README for ancillary files of
"Exact results for $Z_m^{\rm OS}$ and $Z_2^{\rm OS}$ with two mass scales and up to three loops", see Ref. [1]

Together with this paper we provide the following files which contain analytic expressions for the various quantities in `Mathematica` format:

- `master.m` contains the 27 master integrals $M_1, \ldots, M_{16}, M_{18}, \ldots, M_{28}$ as `resM1`, ..., `resM28`. Note that M_{17} is obtained from Eq. (6) of Ref. [1].
- `zmZmZ2.m` contains exact results for the expressions `zmos`, `ZmOS` and `Z2OS` in the on-shell scheme. Here the variable `x` corresponds to `xOSOS`.
- `expansions/` is a direcory which contains the expansions for z_m and $Z_2^{\rm OS}$ in the three limits $x \to 0$, $x \to 1$ and $x \to \infty$ for various combinations of on-shell and $\overline{\rm MS}$ masses for m_1 and m_2.
- `zmZ2_eval.m` provides the functions `zmnum[x,m1,mu1,mu2,[,scheme]]` and `Z2OSnum[x,m1,mu1,mu2[,scheme]]` which can be used for the numerical evaluation of z_m and $Z_2^{\rm OS}$. In the case of `zmnum` the option `scheme` may take the values `"OSOS"`, `"MSOS"`, `"OSMS"` and `"MSMS"`, where the first (last) two letters refer to the scheme of m_2 (m_1). In the case of `Z2OSnum` the values `"OSOS"` and `"MSOS"` are allowed. Depending on the value of x and the specified scheme the corresponding results from `expansions/` are loaded.
- `toGINAC.m` provides rules which maps the iterated integrals `GL[{...}, x]` and HPLs `H[..., x]` to Goncharov polylogarithms which allows for a numerical evaluation with `GiNaC` [2].

For the meaning of the symbols we refer to Tab. 1. The exact expressions in `zmZmZ2.m` contain in addition the iterated integrals `GL[{...}, x]` $= I(\{...\}), x)$ and the HPLs (`H[..., x]`), both in the notation of `HamonicSums` [3].

api	lmm1	cf	ca	tr	nl	nm	nh
$\alpha_s(\mu)/\pi$	$\log(\mu^2/(m_1^{\rm OS})^2)$	C_F	C_A	T_F	n_l	n_m	n_h
x	xOSOS	xMSOS	xOSMS	xMSMS	mu1	mu2	
x	x	$x_f(\mu_f)$	$x_q(\mu_f)$	$x_{fq}(\mu_f, \mu)$	μ	μ_f	
m1OS	m1MS	m2OS	m2MS				
$m_1^{\rm OS}$	$\overline{m}_1(\mu)$	$m_2^{\rm OS}$	$\overline{m}_2(\mu_f)$				

Table 1: Meaning of the symbols used in the `Mathematica` expressions.

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