

Estimating the impact of mixed QCD-electroweak corrections on the W -mass determination at the LHC

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We study the impact of the recently computed mixed QCD-electroweak corrections to the production of W and Z bosons at the LHC on the value of the W mass extracted from the transverse momentum distribution of charged leptons from W decays. Using the average lepton transverse momenta in W and Z decays as simplified observables for the determination of the W mass, we estimate that mixed QCD-electroweak corrections can shift the extracted value of the W mass by up to $\mathcal{O}(20)$ MeV, depending on the kinematic cuts employed to define fiducial cross sections for Z and W production. Since the target precision of the W -mass measurement at the LHC is $\mathcal{O}(10)$ MeV, our results emphasize the need for fully-differential computations of mixed QCD-electroweak corrections and a careful analysis of their potential impact on the determination of the W mass.

I. INTRODUCTION

The measurement of the W boson mass at the LHC is a Holy Grail of precision hadron collider physics. It is believed that the W mass can be extracted from LHC data with an uncertainty of about 10 MeV [1]. If this happens, the precision of the direct measurement will match the precision that has already been achieved for the W mass extracted from global electroweak fits using the renormalizability of the Standard Model [2]. A comparison of direct and indirect determinations of the W mass has the potential to further stress-test the consistency of the Standard Model at the quantum level and, perhaps, reveal unknown contributions to precision electroweak observables. Until now such a comparison has been limited by the uncertainties of direct determinations. Indeed, the W mass was measured at LEP and Tevatron with an uncertainty of 33 MeV [3] and 16 MeV [4], respectively. Recently, the ATLAS Collaboration reported a measurement of m_W with an uncertainty of 19 MeV [5]. To improve on this result, both an exquisite control of experimental systematics and a thorough investigation of all possible sources of theory uncertainties are necessary.

In general, measurements of particle masses at colliders rely on correlations between them and selected kinematic observables. A classic example of such an observable,

which has been employed to measure m_W for many years, is the so-called transverse W mass¹, which has a sharp edge at m_W . The observation of such an edge provides one with immediate information about the value of the W mass which depends only weakly on the theoretical description of W production in hadron collisions and its subsequent decay. Nevertheless, even in this case ultra-high precision on the W -mass measurement calls for a detailed understanding of e.g. the uncertainty with which the missing energy can be determined, the effects of the finite width of the W boson in theoretical modeling of W production, and so on.

Another important observable that is used for the W -mass measurement is the transverse momentum distribution of charged leptons from the decay $W \rightarrow l\nu_l$. Features of this distribution are correlated with m_W and, in comparison to the transverse mass, it is under better experimental control. As a consequence, the p_\perp^l distribution plays quite a prominent role in high-precision m_W determinations. Indeed, the recent ATLAS extraction of the W mass at the LHC [5] was mostly driven by the measurement of the charged lepton p_\perp^l distribution.

Unfortunately, the p_\perp^l distribution is quite sensitive to the theoretical description of W production and decay, including the modeling of the transverse momentum spectrum of the W boson, control of the parton distribution functions, and a detailed understanding of QCD and QED radiation, both from the initial and the final state. Although it is well understood how to describe the charged lepton p_\perp^l distribution using the framework of collinear factorization in QCD, the challenge arises from

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¹ For a definition, see e.g. Ref. [5].

the extraordinary precision of the planned W -mass measurement. Indeed, as we already mentioned, the W mass is expected to be measured with a precision of about $\mathcal{O}(10)$ MeV or 0.01 percent. It is perfectly clear that existing theoretical approaches, be they fixed order computations or parton showers or resummations, are not suitable for the description of *any* hadron-collider observable with such precision.

This problem is usually overcome by exploiting similarities between the production of Z and W bosons in hadron collisions and by making use of the fact that the mass of the Z boson has been measured very precisely at LEP. The extraction of the W mass from studies of the lepton distribution p_{\perp}^l in the process $pp \rightarrow W + X \rightarrow l\nu_l + X$ relies on these considerations and makes use of the fact that a similar distribution in the process $pp \rightarrow Z + X \rightarrow l\bar{l} + X$ can be used for calibration purposes. The underlying theoretical assumption is that QCD effects in these two processes are strongly correlated and, as a consequence, a theoretical model “tuned” to describe the p_{\perp}^l distribution in the Z sample can be used with minimal modifications to obtain precise predictions for the p_{\perp}^l distribution in the W case. This is the approach on which the analysis of Ref. [5] as well as earlier measurements of the W mass at the Tevatron are based.

Clearly, if one relies on using Z boson production for the calibration, all effects that distinguish between the Z and W cases must be estimated theoretically. As we already mentioned, QCD corrections are expected to be largely similar for W and Z production, although even in this case the impact of different quark flavors in the initial state [6–11] as well as of the $gg \rightarrow Zg$ contribution that exists in Z production but not in the W case must be investigated.

On the other hand, it is also clear that electroweak (EW) corrections may affect the production of W and Z bosons differently, potentially leading to uncorrelated effects of these corrections on the p_{\perp}^l spectra in Z and W samples. If this does happen, any measurement of the W mass that relies of the similarity of Z and W kinematic distributions will be affected.

These considerations motivated extensive studies of the NLO electroweak corrections [12–20] to the Z and W production processes, as well as effects related to multiple photon emissions [21–27] in Z and W decays. Their impact on the W -mass determination has been studied in detail, see Ref. [28] for a comprehensive review.

It was also recognized long ago that for the target precision of the W -mass measurement one has to go beyond NLO electroweak corrections and account for *mixed* QCD-electroweak effects. Approximate $\mathcal{O}(\alpha_s\alpha_W)$ corrections are available in parton showers using a factorized approach [29–31], and their impact on the W -mass determination was also studied in Ref. [28]. However, genuine mixed QCD-EW corrections were, until recently, only known for initial-state QCD radiation and final-

state photon emission [32, 33] which are expected to give the dominant contribution to the full QCD-EW corrections. Their impact on W -mass determinations has been studied in Refs. [28, 33].

The computation of the remaining mixed QCD-EW corrections to the Z and W production processes was recently completed [34–41]. The goal of this note is to estimate how these corrections affect the value of the W mass extracted from the transverse momentum distribution of a charged lepton.

Although in the experimental analyses [4, 5, 42] the mass of the W boson is determined from fits to templates of p_{\perp}^l distributions, here we adopt a simplified approach that allows us to *estimate* the resulting mass shift in a simple and transparent way. We believe that the simplicity and transparency of our analysis justifies its use in a theoretical paper but we emphasize that, should corrections turn out to be non-negligible, a more refined study of the impact of mixed QCD-EW effects on the W -mass extraction that better reflects the details of experimental analyses will be required.

To estimate the impact of mixed QCD-electroweak corrections on the W -mass measurement we make use of the fact that the average transverse momentum $\langle p_{\perp}^{l,V} \rangle$ ($V = Z, W$) is correlated with the mass of the respective gauge boson. Indeed, it is straightforward to compute $\langle p_{\perp}^{l,V} \rangle$ at leading order in perturbative QCD. The result, as a function of the lower cut on the lepton transverse momentum p_{\perp}^{cut} , is

$$\langle p_{\perp}^{l,V} \rangle = m_V f\left(\frac{p_{\perp}^{\text{cut}}}{M_V}\right), \quad (1)$$

where

$$f(r) = \frac{3}{32} \frac{r(5-8r^2)}{1-r^2} + \frac{15}{64} \frac{\arcsin(\sqrt{1-4r^2})}{(1-r^2)\sqrt{1-4r^2}}. \quad (2)$$

The function $f(r)$ quantifies the dependence of the average momentum $\langle p_{\perp}^{l,V} \rangle$ on the p_{\perp}^{cut} ; if no cut is imposed, we obtain $\langle p_{\perp}^{l,V} \rangle = m_V f(0) = 15\pi/128 m_V$.

We note that for physical values of r , $0 < r < 0.5$, the function $f(r)$ does not change strongly, $0.368 < f(r) < 0.5$. Therefore, we expect that either the selection of cuts can be optimized to enhance the similarity of the p_{\perp}^l distributions in W and Z production, or that the effect of cuts can be adequately predicted in perturbation theory. Hence, we write the following formula for the W mass extracted from measurements of average values of lepton transverse momenta as

$$m_W^{\text{meas}} = \frac{\langle p_{\perp}^{l,W} \rangle_{\text{meas}}}{\langle p_{\perp}^{l,Z} \rangle_{\text{meas}}} m_Z C_{\text{th}}. \quad (3)$$

The *theoretical* correction factor C_{th} is determined by comparing the value of the W mass obtained by follow-

	$V = Z$			$V = W^+$		
	$\mu = m_Z/4$	$\mu = m_Z/2$	$\mu = m_Z$	$\mu = m_W/4$	$\mu = m_W/2$	$\mu = m_W$
$F_V(0, 0; 1)$, [pb]	1273	1495	1700	7434	8810	10083
$F_V(1, 0; 1)$, [pb]	570.2	405.4	246.9	3502	2533	1580
$F_V(0, 1; 1)$, [pb]	$-5810 \cdot 10^{-3}$	$-6146 \cdot 10^{-3}$	$-6073 \cdot 10^{-3}$	$-1908 \cdot 10^{-3}$	$3297 \cdot 10^{-3}$	$10971 \cdot 10^{-3}$
$F_V(1, 1; 1)$, [pb]	$-2985 \cdot 10^{-3}$	$-2033 \cdot 10^{-3}$	$-1236 \cdot 10^{-3}$	$-8873 \cdot 10^{-3}$	$-7607 \cdot 10^{-3}$	$-7556 \cdot 10^{-3}$
$F_V(0, 0; p_\perp^e)$ [GeV · pb]	42741	50191	57073	220031	260772	298437
$F_V(1, 0; p_\perp^e)$ [GeV · pb]	23418	17733	12221	124487	95132	66090
$F_V(0, 1; p_\perp^e)$ [GeV · pb]	-182.85	-192.77	-189.11	74.53	243.54	484.82
$F_V(1, 1; p_\perp^e)$ [GeV · pb]	-163.87	-125.22	-92.05	-553.87	-482.0	-448.0

Table I: Inclusive cross sections and first moments of the positron transverse momentum distributions in $pp \rightarrow W^+ \rightarrow \nu e^+$ and $pp \rightarrow Z \rightarrow e^- e^+$ at the 13 TeV LHC. Results are shown at leading order, for the next-to-leading order QCD and EW corrections, and for the mixed QCD-electroweak corrections. See text for details.

ing this procedure within a particular theoretical framework with the actual W mass m_W used as an input in a theoretical calculation. Therefore

$$C_{\text{th}} = \frac{m_W}{m_Z} \frac{\langle p_\perp^{l,Z} \rangle^{\text{th}}}{\langle p_\perp^{l,W} \rangle^{\text{th}}}. \quad (4)$$

If the theoretical framework used to compute C_{th} changes, for example because a more refined theoretical prediction for $\langle p_\perp^l \rangle$ becomes available, there is a shift in the extracted value of the W mass m_W^{meas} . It evaluates to

$$\frac{\delta m_W^{\text{meas}}}{m_W^{\text{meas}}} = \frac{\delta C_{\text{th}}}{C_{\text{th}}} = \frac{\delta \langle p_\perp^{l,Z} \rangle^{\text{th}}}{\langle p_\perp^{l,Z} \rangle^{\text{th}}} - \frac{\delta \langle p_\perp^{l,W} \rangle^{\text{th}}}{\langle p_\perp^{l,W} \rangle^{\text{th}}}. \quad (5)$$

This equation shows clearly the role that the Z boson observables play in Eqs.(3,4). Indeed, it follows from Eq.(5) that all effects that influence the lepton transverse momentum distributions in Z and W production and decay in a similar way do not result in a shift in the measured value of the W mass. However, if this is not the case, a shift in the extracted value m_W^{meas} arises.

Eq.(5) provides the basis for our estimate of the impact of the mixed QCD-electroweak corrections on the determination of the W mass. Indeed, the calculations reported in Refs. [36, 39] allow us to compute average lepton transverse momenta in Z and W production with and without mixed QCD-electroweak corrections. Using this information, we construct quantities that appear on the right hand side of Eq.(5) and estimate the shift in the extracted value of the W mass.

Before presenting the results, we briefly discuss the setup of the calculation. We use the same input parameters as described in Refs. [36, 39]. In particular, we adopt the G_μ renormalization scheme and use $G_F = 1.16639 \cdot 10^{-5} \text{ GeV}^{-2}$, $m_Z = 91.1876 \text{ GeV}$, $m_W = 80.398 \text{ GeV}$, $m_H = 125 \text{ GeV}$ and $m_t = 173.2 \text{ GeV}$. We work in the narrow-width approximation and consider all quarks but the top quark to be massless.² For definiteness, we

consider decays $Z \rightarrow e^- e^+$ and $W^+ \rightarrow \nu_e e^+$ and consider the electrons as being massless. We employ the NNLO NNPDF3.1luxQED [43–45] parton distributions with $\alpha_s(m_Z) = 0.118$. For our analysis, we focus on Z and W^+ production at the 13 TeV LHC and study the transverse momentum distribution of the positron e^+ . Since the contribution of QCD initial-state and EW final-state corrections to the full mixed QCD-EW result and its impact on the W -mass determinations is known [32, 33], we do not consider corrections to the $W \rightarrow \nu_e e^+$ and $Z \rightarrow e^- e^+$ decay subprocesses. In other words, for our estimates we only consider mixed QCD-EW corrections to the *production* sub-processes $pp \rightarrow W/Z$. As we have already said, this is the only mixed QCD-electroweak contribution whose impact on the W -mass determination is currently unknown.

For the sake of clarity, we begin by considering inclusive quantities and do not apply any kinematic cuts. We write the differential cross sections for Z and W production as

$$d\sigma_{Z,W} = \sum_{i,j=0} \alpha_s^i \alpha_W^i d\sigma_{Z,W}^{i,j}, \quad (6)$$

where α_s and α_W are the strong and electroweak couplings, respectively. We also define weighted integrals

$$F_{Z,W}(i, j, \mathcal{O}) = \alpha_s^i \alpha_W^i \int d\sigma_{Z,W}^{i,j} \times \mathcal{O}, \quad (7)$$

where \mathcal{O} is a particular kinematic variable. With this notation, the average transverse momentum of the positron in the processes $pp \rightarrow Z + X \rightarrow e^- e^+ + X$ and $pp \rightarrow W^+ + X \rightarrow \nu_e e^+ + X$ reads

$$\langle p_\perp^{e^+,V} \rangle^{\text{th}} = \frac{\sum_{ij} F_V(i, j, p_\perp^{e^+})}{\sum_{ij} F_V(i, j, 1)}. \quad (8)$$

² We neglect the contribution of Feynman diagrams with internal

top quarks in the calculation of mixed QCD-electroweak two-loop corrections. Our result then only depends on m_t through the renormalization procedure, see Ref. [36] for details.

In Table I we report results for F_V when no fiducial cuts are applied.

To study the impact of mixed QCD-EW corrections on the W -mass determination, we use Eq.(5). We determine the shifts $\delta\langle p_{\perp}^{e^+,V} \rangle^{\text{th}}$ by computing $\langle p_{\perp}^{e^+,V} \rangle^{\text{th}}$ in Eq.(8) with mixed QCD-electroweak contributions (i.e. with the $F_V(1, 1, \dots)$ terms). We then take the difference of this result with respect to the result including both the NLO QCD and NLO EW corrections. Using the results presented in Table I, we find

$$\frac{\delta m_W^{\text{meas}}}{m_W^{\text{meas}}} = -0.93_{+0.29}^{-0.22} \times 10^{-4}. \quad (9)$$

To compute the central value, we have set both the renormalization and factorization scales to $\mu = m_V/2$. The upper (lower) value corresponds to $\mu = m_V$ and $\mu = m_V/4$, respectively.

Using $m_W^{\text{meas}} = 80.398$ GeV in Eq.(9), we find that the value of the W boson mass extracted from the $\langle p_{\perp}^{e^+} \rangle$ distribution without accounting for mixed QCD-electroweak corrections exceeds the true value by $\mathcal{O}(7)$ MeV. This result is only mildly affected by PDFs uncertainties: using a compressed NNPDF3.1luxQED set, obtained along the lines described in Refs. [46, 47], we find that uncertainties in parton distribution functions may change the above estimate of the mass shift by about 1 MeV.

It is interesting to point out that if we use this analysis to study the impact of *electroweak* corrections to the production processes $pp \rightarrow Z$ and $pp \rightarrow W^+$ on the value of the W mass, we find a very small shift of about $\mathcal{O}(1)$ MeV provided that we use the NLO QCD calculation as a baseline. This result shows that mixed QCD-electroweak corrections have *larger impact* on the W -mass measurement than the electroweak ones. There seem to be two reasons for that. The first reason is that electroweak and mixed QCD-electroweak corrections to observables in W and Z production are comparable and do not quite follow the standard hierarchy where the electroweak corrections are expected to be larger than the mixed ones. This feature can be seen in Table I, and was also previously noted in Refs. [36, 39] where it was pointed out that the use of the so-called G_{μ} renormalization scheme reduces electroweak corrections significantly. The second reason for the tiny shift in the extracted value of the W mass caused by the electroweak corrections is a very strong cancellation between the first and the second terms on the right hand side of Eq.(5). This means that electroweak corrections cause nearly identical relative changes in the average transverse momenta of charged leptons in decays of Z and W bosons, so that the significance of these corrections is substantially reduced.

To elaborate on this point further, we note that if we only compute relative changes to the average transverse momentum of the lepton coming from the W decay and set the term $\delta\langle p_{\perp}^{e^+,Z} \rangle / \langle p_{\perp}^{e^+,Z} \rangle$ in Eq.(5) to zero, we find

that electroweak corrections induce a $\mathcal{O}(-31)$ MeV shift in m_W . If we do the same for mixed QCD-electroweak corrections, this mass shift turns out to be $\mathcal{O}(54)$ MeV. These results imply that *i*) the magnitude of EW and QCD-EW corrections to the average lepton transverse momenta are indeed comparable; *ii*) there are significant correlations between corrections to average $p_{\perp}^{e^+}$ in Z and W production and *iii*) these correlations are *slightly* stronger for electroweak than for mixed QCD-electroweak corrections leading to *significantly* larger shifts in m_W^{meas} in the latter case.

We can easily extend the calculation that we just described to include kinematic restrictions applied in experimental analyses. As an example, we re-compute the average transverse momenta of the charged leptons using kinematic cuts inspired by the ATLAS analysis [5]. In the case of W boson production, we require that the transverse momentum of the charged lepton and the missing transverse momentum, which we identify with the transverse momentum of the neutrino, satisfy $p_{\perp}^{e^+} > 30$ GeV and $p_{\perp}^{\text{miss}} > 30$ GeV, and that the rapidity of the charged lepton is bounded by $|\eta_{e^+}| < 2.4$. We also require that the transverse mass of the positron-neutrino system is larger than 60 GeV. In the case of the Z boson, we select electrons and positrons with transverse momenta larger than 25 GeV and require that their rapidities are within the interval $|\eta_{e^{\pm}}| < 2.4$.

Repeating the computation described above for fiducial cross sections, we find larger shifts in the W mass due to mixed QCD-electroweak corrections. Specifically, we obtain

$$\delta m_W^{\text{meas}} = -17 \pm 2 \text{ MeV}, \quad (10)$$

where the central value is for $\mu = m_V/2$ and the uncertainty is obtained from a three-point scale variation. Although electroweak corrections also increase if fiducial cuts are applied, they are still small; we estimate that they change the measured value of the W mass by only about 3 MeV.

Although a detailed study of the impact of fiducial cuts on the W -mass extraction is beyond the scope of this simple analysis, it is interesting to investigate how the somewhat larger $\mathcal{O}(17)$ MeV shift comes about. The key reason for this is that the transverse momenta that play a role in the analysis are determined by ratios $p_{\perp}^{e^+}/M_V$, see Eq.(1). The ATLAS collaboration applies a *higher* $p_{\perp}^{e^+}$ cut to the (lighter) W boson sample than to the (heavier) Z boson sample. Effectively, this choice of cuts gives higher weight to the high- $p_{\perp}^{e^+}$ region in the W case as compared to the Z case. Since radiative corrections in the W case extend to a wider range beyond the Jacobian peak, this leads to a (small) decorrelation of the transverse momentum distributions from Z and W production [48] which is sufficient, however, to cause a shift in m_W that appears to be significant given the target precision.

If the large shift in the W mass in Eq.(10) is caused by an experimentally motivated but “unfortunate” choice of cuts, one can ask whether it possible to choose cuts in such a way that the similarity of $p_{\perp}^{e^+}$ -distributions in Z and W samples is actually enforced. To answer this question, we proceed as follows: we start with the baseline fiducial region described above, but for the W^+ case we *decrease* the cuts on the transverse momentum of the positron and on the missing transverse energy until the theoretical correction factor C_{th} in Eq. (4) becomes $C_{\text{th}} = 1$ at leading order. This leads to a p_{\perp} cut of 25.44 GeV. Using this set of cuts, we find that both the EW and the mixed QCD-EW corrections to the $p_{\perp}^{e^+}$ spectra in Z and W production become more strongly correlated. Specifically, we observe that mixed QCD-EW corrections shift the W mass by only

$$\delta m_W^{\text{meas}} = -1 \pm 5 \text{ MeV}, \quad (11)$$

where again the central value corresponds to $\mu = m_V/2$ and the uncertainty is obtained from a three-point scale variation. For comparison, electroweak corrections in this case shift the W mass by $\mathcal{O}(-3)$ MeV.

In conclusion, we have applied a simple and theoretically clean procedure to estimate the impact of the recently computed mixed QCD-EW corrections [36, 39] on the W -mass extraction at the LHC. Similar to the experimental analyses [4, 5, 42], we used the transverse momentum distribution of a charged lepton from W decays as an observable from which the W mass can be inferred. However, instead of using the full distribution, we focused our analysis on its first moment, i.e. on the average p_{\perp} of the charged lepton.

The key element of the experimental analysis is the use of the lepton p_{\perp} distribution in Z production and the known mass of the Z boson as a constraint to be employed in the extraction of the W mass. The idea is that all effects that impact Z and W production in a similar way play no role in the W -mass extraction if the Z sample is used to normalize the W sample. Hence, the important question is not by how much lepton distributions in Z and W production are affected by various radiative corrections but rather if they are affected in a correlated fashion or not.

Our calculations and analyses indicate that there is no simple answer to this question in the sense that selection criteria applied to Z and W samples do matter [48]. Indeed, we observe that when no cuts are applied to lepton p_{\perp} distributions or when the $p_{\perp}^{l,W,Z}$ cuts are chosen in a way that roughly respect the ratio of W and Z masses, shifts in m_W caused by the mixed QCD-electroweak corrections to the production process appear to be below the LHC target precision of $\mathcal{O}(10)$ MeV. On the other hand, with a choice of cuts more aligned with experimental practices we find that mixed QCD-electroweak corrections cause bigger shifts in m_W . For example, we estimate that the cuts employed by the ATLAS collaboration in their recent extraction of the W mass [5] may lead to a shift of about $\mathcal{O}(17)$ MeV due to unaccounted mixed QCD-electroweak effects in the production process.

We stress that these results are only *estimates*: given the simplified nature of our analysis, we cannot insist that shifts in the W mass described above should be applied to results of actual measurements. Nevertheless, we believe that the size of the effects found here warrants further, more in-depth studies, which should ideally go hand-in-hand with the actual experimental analyses. Natural avenues of investigation include using more differential information rather than just the first moment of the lepton p_{\perp} distribution, or quantifying how much of these effects are actually captured by the simulation tools that are currently used by the experimental collaborations. We believe that such studies are mandatory to make a convincing case for $\mathcal{O}(10)$ MeV precision on W -mass extractions at the LHC.

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