensionless coupling $g_{lf}^V = m_A \frac{v_{lf}}{\Lambda}$ of vector dark photon with fermion are given in Fig. 2. Contribution induced by LFV coupling and effect and light particle propagating in the loop is negative and cannot be considered as a limit.
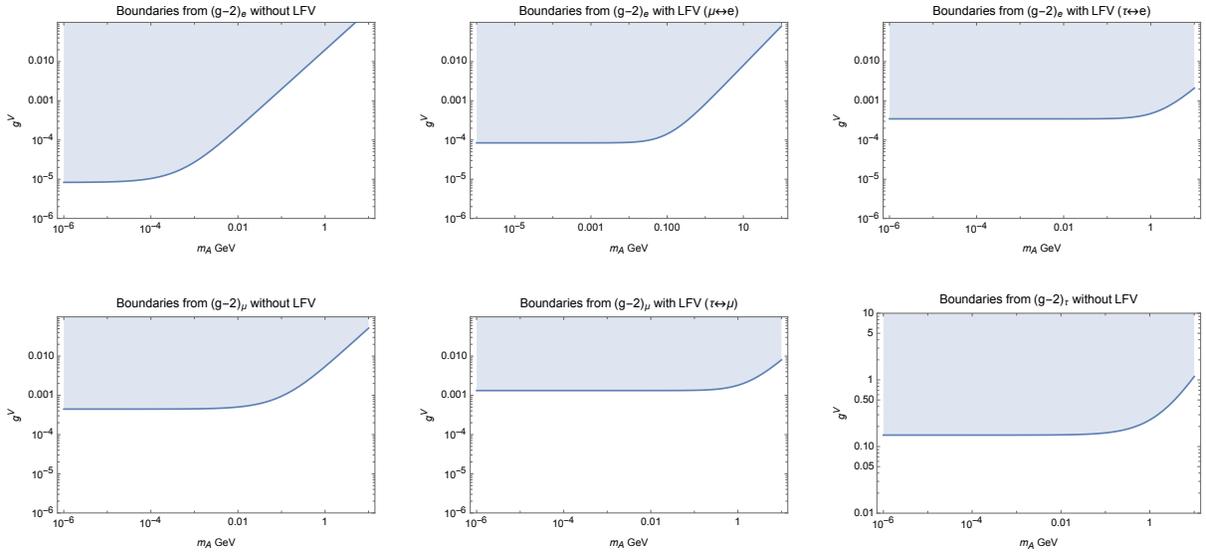
FIG. 2: Upper boundaries on the couplings g^V as function of dark photon mass m_A' derived using data on leptonic $(g-2)$ coupling. The shaded area is prohibited by data on $(g-2)$.
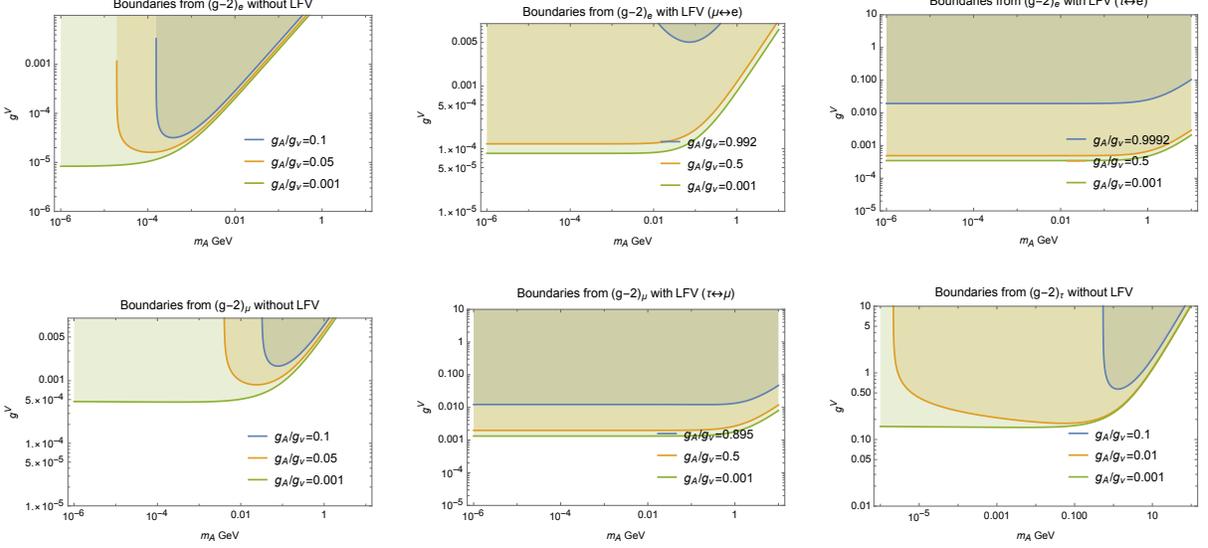
FIG. 3: Bounds from $(g-2)$ of leptons on the couplings of dark boson in the vector+axial-vector channel in dependence on different ratio of the channel constants d_V and d_A. The shaded area is prohibited from $(g-2)$ lepton data.

B. Axial-vector contribution

The contribution to AMM from axial-vector channel is

$$
\delta a_f^{AV} = -\frac{(g_{fl}^{AV} g_{lf}^{AV})}{4\pi^2} \left(\frac{m_f}{m_l}\right) \int_0^1 dx \ x(1-x) \frac{\frac{m_f}{m_l}(1+x)+2}{\frac{m_A^2}{m_l^2}x + (1-x)(1-\frac{m_f^2}{m_l^2}x)} \ ,
\tag{23}
$$

where $g_{lf}^{AV} = m_A \frac{a_{lf}}{\Lambda}$ is axial-vector dimensionless coupling which, in general, is not equal to g_{lf}^V. If $d_{L_{ij}}$ and $d_{R_{ij}}$ coupling constants are positive and real when axial-vector coupling should be less than vector coupling on magnitude.

The contribution from axial-vector channel is negative. In this case we obtain that sum of vector and axial-vector channels is analogue of the contribution of electroweak Z-boson to $(g-2)$, which is negative. In case, when g^{AV} couplings are suppressed, vector contribution is dominant.

In Fig. 3 we show boundaries for couplings g_{lf}^V as function of m_A' and in dependence on ratio g^{AV}/g^V. It is very important to note that the combination of vector and negative axial-vector contributions to $(g-2)_\mu$ makes limits less stringent in comparison with the case of using pure vector term. This is a chance for existence of light dark vector particles in more wide band of possible masses and couplings. It is worth noting that for the case of boundary without LFV difference between g_{lf}^V and g_{lf}^{AV} couplings can be less but allowable area will be wide. In the case when we have heavy particle propagating in the loop, negative contribution from axial-vector part are strongly suppressed.

C. Scalar contribution

As we stressed before, scalar contribution is defined by new boson σ occurs only when LFV effects are taken into account and has the following form

$$
\delta a_f^S = \frac{g_{fl}^S g_{lf}^S}{8\pi^2} \left(\frac{m_f}{m_l}\right) \int_0^1 dx \ (1-x)^2 \frac{1-\frac{m_f}{m_l}x}{\frac{m_S^2}{m_l^2}x + (1-x)(1+\frac{m_f^2}{m_l^2}x)} \ ,
\tag{24}
$$

where m_S and g_{fl}^S are the mass and coupling of a new scalar boson introduced before. In the case when particle propagating in the loop is lighter then initial particle, the contribution is negative and we cannot constraint couplings
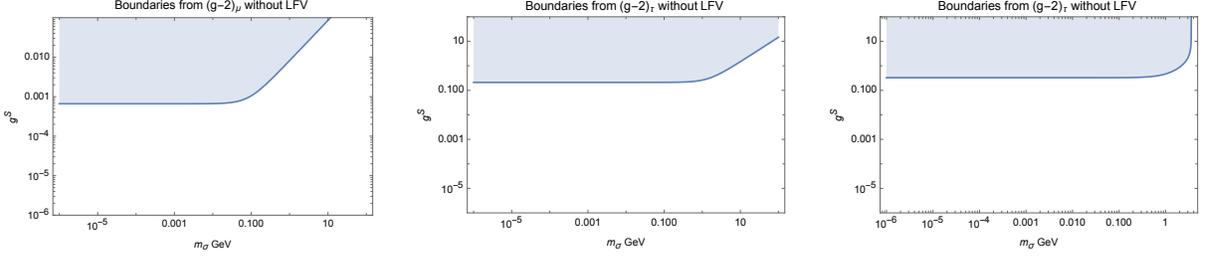
FIG. 4: Bounds from $g - 2$ of leptons for couplings g_{ij}^{S} of scalar boson with fermions in dependence on m_σ. The shaded area is prohibited from $(g - 2)$ lepton data.
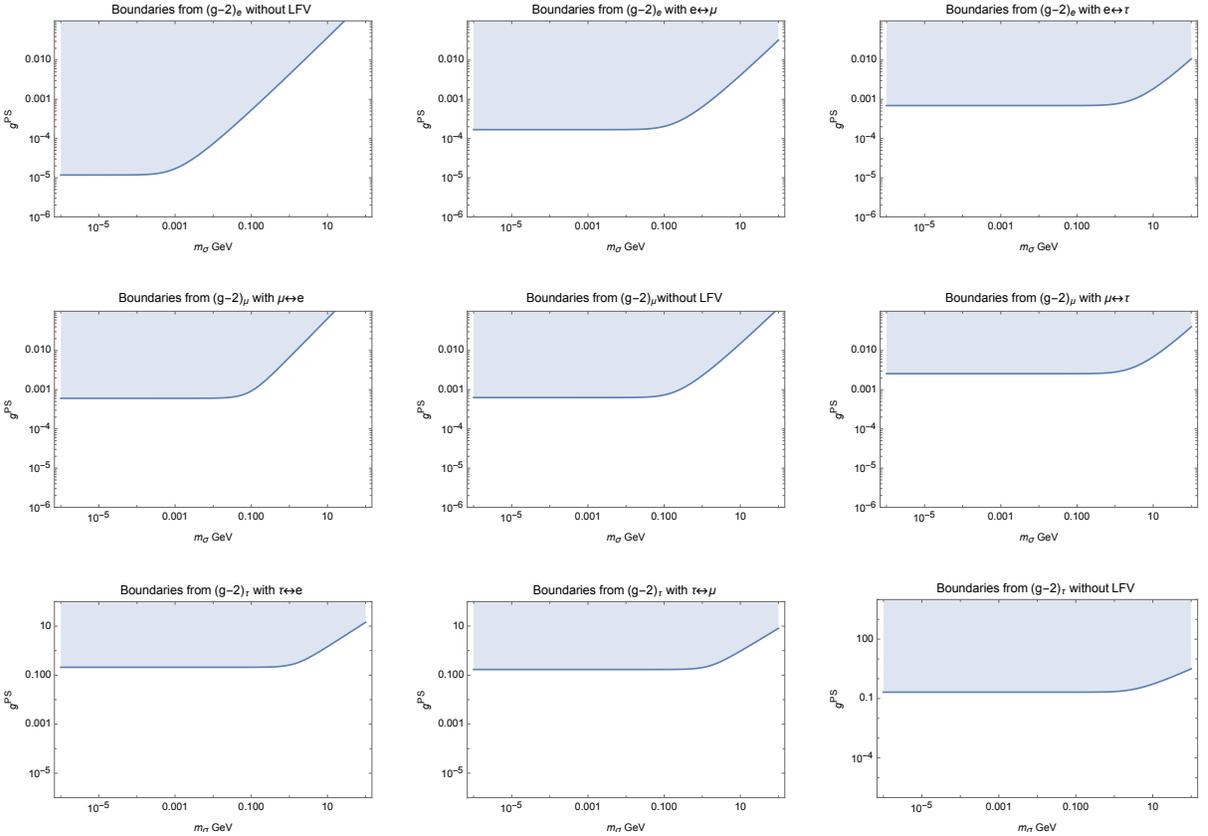
FIG. 5: Bounds from $g - 2$ of leptons on the couplings g_{ij}^{PS} of scalar boson with fermions in dependence on m_σ. The shaded area is prohibited from $(g - 2)$ lepton data.

and should use boundary from $(g - 2)$ of more heavy leptons. Scalar dimensionless coupling boundaries from $(g - 2)$ of lepton are presented in Fig. 4.

D. Pseudoscalar contribution

Pseudoscalar dimensionless coupling boundaries from $(g - 2)$ of lepton are presented in Fig.5. Upper boundary from τ lepton $(g - 2)$ is huge. It is connected with the difficulty of measurements of this value for τ lepton. Mass of τ lepton is more heavy them the masses other leptons and main band of mass range of new scalar meson because the boundary for lepton conserving coupling and coupling which include LFV effects have the same restriction.

The form of pseudoscalar contribution to $(g-2)$ of lepton from new σ DSB is:

$$\delta a_f^{PS} = \frac{(g_{fl}^{PS})^2}{8\pi^2} \left(\frac{m_f}{m_l}\right) \int_0^1 dx \, (1-x)^2 \frac{1 + \frac{m_f}{m_l}x}{\frac{m_S^2}{m_l^2}x + (1-x)(1+\frac{m_f^2}{m_l^2}x)}, \tag{25}$$

which have a difference with scalar only in sign at term $\frac{m_f}{m_l}x$ in numerator. The light lepton state in loop non play a role in dependence of AMM of one and we have a similar behavior for different couplings from same AMM quantity.

IV. NUCLEAR LEPTON CONVERSION

Here we estimate our LFV couplings between dark photon and leptons using data (uppers limits) of the SINDRUM II searches for $\mu - e$ conversion on ^{198}Au [59]. Together with these limits we consider the limits corresponding to the future experiment PRISM/PRIME [60] with titanium ^{48}Ti target aiming at the sensitivity of 10^{-18}. We have for these two cases:

$$R_{\mu e}^{\mathrm{Au}} \le 4.3 \times 10^{-12} \ [59]: \quad \alpha_{A'}^{V(A)} \left(\frac{1\,\mathrm{GeV}}{\Lambda_{\mathrm{LFV}}}\right)^2 \le 8.5 \times 10^{-13}, \tag{26}$$

$$R_{\mu e}^{\mathrm{Ti}} \lesssim 10^{-18} \qquad [60]: \quad \alpha_{A'}^{V(A)} \left(\frac{1\,\mathrm{GeV}}{\Lambda_{\mathrm{LFV}}}\right)^2 \le 1.6 \times 10^{-15}, \tag{27}$$

where $\alpha_{A'}^{V(A)}$ is defined from effective Lagrangian which describes nucleon-lepton LFV coupling

$$\mathcal{L}_{\mathrm{eff}}^{\ell N} = \bar{N}\gamma^\mu N \ \bar{e} \left[\alpha_{A'}^V \gamma_\mu + \alpha_{A'}^A \gamma_\mu \gamma_5\right] \mu + \mathrm{H.c.} \tag{28}$$

These couplings $\alpha_{A'}^{V(A)}$ are related to couplings g_{ij}^V of dark photon as [see details in Ref. [61]]:

$$\alpha_{A'}^{V(A)} \simeq g_{NN}^{V(A)} g_{e\mu}^{V(A)} \frac{m_A^2}{m_A^2 + m_\mu^2}, \tag{29}$$

where $g_{NN}^{V(A)}$ is the dark photon A'-nucleon coupling, which we can put equal $g_{NN}^{V(A)} = m_A/\Lambda$ in flavor conservation scenario as for lepton. The vector couplings with leptons were defined before. Using this definition and approximation proposed in [61], we can get boundary for LFV coupling as

$$|v_{e\mu}| \simeq \begin{cases} 8.5 \times 10^{-13} \left[\frac{m_A}{\Lambda}\right]^{-2} \left(\frac{m_A^2}{m_A^2 + m_\mu^2}\right)^{-1} & \mathrm{SINDRUM} \\[2ex] 1.6 \times 10^{-15} \left[\frac{m_A}{\Lambda}\right]^{-2} \left(\frac{m_A^2}{m_A^2 + m_\mu^2}\right)^{-1} & \mathrm{PRISM/PRIME} \end{cases} \tag{30}$$

Similar limits we can establish for the t-channel scalar boson exchange and obtain boundary for a_{ij} LFV couplings from $\mu - e$ conversion restriction using that $g_{NN}^{PS} = 2m_N/\Lambda$, where m_N is the nucleon mass:

$$|a_{e\mu}| \simeq \begin{cases} 8.5 \times 10^{-13} \left[\frac{m_\mu m_N}{\Lambda^2}\right]^{-1} \left(\frac{m_\sigma^2}{m_\sigma^2 + m_\mu^2}\right)^{-1} & \mathrm{SINDRUM} \\[2ex] 1.6 \times 10^{-15} \left[\frac{m_\mu m_N}{\Lambda^2}\right]^{-1} \left(\frac{m_\sigma^2}{m_\sigma^2 + m_\mu^2}\right)^{-1} & \mathrm{PRISM/PRIME} \end{cases} \tag{31}$$

Mass dependence of DGB coupling $g_{ij}^V = \frac{v_{ij} m_A}{\Lambda}$ gives mass square suppression for upper limits $|v_{e\mu}|$ from lepton conversion by DGB exchange. The case DSB from Stueckelberg portal has other suppression factor which is not connected with mass of mediator state.

V. LFV DECAYS $l_i \to l_k\gamma$

Rare LFV decays $l_i \to l_k\gamma$ due to LFV effect generated by dark vector or scalar bosons vanish. Scalar boson have two different types of Feynman diagrams which generate this rare decay. First is analog Barr-Zee diagram by accounting scalar two photon transition [see diagrams (a-b) in Fig. 6] and second type is triangle diagram with two additional diagrams which needed for fulfillment of gauge invariance [see diagrams (c-e) in Fig. 6]. Dark photon A' also gives contribution to this rare LFV decay [see diagrams in Fig. 6(c-e)].

In general matrix element describing this LFV process can be parameterized as

$$iM_{ik} = ie\epsilon^\mu(q)\bar{u}_i(p_2, m_e)\left[\frac{i}{2m_i}\sigma_{\mu\nu}q^\nu F_M^{LFV} + \frac{i}{2m_i}\sigma_{\mu\nu}q^\nu\gamma_5 F_D^{LFV}\right]u_k(p_1, m_\mu). \tag{32}$$

Square of the amplitude is

$$|M|^2 = \frac{1}{m_i^2}((F_M^{LFV})^2 + (F_D^{LFV})^2)(m_i^4 - 2m_k^2 m_i^2 + m_k^4). \tag{33}$$

Using approximation $m_e \ll m_\mu \ll m_\tau$, decay width for this rare decay is

$$\Gamma(l_i \to el_k) = \frac{1}{2m_i}\int \frac{d^3q\, d^3p_2}{4E_2 E_q(2\pi)^6}|M|^2(2\pi)^4\delta^{(4)}(p_1 - p_2 - q) \tag{34}$$
$$= \frac{m_i}{8\pi}e^2\left(|F_M^{LFV}|^2 + |F_D^{LFV}|^2\right),$$

where F_M and F_D are dimensionless form-factors in analogy of magnetic and dipole which goes from different channel of interaction and with different intermediate leptons in loops.

Contributions induced by diagrams in Figs. 6(a) and 6(b) for CP even and CP-odd vertices are connected with CP-even and CP-odd couplings with gauge bosons of SM [see Eq. 12]. These contributions have the form

$$F_M = -\frac{m_i}{8\pi^2}g_{ik}^{PS}g_{\sigma\gamma\gamma}h_{\sigma\gamma\gamma}^1(x_\mu) = -\frac{m_i^2}{8\pi^2\Lambda}a_{ik}g_{\sigma\gamma\gamma}h_{\sigma\gamma\gamma}^1(x_\mu),$$
$$F_D = -\frac{m_i}{8\pi^2}g_{ik}^S g_{\sigma\gamma\gamma}h_{\sigma\gamma\gamma}^1(x_\mu) = -\frac{m_i^2}{8\pi^2\Lambda}v_{ik}g_{\sigma\gamma\gamma}h_{\sigma\gamma\gamma}^1(x_\mu) \tag{35}$$

for CP-even coupling and

$$F_M = -\frac{m_i}{8\pi^2}g_{ik}^S\bar{g}_{\sigma\gamma\gamma}h_{\sigma\gamma\gamma}^2(x_\mu) = -\frac{m_i^2}{8\pi^2\Lambda}v_{ik}\bar{g}_{\sigma\gamma\gamma}h_{\sigma\gamma\gamma}^2(x_\mu),$$
$$F_D = -\frac{m_i}{8\pi^2}g_{ik}^{PS}\bar{g}_{\sigma\gamma\gamma}h_{\sigma\gamma\gamma}^2(x_\mu) = -\frac{m_i^2}{8\pi^2\Lambda}a_{ik}\bar{g}_{\sigma\gamma\gamma}h_{\sigma\gamma\gamma}^2(x_\mu), \tag{36}$$

for CP-odd coupling, where $h_{\sigma\gamma\gamma}^i(x_\mu)$ is sum of different contributions from diagrams in Figs. 6(a) and 6(b) for CP even and CP-odd case of vertex interaction scalar with SM photon in Barr-Zee diagram contribution. We remind that $g_{\sigma\gamma\gamma}$ and $\bar{g}_{\sigma\gamma\gamma}$ couplings are gone from fermion loop. When this contribution to decay width is α^2 suppressed for CP-even type coupling contribution and α^2 and LFV suppressed for CP-odd coupling.
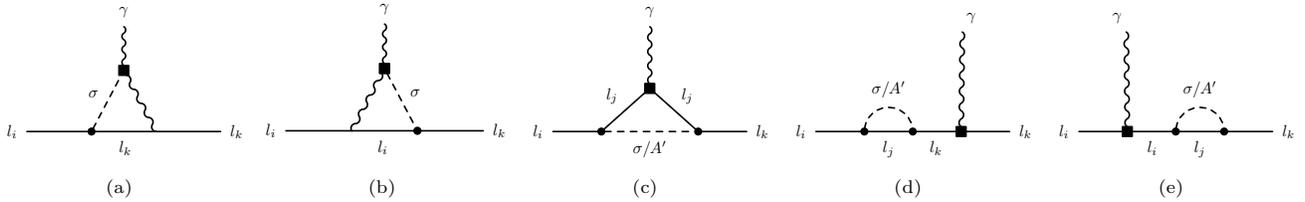
FIG. 6: Feynman diagrams of gauge invariant matrix elements of interaction lepton with external electromagnetic field accounting LFV effect generated by new scalar and vector fields.

The contributions to $l_i \to l_k \gamma$ rare decay involving only one LFV coupling where [case when index $j = i$ or $j = k$ in diagrams in Figs. 6(c)-(e)] are leading and have the form:

$$\begin{aligned}
F_M &= -\frac{1}{16\pi^2} g_{ik}^{PS} \left[g_{ii}^{PS} h_2^S(x_\mu) + g_{kk}^{PS} h_3^S(x_\mu) \right] = -\frac{2m_i}{16\pi^2 \Lambda^2} a_{ik} \left[m_i a_{ii} h_2^S(x_\mu) + m_k a_{kk} h_3^S(x_\mu) \right] , \\
F_D &= -\frac{1}{16\pi^2} g_{ik}^S \left[g_{ii}^{PS} h_2^S(x_\mu) + g_{kk}^{PS} h_3^S(x_\mu) \right] = -\frac{2m_i}{16\pi^2 \Lambda^2} v_{ik} \left[m_i a_{kk} h_2^S(x_\mu) - m_k a_{ii} h_3^S(x_\mu) \right] ,
\end{aligned} \tag{37}$$

and for the process $\mu \to e\gamma$ with τ lepton in loop with double LFV coupling

$$\begin{aligned}
F_M &= -\frac{1}{16\pi^2} \left(\frac{m_\mu}{m_\tau} \right) \left[g_{\mu\tau}^{PS} g_{\tau e}^{PS} + g_{\mu\tau}^S g_{\tau e}^S \right] h_1^S(x_\tau) = -\frac{m_\mu m_\tau}{16\pi^2 \Lambda^2} \left[a_{\mu\tau} a_{\tau e} - v_{\mu\tau} v_{\tau e} \right] h_1^S(x_\tau), \\
F_D &= -\frac{1}{16\pi^2} \left(\frac{m_\mu}{m_\tau} \right) \left[g_{\mu\tau}^S g_{\tau e}^{PS} + g_{\mu\tau}^{PS} g_{\tau e}^S \right] h_1^S(x_\tau) = -\frac{m_\mu m_\tau}{16\pi^2 \Lambda^2} \left[a_{\mu\tau} v_{\tau e} - v_{\mu\tau} a_{\tau e} \right] h_1^S(x_\tau).
\end{aligned} \tag{38}$$

For dark photon vector boson channel we also have a contribution to LFV lepton decay. Dimensionless parameters g_{ij}^V and g_{ij}^{AV} are defined before. They don't have dependence from lepton mass (it makes them different from scalar couplings) and are proportional to the factor m_A/Λ:

$$\begin{aligned}
F_M &= -\frac{1}{16\pi^2} g_{ik}^V \left[g_{ii}^V h_2^V(x_\mu) + g_{kk}^V h_3^V(x_\mu) \right] = -\frac{m_A^2}{16\pi^2 \Lambda^2} v_{ik} \left[v_{ii} h_2^V(x_\mu) + v_{kk} h_3^V(x_\mu) \right] , \\
F_M &= -\frac{1}{16\pi^2} g_{ik}^{AV} \left[g_{ii}^{AV} h_2^V(x_\mu) + g_{kk}^{AV} h_3^V(x_\mu) \right] = -\frac{m_A^2}{16\pi^2 \Lambda^2} a_{ik} \left[a_{ii} h_2^V(x_\mu) + a_{kk} h_3^V(x_\mu) \right]
\end{aligned} \tag{39}$$

and

$$\begin{aligned}
F_D &= \frac{1}{16\pi^2} g_{ik}^V \left[g_{ii}^{AV} h_2^V(x_\mu) + g_{kk}^{AV} h_3^V(x_\mu) \right] = \frac{m_A^2}{16\pi^2 \Lambda^2} v_{ik} \left[a_{\mu\mu} h_2^V(x_\mu) + a_{kk} h_3^V(x_\mu) \right] , \\
F_D &= \frac{1}{16\pi^2} g_{ik}^{AV} \left[g_{ii}^V h_2^V(x_\mu) + g_{kk}^V h_3^V(x_\mu) \right] = \frac{m_A^2}{16\pi^2 \Lambda^2} a_{ik} \left[v_{ii} h_2^V(x_\mu) + v_{kk} h_3^V(x_\mu) \right]
\end{aligned} \tag{40}$$

for $\mu \to e\gamma$ process with τ lepton in loop with double LFV coupling

$$\begin{aligned}
F_M &= -\frac{1}{16\pi^2} \left(\frac{m_\mu}{m_\tau} \right) \left[g_{\mu\tau}^V g_{\tau e}^V h_1^V(x_\tau) + g_{\mu\tau}^{AV} g_{\tau e}^{AV} h_1^V(x_\tau) \right] = -\frac{m_A^2}{16\pi^2} \left(\frac{m_\mu}{m_\tau} \right) \left[v_{\mu\tau} v_{\tau e} h_1^V(x_\tau) + a_{\mu\tau} a_{\tau e} h_1^V(x_\tau) \right] , \\
F_D &= \frac{1}{16\pi^2} \left(\frac{m_\mu}{m_\tau} \right) \left[g_{\mu\tau}^V g_{\tau e}^{AV} h_1^V(x_\tau) + g_{\mu\tau}^{AV} g_{\tau e}^V h_1^V(x_\tau) \right] = \frac{m_A^2}{16\pi^2} \left(\frac{m_\mu}{m_\tau} \right) \left[v_{\mu\tau} a_{\tau e} h_1^V(x_\tau) + a_{\mu\tau} v_{\tau e} h_1^V(x_\tau) \right] ,
\end{aligned} \tag{41}$$

where $x_i = m_i^2/m_S^2$ for scalar channel and $x_i = m_i^2/m_A^2$ for dark vector channel, $h_i^S(x_i)$ and $h_i^V(x_i)$ are loop integration functions arising from form factors at certain limit presented in Appendix B, full expressions for the form factors are also presented in this Appendix B. We note that form factors h_i generated by diagrams in Figs. 6(c)-(e) do not have divergences unlike contribution due to the Barr-Zee like diagrams displayed in Figs. 6(a) and 6(b) having CP-even coupling with photon. Herewith we note that contributions from Figs. 6(d) and 6(e) is needed for cancellation divergences a contribution from diagram in Fig. 6(c) and for gauge invariance of interaction photon with lepton trough loop diagrams with intermediate DSB or DGB states.

Similarly we can write the contribution to F_M and F_D form factors of the for $\tau \to \mu\gamma$ and $\tau \to e\gamma$ LFV rare decays in the case when initial/final leptons are differed from a lepton propagating in the loop:

$\tau \to \mu\gamma$ process

$$\begin{aligned}
F_M &= -\frac{1}{16\pi^2} (g_{\tau e}^{PS} g_{e\mu}^{PS} + g_{\tau e}^S g_{e\mu}^S) h_3^S(x_\tau) = -\frac{m_\tau m_\mu}{16\pi^2 \Lambda^2} (a_{\tau e} a_{\mu e} + v_{\tau e} v_{\mu e}) h_3^S(x_\tau) , \\
F_D &= -\frac{1}{16\pi^2} (g_{\tau e}^{PS} g_{e\mu}^S + g_{\tau e}^S g_{e\mu}^{PS}) h_3^S(x_\tau) = \frac{m_\tau m_\mu}{16\pi^2 \Lambda^2} (a_{\tau e} v_{\mu e} + v_{\tau e} a_{\mu e}) h_3^S(x_\tau)
\end{aligned} \tag{42}$$

from scalar and

$$\begin{aligned}
F_M &= -\frac{1}{16\pi^2} \left[g_{\tau e}^V g_{e\mu}^V + g_{\tau e}^{AV} g_{e\mu}^{AV} \right] h_3^V(x_\tau) = -\frac{m_A^2}{16\pi^2 \Lambda^2} \left[v_{\tau e} v_{e\mu} + a_{\tau e} a_{e\mu} \right] h_3^V(x_\tau), , \\
F_D &= \frac{1}{16\pi^2} \left[g_{\tau e}^V g_{e\mu}^{AV} + g_{\tau e}^{AV} g_{e\mu}^V \right] h_3^V(x_\tau) = \frac{m_A^2}{16\pi^2 \Lambda^2} \left[v_{\tau e} a_{e\mu} + a_{\tau e} v_{e\mu} \right] h_3^V(x_\tau)
\end{aligned} \tag{43}$$

from vector A' boson exchange.

$\tau \to e\gamma$ process

$$
\begin{aligned}
F_M &= -\frac{1}{16\pi^2}(g^{PS}_{\tau\mu}g^{PS}_{e\mu} + g^S_{\tau\mu}g^S_{e\mu})h^S_3(x_\tau) = -\frac{m_\tau m_\mu}{16\pi^2\Lambda^2}(a_{\tau\mu}a_{e\mu} - v_{\tau\mu}v_{e\mu})h^S_3(x_\tau), , \\
F_D &= -\frac{1}{16\pi^2}(g^{PS}_{\tau\mu}g^S_{e\mu} + g^S_{\tau\mu}g^{PS}_{e\mu})h^S_3(x_\tau) = -\frac{m_\tau m_\mu}{16\pi^2\Lambda^2}(a_{\tau\mu}v_{e\mu} - v_{\tau\mu}a_{e\mu})h^S_3(x_\tau)
\end{aligned}
\tag{44}
$$

from scalar and

$$
\begin{aligned}
F_M &= -\frac{1}{16\pi^2}\left[g^V_{\tau\mu}g^V_{\mu e} + g^{AV}_{\tau\mu}g^{AV}_{\mu e}\right]h^V_3(x_\tau) = -\frac{m^2_A}{16\pi^2\Lambda^2}\left[v_{\tau\mu}v_{\mu e} + a_{\tau\mu}a_{\mu e}\right]h^V_3(x_\tau), , \\
F_D &= \frac{1}{16\pi^2}\left[g^V_{\tau\mu}g^{AV}_{\mu e} + g^{AV}_{\tau\mu}g^V_{\mu e}\right]h^V_3(x_\tau) = \frac{m^2_A}{16\pi^2\Lambda^2}\left[v_{\tau\mu}a_{\mu e} + a_{\tau\mu}v_{\mu e}\right]h^V_3(x_\tau),
\end{aligned}
\tag{45}
$$

from vector A' boson exchange.

Here we find a full agreement for scalar contribution with results reported in Ref. [42] with precision to a factor 2 that is hidden in the definition of form factors. It is same because we use the same idea of familon as in Refs. [34, 42] for ALPs physics. Gordon identities which are used here for LFV process are listed in Appendix A.

VI. ANALYSIS OF CURRENT LIMITS

Current limits for the branchings of the LFV lepton decays $l_i \to l_k\gamma$ are [58]

$$
\begin{aligned}
\mathrm{Br}(\mu \to e\gamma) &< 4.2 \times 10^{-13}, \\
\mathrm{Br}(\tau \to e\gamma) &< 3.3 \times 10^{-8}, \\
\mathrm{Br}(\tau \to \mu\gamma) &< 4.4 \times 10^{-8}.
\end{aligned}
\tag{46}
$$

Future experimental prospects aim to measure these quantities with more high precision in order to low the upper boundary by one order of magnitude.

We analyze LFV couplings by focusing onto scenario of lepton-flavor universality for lepton conserving couplings a_{ii} or v_{ii}. For estimation of bounds we put a scale of New Physics Λ equal to 1 TeV. Scalar and vector contributions to LFV rare decay widths are considered independently. The results for the lepton-flavor universality scenario using $\mu \to e\gamma$ and $\tau \to e\gamma$ processes are presented in Fig. 7 in case of equal a_{ii} couplings. Bounds from $\tau \to \mu\gamma$ are the same as for $\tau \to e\gamma$ because in approximation $m_e \ll m_\mu \ll m_\tau$ contributions from the loops are same. We consider independent contributions from diagrams in Fig.6(c)-(e). Contributions from the Barr-Zee diagram are suppressed as pointed before. The contributions of heavy lepton state in loop was omitted here in the analysis of certain couplings because these contributions have double LFV suppression.

The peaks in the Fig. 7 is connected to behavior of loop integrals $h_i(x)$ near the point $x = 1$ where vector boson production in vicinity of the threshold. For solution of this problem one needs to include the decay width of dark vector boson to lepton pair with $\tau_A^{-1} \sim (v_{ij}/\Lambda)^2$ in the Breit-Wigner propagator.

Boundaries to LFV couplings on Fig. 7 include restriction from lepton $(g-2)$ and rare LFV $l_i \to l_k\gamma$ lepton decays. In case of $e - \mu$ LFV transition, left pictures in Figs. 7, we add bound from $e - \mu$ conversion. Suppression at heavy masses for $e - \mu$ conversion is due of heavy bosons exchange in the t-channel. Inclusion of boundaries from $e - \mu$ conversion gives suppression from scalar a_{ij} coupling in all ranges of mass. The dark vector photon keeps a window for huge LFV couplings v_{ij} at light masses of one.

Boundaries for a_{ij} from leptonic $(g - 2)$ are strong for more massive lepton propagating in the loop and less suppressed lepton conserving couplings. From LFV decays $l_i \to l_k\gamma$ the behavior of bound is similar but suppression is more strong. It also occurs for scalar channel and dark vector boson channels. Wherein that vector and scalar coupling have the same hermitian matrices v_{ij} and a_{ij}. The boundary from the vector channel gives limits for scalar LFV dimensionless couplings. Such interference of couplings is due to the Stueckelberg mechanism.

Using boundary from rare decays $l_i \to l_k\gamma$ in universal lepton conserving couplings scenario for $a_{ii} = 1$ or $v_{ii} = 1$ we can deduce possible lepton contribution to $(g - 2)$ in dependence on masses of new scalar or dark vector bosons. We estimate contribution to AMM due to sum of the loops with lightest leptons μ and e and with taking into account lepton flavor conservation (LFC) couping in conserving scenario and LFV coupling are constrained from LFV lepton decay $l_i \to l_k\gamma$

$$
\Delta a_{l_i} = (\Delta a_{l_i})_{\mathrm{LFC}} + (\Delta a_{l_i})_{\mathrm{LFV}}.
\tag{47}
$$
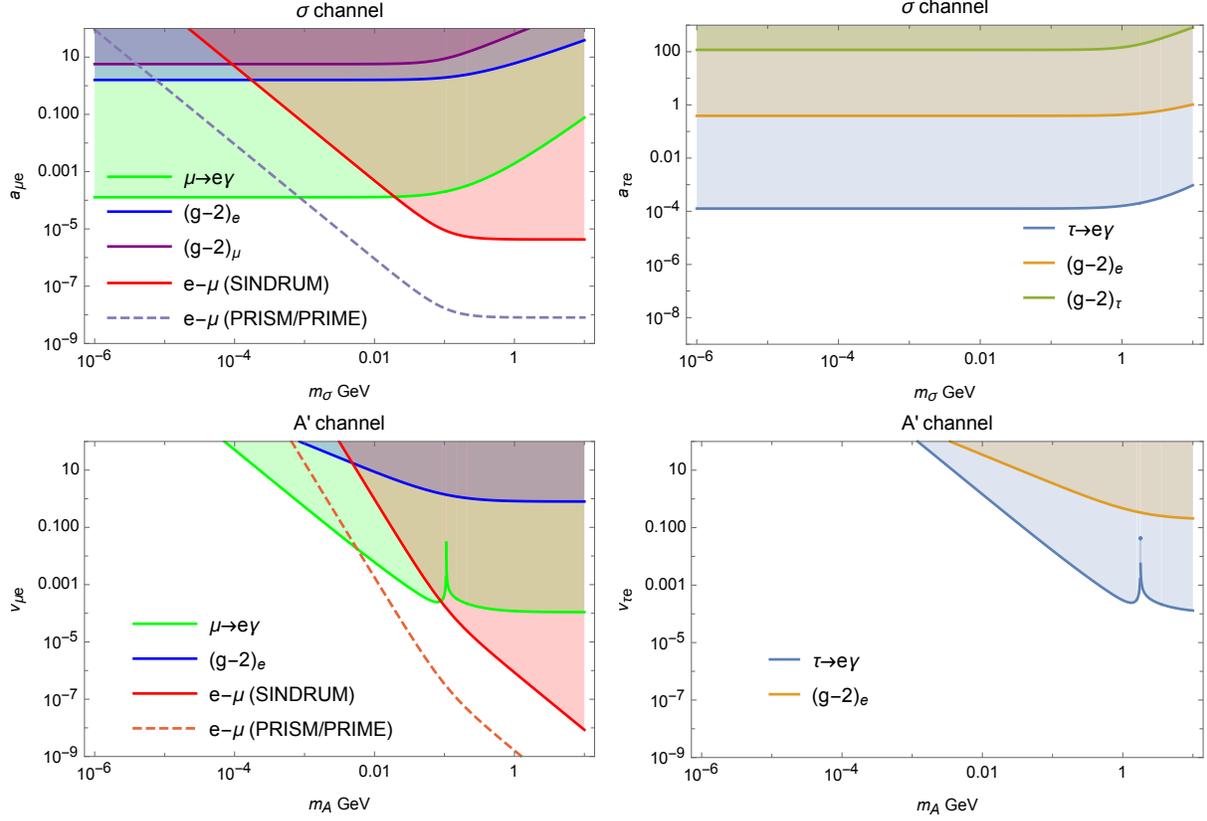
FIG. 7: Bounds from $g-2$ of leptons, from $\mu \to e\gamma$ and $\tau \to e\gamma$ LFV decay width and lepton conversion to interaction couplings to a_{ij} scalar boson coupling and v_{ij} dark vector A' boson with fermions in dependence from m_σ or m_A. Bound is constructed for the scale $\Lambda = 1$ TeV and at assumption that $a_{ii} = 1$ or $v_{ii} = 1$. The shaded area is prohibited from data.
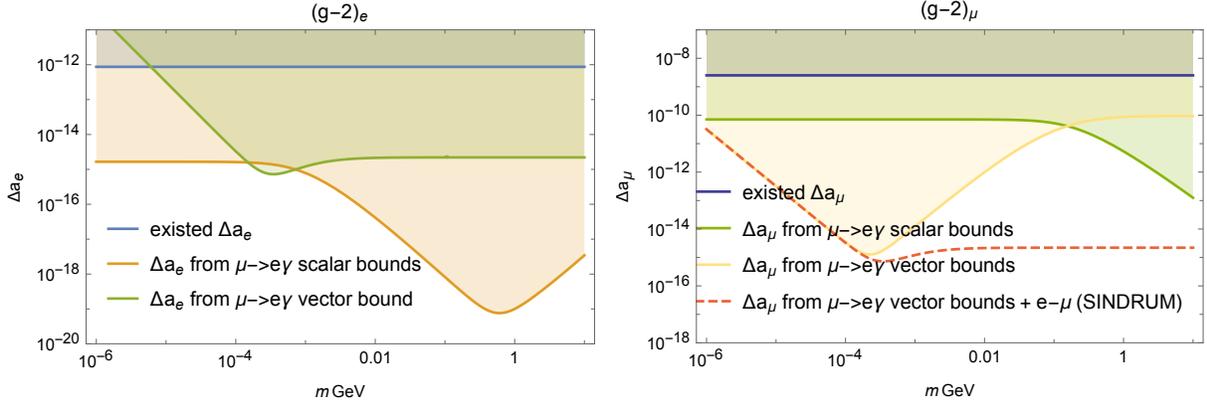
FIG. 8: Estimate of contribution to lepton AMM in dependence on masses of scalar boson or dark photon. Boundary is made for the scale $\Lambda = 1$ TeV and in assumption that $a_{ii} = 1$ or $v_{ii} = 1$ with taking into account restriction for LFV couplings.

For vector contribution we also include restriction from $\mu - e$ conversion. In case of $(g-2)$ of electron, restriction from conversion it makes no sense. For $(g-2)$ of muon, vector channel has a suppression at mass of dark photon larger than 10^{-3} GeV. Such additional restriction for $(g-2)$ of muon gives the same behavior of vector channel for this anomaly as for the case of $(g-2)$ of electron. The dependence is presented in Fig. 8.

As mentioned in Ref. [33], a various of lepton conserving couplings we can tuck up mass (or mass and LFC couplings) when difference between theory and experimental data of $(g-2)$ of leptons due to contributions from ALPs or new

scalar in our case. In the case where we take into account dark vector photon, the problem of lepton $(g-2)$ anomaly can be solved by accounting for contributions from vector channel and scalar can have more suppressed coupling in dependence of mass.

VII. CONCLUSIONS

We constructed a phenomenological Lagrangian approach which combines SM and DM sectors based on the Stueckelberg mechanism for generation mass of dark $U_D(1)$ gauge boson (or dark photon). The DM sector contains dark photon, dark scalar, and generic dark fermion fields. DM scalar could be identified as the Stueckelberg ALPs. Couplings of DSB and DGB with fermions of SM have interference between v_{ij} and a_{ij} and coupling of DGB with fermions is proportional to ratio mass DGB to scale of New Physics Λ. Stueckelberg Portal opens new possibilities for study of phenomenology of BSM Physics and can be important for running and planning experiments at world-wide facilities (e.g., for the NA64 Experiment at SPS CERN [2, 3]).

We derived boundaries on the effective couplings of our Lagrangian using data on lepton AMMs, LFV lepton decays $l_i \to \gamma l_k$, and $\mu - e$ conversion. It is known that the latter are very useful because they give more stringent limits on the couplings of effective Lagrangian. We also founded that the $(g-2)$ anomaly can be preferably solved by light dark photon then by scalar boson.

In future we plan to study a possible role of the Stueckelberg portal in different rare LFV processes including semileptonic decays. In forthcoming paper we also plan to extend this portal by inclusion of the $SU_D(2)$ gauge sector and matter fields.

Appendix A: Gordon Identities

The Gordon identities the matrix elements describing the coupling of external gauge field with fermions having different masses read:

$$i\sigma_{\mu\nu}q^\nu = -P_\mu + (m_i + m_j)\gamma_\mu; \qquad i\sigma_{\mu\nu}P^\nu = -q_\mu + (m_j - m_i)\gamma_\mu; \tag{A1}$$

$$i\sigma_{\mu\nu}q^\nu\gamma_5 = -P_\mu\gamma_5 + (m_j - m_i)\gamma_\mu\gamma_5; \qquad i\sigma_{\mu\nu}P^\nu\gamma_5 = -q_\mu\gamma_5 + (m_i + m_j)\gamma_\mu\gamma_5. \tag{A2}$$

Appendix B: Loop function occuring in the form factors

In this Appendix, we present the analytical expressions of the loop integrals occuring in the amplitude of the LFV decays $l_i \to \gamma l_k$ for different new particle channel and lepton propagating in the loop. Full results for the form factors in form of Feynman integrals by Feynman are presented too. All results for the form factors have been numerically and analytically cross-checked using the Mathematica Package-X [62] and packages FeynHelpers [63] and FeynCalc [64].

Loop function from CP-even and CP-odd coupling of scalar field with photons Fig.6(a) and (b):

$$h^1_{\sigma\gamma\gamma} = 2\ln\left(\frac{\Lambda^2}{m_S^2}\right) - \frac{\ln x}{x-1} - (x-1)\ln\left(\frac{x}{x-1}\right) - 2,$$

$$h^2_{\sigma\gamma\gamma} = \frac{1}{2}\left(1 - \ln(x) + \frac{x\ln(x)}{x-1} - (x-1)\ln\left(\frac{x}{x-1}\right)\right), \tag{B1}$$

where $x = m_S^2/m_\mu^2$.

Full loop function occuring in the form factors read:

Scalar or pseudoscalar case:

$$F^{PS/S} = 2m_i \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{(m_f x_2 + m_i x_1)(x_1 + x_2 - 1) \pm m_l(x_1 + x_2)}{(x_1 + x_2)(m_f^2 x_2 + m_i^2 x_1 + m_l^2) - m_f^2 x_2 - m_i^2 x_1 - m_S^2(x_1 + x_2 - 1)}. \tag{B2}$$

Vector intermediate state case:

$$F^{V/AV} = 2m_i \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{2(x_1 + x_2 - 1)(m_f(x_2 - 1) + m_i(x_1 - 1) \pm 2m_l)}{(x_1 + x_2)(m_i^2 x_2 + m_f^2 x_1 + m_l^2) - m_e^2 x_2 - m_i^2 x_1 - m_S^2(x_1 + x_2 - 1)}. \tag{B3}$$

Here, m_i is a mass off initial lepton state, m_f is a mass of final lepton state, and m_l is mass of loop lepton.
In the approximation when $m_e \ll m_\mu \ll m_\tau$ the loop integrals reduce to simple functions h^S.
Case: scalar intermediate state and τ lepton in the loop

$$h_1^S(x) = \frac{3x^2 - 4x + 2x^2 \ln(x) + 1}{(1-x)^3}, \tag{B4}$$

where $x = m_S^2/m_\tau^2$.
Case: scalar intermediate state and muon in the loop

$$h_2^S(x) = \frac{2x^2 - (x-3)x^2 \ln(x) - 3x + 1}{1 - x} - 2x\sqrt{x^2 - 4x} \ln\left(\frac{\sqrt{x} + \sqrt{x-4}}{2}\right), \tag{B5}$$

where $x = m_S^2/m_\mu^2$.
Case: scalar intermediate state and electron in the loop

$$h_3^S(x) = \left(1 - 2x - 2(x-1)x \ln\left(\frac{x-1}{x}\right)\right), , \tag{B6}$$

where $x = m_S^2/m_\mu^2$.
Case: vector intermediate state and τ lepton in the loop

$$h_1^V(x) = 4\frac{x^2 - 1 - 2x \ln(x)}{(1-x)^3}, \tag{B7}$$

where $x = m_S^2/m_\tau^2$.
Case: vector intermediate state and muon in the loop

$$\begin{aligned} h_2^V &= 2\left(2\operatorname{Li}_2(1-x) - 2\operatorname{Li}_2\left(-\frac{x}{2} - \frac{1}{2}\sqrt{(x-4)x} + 1\right) + 2\operatorname{Li}_2\left(\frac{2}{x + \sqrt{(x-4)x}}\right) \right. \\ &+ \left. \frac{(x-3)x^2 \ln(x)}{x-1} - 2x + \ln^2\left(\frac{x + \sqrt{(x-4)x}}{2x}\right) - 2\sqrt{x^2 - 4x}\, x \ln\left(\frac{\sqrt{x} + \sqrt{x-4}}{2}\right) + 1\right), \end{aligned} \tag{B8}$$

where $x = m_S^2/m_\mu^2$,
Case: vector intermediate state and electron in the loop

$$h_3^V = 2x - 2(x-1)\, x \ln\left(\frac{x}{x-1}\right) - 1, \tag{B9}$$

where $x = m_S^2/m_\mu^2$.

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