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Modified majoron model for cosmological anomalies

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The vacuum expectation value v_s of a Higgs triplet field Δ carrying two units of lepton number L induces neutrino masses $\propto v_s$. The neutral component of Δ gives rise to two Higgs particles, a pseudoscalar A and a scalar S. The most general renormalizable Higgs potential V for Δ and the Standard-Model Higgs doublet Φ does not permit the possibility that the mass of either A or S is small, of order v_s, while the other mass is heavy enough to forbid the decay $Z \to AS$ to comply with LEP 1 data. We present a model with additional dimension-6 terms in V, in which this feature is absent and either A or S can be chosen light. Subsequently we propose the model as a remedy to cosmological anomalies, namely the tension between observed and predicted tensor-to-scalar mode ratios in the cosmic microwave background and the different values of the Hubble constant measured at different cosmological scales. Furthermore, if Δ dominantly couples to the third-generation doublet $L_\tau = (\nu_\tau, \tau)$, the deficit of ν_τ events at IceCube can be explained. The singly and doubly charged triplet Higgs bosons are lighter than 280 GeV and 400 GeV, respectively, and could be found at the LHC.

INTRODUCTION

Although the Hot Big Bang Model and General Relativity are arguably very robust, they work only if an additional piece is added to the game: inflation. In order to explain the flatness, homogeneity and isotropy of the Universe and the abscence of monopoles and other relics, a period of inflation is crucial.

But inflation is a framework comprising countless different inflationary models. Clearly all of them produce a flat, isotropic, homogeneous and relic-free Universe but each one leaves some specific imprints (as the inhomogeneities pattern of the model at hand in the CMB and structure formation) that can help us find out, which one of the plethora of models in the market is the correct one.

During inflation two types of perturbations are produced: scalar or matter perturbations and tensor (metric) perturbations (gravity waves). Each one can be characterized by its amplitude and the dependence on the scale of such amplitude. However, only a subset of two of these four quantities is independent and therefore all our insight of inflation is reduced to two parameters generally chosen to be the spectral index n_s, *i.e.* the dependence on the scale of the matter perturbations, and the tensor to scalar (amplitude) ratio r. This is the reason why all the inflationary models reduce to lines, points or regions in the $n_s - r$ plane.

As a consequence to discriminate which region is favored by experiments is also to select which inflationary models remain in the game. The theoretical guidance at this stage is crucial. Specific particle physics models with their matter content and interactions should help shed some light on which are the inflationary potentials worth considering, while at the same time making predictions which can be tested elsewhere.

In recent years tensions of cosmological data with the predictions based on the SM and the ΛCDM model have emerged and a light scalar boson ϕ interacting with neutrinos has been considered to alleviate these tensions. Specifically, favored regions for the the ratio r of tensor (metric) to scalar (matter) perturbations inferred from the anisotropies of the cosmic microwave background (CMB) and the spectral index n_s can be significantly modified and therefore the selection rule for successful inflationary models is vastly affected [1–4]. Furthermore, the Hubble constant determined from local measurements disagrees with the value inferred from CMB data and new neutrino interactions might remedy this as well [5, 6].

Historically, the first interest into light scalars interacting with neutrinos was driven by the attempt to build *Majoron* models breaking lepton number spontaneously, with the most natural realization through an SU(2) triplet field [7–9]. These models (and those employing doublet fields) did not comply with LEP 1 data on invisible Z decays and subsequently instead the focus moved to SU(2) singlet fields ϕ [10, 11]. Couplings of singlets to active neutrinos are too tiny to solve the cosmological problems in the favored region with $m_\phi > 30\,\text{keV}$, because coupling constants and/or mixing angles are too small. Thus to date there is no viable model supporting the idea of Refs. [1–6].

In the following we will present a model of neutrinos that can not only modify the allowed region in the $n_s - r$ plane changing this way the inflationary models that survive the experimental scrutiny but also provides a realization of the original Majoron idea [7–9]. Furthermore, it can be tested in forthcoming experiments.

This Letter is organized as follows: In the following section we present a class of Majoron models in which either S or A is light, while the other boson is heavy enough to forbid $Z \to AS$. Next we discuss phenomenological consequences and "smoking gun" features. Finally we conclude.

THE MODEL

An SU(2) triplet Higgs field

$$\Delta = \begin{pmatrix} \frac{\delta^+}{\sqrt{2}} & \delta^{++} \\ v_s + \frac{h_s + i a_s}{\sqrt{2}} & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix} \tag{1}$$

developing a vacuum expectation value (vev) v_s in the neutral component generates Majorana masses of light neutrinos in a natural way via its coupling to the lepton doublets. Electroweak precision data imply $v_s \ll v$, where $v = 174\,\mathrm{GeV}$ is the vev of the doublet Higgs field Φ of the Standard Model (SM). The physical Higgs fields are mixtures of the components of Δ and Φ; in particular there are two extra neutral Higgs bosons, a scalar S and a pseudoscalar P. These approximately coincide with h_s and a_s, respectively. By assigning two units of lepton number L to Δ and choosing an $\mathrm{U}(1)_L$-invariant Higgs potential one arrives at a model which breaks $\mathrm{U}(1)_L$ spontaneously. Then A is a massless Goldstone mode, the *Majoron* [7–9].

Postulating an effective interaction of neutrinos with a light scalar ϕ,

$$L_{\text{eff}} = g_{\alpha\beta} \bar{\nu}_\alpha \nu_\beta \phi, \tag{2}$$

where ν_α generically denotes any of the light left-handed neutrino fields or its charge-conjugate, one can alleviate the tensions in cosmological data mentioned in the introduction while simultaneously modifying the $n_s - r$ region to include well motivated inflationary models which were previously ruled out [1–5]. In this Letter we only consider couplings of ϕ to τ-neutrinos ν_τ, for which terrestrial and astrophysical data do not imply strict bounds and it is not relevant for us whether L is identified with the total lepton number or with L_τ . L_{eff} is not gauge invariant, a meaningful interaction must be formulated in terms of lepton doublets. Identifying ϕ with the Majoron amounts to completing L_{eff} to

$$\begin{aligned} L_y^\Delta &= \frac{y_\tau^\Delta}{2} \overline{L_3^c} \Delta L_3 + h.c. \\ &\supset -\frac{m_{\nu_\tau}^\Delta}{2} (\bar{\nu}_\tau \nu_\tau^c + \overline{\nu_\tau^c} \nu_\tau) \\ &\quad - \frac{y_\tau^\Delta}{4} \left[(h_s + i a_s) \bar{\nu}_\tau \nu_\tau^c \; + \; (h_s - i a_s) \overline{\nu_\tau^c} \nu_\tau) \right], \end{aligned} \tag{3}$$

where $L_3 = (\nu_\tau, \tau)^T$, $L_3^c = (\tau^c, -\nu_\tau^c)^T$, and $m_{\nu_\tau}^\Delta = y_\tau^\Delta v_s$ is the contribution of Δ to the Majorana mass of ν_τ. However, all known Higgs potentials predict that S and A are either both light, with masses around v_s, or both heavy, with masses of order v or larger. This finding holds for the most general renormalizable Higgs potential, irrespective of whether L is broken spontaneously or explicitly. For this reason the original Majoron models were discarded with the advent of LEP 1 data on the invisible Z width which left no room for the decay $Z \to AS$, whose decay rate is entirely fixed by the value of the SU(2) gauge coupling.[#1] Finally, in models in which ϕ in Eq. (2) is a singlet field mixing with a heavy S or A the coupling $g \equiv g_{\tau\tau}$ is suppressed by a tiny mixing angle and is far too small to solve the cosmological tensions. The same remark applies, if a singlet ϕ couples to heavy sterile neutrinos which mix with ν_τ.

For the phenomenological analysis it does not matter whether S or A is the light scalar corresponding to ϕ in Eq. (2) and for definiteness we consider the case $\phi = S \simeq h_s$. Then Eq. (3) entails $g = -y_\tau^\Delta/4$.

Global fits to cosmological data constrain the combination [4]

$$G_{\text{eff}} = \frac{g^2}{m_\phi^2} \; = \; \frac{y_\tau^{\Delta\,2}}{16\, m_S^2}, \tag{4}$$

while successful Big Bang Nucleosythesis (BBN) is sensitive to the mass and coupling of the scalar particle in a different combination. More specifically, BBN is very sensitive to the amount of extra radiation. The observed primordial abundances of deuterium and helium set strong constraints on the coupling and the mass of our scalar particle. These bounds are reflected in figure 1, where it can be seen that two regions are consistent with BBN and at the same time result in a G_{eff} able to change the CMB temperature and polarization spectra. A *heavy* or MeV region with $0.1\,\mathrm{MeV} \le m_S \le 1\,\mathrm{MeV}$ and a *light* or keV region with $m_S \le 0.1$ keV.

Bounds from meson decays are irrelevant in our case (and therefore not shown) as our new interaction concerns only tau neutrinos. We further stress that none of the bounds derived on L_{eff} in Eq. (2) from τ or tauonic Z decays (see eg.g. [12] for a recent study) applies to a complete model like ours: The proper cancellation of infrared singluarities (for $m_\phi \to 0$) requires the inclusion of virtual corrections, which in turn involve also heavy fields (like the charged components of Δ) to cancel ultraviolet divergences.

The Higgs potential of Δ in Eq. (1), $\Phi = (\phi^+, v +$

[#1] The decays $Z \to SS$ and $Z \to AA$ are forbidden by CP invariance of the electroweak gauge interaction.

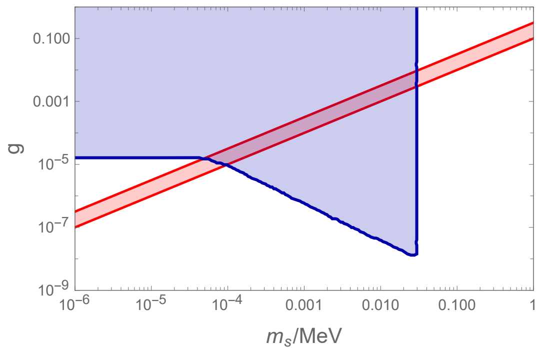

FIG. 1. The contour of the extra radiation $\Delta N_{\rm eff} \equiv N_{\rm eff} - 3 = 0.6$ at a temperature of 1 MeV in the m_S-g plane. The (light-blue) region above the blue line is forbidden by primordial helium and deuterium abundances, thus m_S is in the sub-keV range or $m_S \geq 30\,$keV. The red region corresponds to $10^{-2}/\,{\rm MeV}^2 < G_{\rm eff} < 10^{-1}/\,{\rm MeV}^2$, which is the 2σ CMB favored region for $G_{\rm eff} > 10^{-4}/\,{\rm MeV}^2$ [4].

$\frac{h+ia}{\sqrt{2}})^T$, and $\Phi^c = (v + \frac{h-ia}{\sqrt{2}}, -\phi^-)$ reads:

$$
\begin{aligned}
V =& -\mu^2 \Phi^\dagger\Phi \;-\; \mu_\Delta^2 \,{\rm Tr}\,(\Delta^\dagger\Delta) \;+\; \lambda(\Phi^\dagger\Phi)^2 \\
&+\; \lambda_\Delta \left[{\rm Tr}\,(\Delta^\dagger\Delta)\right]^2 \;+\; \alpha_1\,\Phi^\dagger\Delta^\dagger\Delta\Phi \;+\; \alpha_2\,\Phi^\dagger\Delta\Delta^\dagger\Phi \\
&+\; \alpha_3\,\Phi^\dagger\Phi\,{\rm Tr}\,(\Delta^\dagger\Delta) \;-\; \beta\,\left(\Phi^{c\,\dagger}\Delta^\dagger\Phi + \Phi^\dagger\Delta\Phi^c\right) \\
&+\; \delta_1\,\left(\Phi^{c\,\dagger}\Delta^\dagger\Phi + \Phi^\dagger\Delta\Phi^c\right)^2 \\
&-\; \delta_2\,\left(\Phi^{c\,\dagger}\Delta^\dagger\Phi - \Phi^\dagger\Delta\Phi^c\right)^2 . \qquad (5)
\end{aligned}
$$

V is complete up to terms of dimension 4[#2] and the dimension-6 terms involving $\delta_{1,2}$ are instrumental to lift the mass of either S or A above M_Z. All parameters are chosen real, so that V is invariant under charge conjugation C, and we only consider solutions with real vevs. L is a good symmetry of V for $\beta = \delta_1 - \delta_2 = 0$. The minimization conditions $\partial V/\partial v = 0 = \partial V/\partial v_s$ read:

$$
\begin{aligned}
\mu^2 &= 2\lambda v^2 \,+\, \mathcal{O}\left(\frac{v_s^4}{v^2}\right), \\
\beta &= v_s\left(\frac{m^2}{v^2} + 4\delta_1 v^2 + 2\lambda_\Delta \frac{v_s^2}{v^2}\right) \qquad (6)
\end{aligned}
$$

$$
\text{with} \qquad m^2 \equiv -\mu_\Delta^2 + \alpha_2 v^2 + \alpha_3 v^2 \qquad (7)
$$

The parameter m^2 will govern the mass of the desired light state. Avoiding fine-tuning between different terms in Eq. (7) we must have

$$
\mu_\Delta^2, \alpha_2 v^2, \alpha_3 v^2 \leq \mathcal{O}(m^2), \qquad (8)
$$

Using the minimization conditions in Eq. (6) we trade μ and β for v and v_s in the formulae below for fields and masses. Neglecting $\mathcal{O}\left(\frac{v_s^2}{v^2}\right)$ terms the physical states are

$$
\begin{aligned}
G &= a + 2\frac{v_s}{v}\,a_s, \qquad A \;=\; a_s - \frac{v_s}{v}\,a, \qquad (9) \\
H &= h - \frac{2\mu_\delta^2 + 8\delta_1 v^4}{m^2 - 4\lambda v^2 + 4\delta_1 v^4}\,\frac{v_s}{v}\,h_s, \\
S &= h_s + \frac{2\mu_\delta^2 + 8\delta_1 v^4}{m^2 - 4\lambda v^2 + 4\delta_1 v^4}\,\frac{v_s}{v}\,h, \qquad (10) \\
G^+ &= \phi^+ + \sqrt{2}\frac{v_s}{v}\,\delta^+, \quad H^+ \;=\; \delta^+ - \sqrt{2}\frac{v_s}{v}\,\phi^+, \quad (11) \\
H^{++} &= \delta^{++}. \qquad (12)
\end{aligned}
$$

Here G and G^+ are the massless Goldstone bosons eaten by Z and W^+, respectively. Neglecting subdominant terms the desired (squared) Higgs masses read

$$
\begin{aligned}
m_A^2 &= 4\delta_2 v^4 + m^2 + 2\lambda_\Delta v_s^2 \qquad (13) \\
m_S^2 &= 4\delta_1 v^4 + m^2 + 6\lambda_\Delta v_s^2 \qquad (14) \\
m_H^2 &= 4\lambda v^2 = (125\,{\rm GeV})^2, \qquad (15) \\
m_{H^{++}}^2 &= 2m_{H^+}^2 = \alpha_1 v^2. \qquad (16)
\end{aligned}
$$

We start our discussion with the role of ${\rm U}(1)_L$ symmetry in V. For this it is helpful to use Eq. (6) to trade $m^2 + 2\lambda_\Delta v_s^2$ for β in m_A^2 in Eq. (13) to find

$$
m_A^2 = 4(\delta_2 - \delta_1)v^4 + \frac{\beta v^2}{v_s}. \qquad (17)
$$

For exact ${\rm U}(1)_L$ symmetry with $\beta = 0$ and $\delta_1 = \delta_2$ we verify that A is the massless Goldstone boson of this broken symmetry. This solution corresponds to the vanishing of the bracket in Eq. (6). (For the second solution with $v_s = 0$ the symmetry is unbroken and Eq. (17) does not hold.) An interesting case is $\beta = \delta_1 = 0$ with $\delta_2 \neq 0$: ${\rm U}(1)_L$ is explicitly broken by a higher-dimensional term only: The minimization equation (6) is not sensitive to this term and features spontaneous ${\rm U}(1)_L$ as in the original, renormalizable Majoron models. The phenomenological effect of $\delta_2 \neq 0$ is to render m_A massive, with the possibility of $m_A > M_Z$, and we may view this case as *spontaneous symmetry breaking without Goldstone.*

For fixed $G_{\rm eff}$ the perturbativity limit $y_\tau^{\Delta\,2} \lesssim 2$ entails an upper limit on m_S through Eq. (4). In the scenario with spontaneous ${\rm U}(1)_L$ breaking (where $\beta = 0$) one necessarily has $m \lesssim v_s$. As a consequence, $m_{\nu_\tau}^\Delta = y_\tau^\Delta v_s \lesssim 1\,$eV pushes $y_\tau^{\Delta\,2}$ and m_S far below their otherwise theoretically allowed upper bounds. Specifically, $m_S \lesssim 10\,$keV for $G_{\rm eff} \geq 10^{-4}\,{\rm MeV}^{-2}$ and the BBN constraint of figure 1 tightens this to $m_S \lesssim 0.3\,$keV. In the scenario, with explicit ${\rm U}(1)_L$ breaking, however, one easily infers from Eq. (6) that one can choose m (and thereby m_S) and v_s independently thanks to the free parameter β.

It is easy to find a UV completion generating the dim-6 terms in Eq. (5): Consider heavy real scalar singlets $\chi_{1,2}$

[#2] The term $\lambda'_\Delta\,{\rm Tr}\,(\Delta^\dagger\Delta^\dagger){\rm Tr}\,(\Delta\Delta)$ is phenomenologically irrelevant.

coupling as

$$V_s = (\rho_1 + i\sigma_1)\chi_1 \Phi^{c\,\dagger}\Delta^\dagger\Phi + (\rho_2 + i\sigma_2)\chi_2 \Phi^{c\,\dagger}\Delta^\dagger\Phi + \text{H.c.} \tag{18}$$

with real $\rho_{1,2}$, $\sigma_{1,2}$. Under charge conjugation we have $\Delta \leftrightarrow \Delta^*$, $\Phi \leftrightarrow \phi^*$. Choosing further $\chi_1 \leftrightarrow \chi_1$, $\chi_2 \leftrightarrow -\chi_2$ and demanding C invariance of V_s implies $\sigma_1 = \rho_2 = 0$. Integrating out the heavy singlet fields gives $\delta_1 = \rho_1^2/m_{\chi_1}^2$ and $\delta_2 = \sigma_2^2/m_{\chi_2}^2$, and e.g. $m_{\chi_1} \gg m_{\chi_2}$ produces the scenario with heavy A and light S. Instead of invoking C symmetry one can also work with L, by assigning $L = 1$ to $\chi = \chi_1 + i\chi_2$ enforcing $\rho_2 + i\sigma_2 = i(\rho_1 + i\sigma_1)$. The UV sector must be more baroque and break $L_{(\tau)}$ in a way to produce the desired mass hierarchy. Loop effects and/or a small vev of χ_1 may render $\beta \neq 0$. However, we consider it as an advantage that V in Eq. (5) allows us to fully study the low-energy phenomenology (including loop effects where needed) without specifying the UV completion.

PHENOMENOLOGY

Studies of perturbativity for the SM [13] and 2HDM [14, 15] have shown that self-couplings should be smaller than ≈ 5. Applying this bound to α_1 in Eq. (16) implies that $m_{H^{++}} \lesssim 400\,\text{GeV}$ and $m_{H^+} \lesssim 280\,\text{GeV}$. Current collider bounds are much weaker, because the production of these heavy charged Higgs bosons is an electroweak gauge process (e.g. vector boson fusion at the LHC). Favorable decays are $H^{++} \to \tau^+\tau^+$ and $H^+ \to \tau^+\nu_\tau$, unless y_τ^Δ is too small. In the latter case one must resort to gauge-coupling driven decays like $H^+ \to W^+S, W^+A$. For a cut-off scale of $\Lambda \sim 0.5\,\text{TeV}$ (and $\mathcal{O}(1)$ couplings in the UV completion) we have $\delta_2 v^2 \sim 0.1$ and Eq. (14) gives $m_A \sim 120\,\text{GeV}$. A is produced through gauge interactions, and now a small y_τ^Δ is welcome to suppress the decay into neutrinos. Detection through $A \to ZS$ will fail if $M_A - M_Z$ is smaller than the trigger threshold for missing transverse momentum. It is therefore advisable to focus on the searches for the charged bosons.

The model can be tested by its astrophysical signatures as well. Depending whether our scalar field is in the MeV or keV range different signals can be expected. As mentioned before, CMB expriments are sensitive only to G_{eff} and therefore both ranges give exactly the same phenomenology cosmology-wise. This is not the case regarding astrophysical experiments. For scalars in the MeV range, the interaction introduced will make high energy ($\sim$TeV) tau neutrinos from astrophysical sources scatter resonantly with the CMB tau neutrinos and therefore a deficit of tau neutrinos can be expected. More precisely a dip in the tau neutrino spectrum corresponding to the resonant energy [4]

$$E_{\text{resonant}} \simeq \frac{m_\phi^2}{2m_{\nu_\tau}} \tag{19}$$

is to be expected in experiments like IceCube and KM3Net[#3]. Remarkably, IceCube seems to be seeing a deficit in tau neutrinos although the effect is not significant yet.

For scalars in the keV range the resonant energies involved make it ideal to detect such interactions in experiments sensitive to the diffuse supernova neutrino background, like T2HK. A detailed analysis of both signals will be given elsewhere.

CONCLUSIONS

The possibility of a light scalar particle interacting with neutrinos receives a lot of attention to alleviate several tensions in cosmological data. The particle physics community is interested in this kind of interaction as a means to break lepton number L spontaneously, providing a natural framework for neutrino Majorana masses. So far all cosmological analyses have employed the effective interaction of Eq. (2), which violates electroweak SU(2) symmetry. In this Letter we have presented a viable realization of a model of a scalar interacting with neutrinos, by complementing the original SU(2) triplet models (featuring spontaneous or explicit L violation) with higher-dimensional terms and devising possible UV completions generating these terms. With our lagrangian one can now consistently calculate constraints from Z, charged lepton, and meson decays, which was not possible with Eq. (2). To bypass these constraints we have discussed the cosmological and astrophysical implications for the case that the light scalar couples dominantly to tau neutrinos. Depending on the mass range of the scalar particle, characteristic signals are possible at the IceCube or T2HK experiments. The LHC can search for the charged members of the SU(2) triplet, whose masses must be below 400 GeV. For appropriate choices of the parameters our Higgs potential conserves L at the level of the renormalizable terms and shares essential features of the orginal Majoron model [9].

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[#3] Note that when the tau neutrino mass drops below the neutrino CMB temperature the resonance energy becomes independent of the neutrino mass. In this case the resonance falls into the IceCube sensitivity range, unless the mass of the neutrino is close to the current cosmological limit

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[1] I. M. Oldengott, T. Tram, C. Rampf and Y. Y. Wong, JCAP **11** (2017), 027 doi:10.1088/1475-7516/2017/11/027 [arXiv:1706.02123 [astro-ph.CO]].

[2] L. Lancaster, F. Y. Cyr-Racine, L. Knox and Z. Pan, JCAP **07** (2017), 033 doi:10.1088/1475-7516/2017/07/033 [arXiv:1704.06657 [astro-ph.CO]].

[3] C. D. Kreisch, F. Y. Cyr-Racine and O. Doré, [arXiv:1902.00534 [astro-ph.CO]].

[4] G. Barenboim, P. B. Denton and I. M. Oldengott, Phys. Rev. D **99** (2019) no.8, 083515 doi:10.1103/PhysRevD.99.083515 [arXiv:1903.02036 [astro-ph.CO]].

[5] N. Blinov, K. J. Kelly, G. Z. Krnjaic and S. D. McDermott, Phys. Rev. Lett. **123** (2019) no.19, 191102 doi:10.1103/PhysRevLett.123.191102 [arXiv:1905.02727 [astro-ph.CO]].

[6] M. Escudero and S. J. Witte, Eur. Phys. J. C **80** (2020) no.4, 294 doi:10.1140/epjc/s10052-020-7854-5 [arXiv:1909.04044 [astro-ph.CO]].

[7] Y. Chikashige, R. N. Mohapatra and R. Peccei, Phys. Rev. Lett. **45** (1980), 1926 doi:10.1103/PhysRevLett.45.1926

[8] Y. Chikashige, R. N. Mohapatra and R. Peccei, Phys. Lett. B **98** (1981), 265-268 doi:10.1016/0370-2693(81)90011-3

[9] G. Gelmini and M. Roncadelli, Phys. Lett. B **99** (1981), 411-415 doi:10.1016/0370-2693(81)90559-1

[10] C. P. Burgess and J. M. Cline, Phys. Rev. D **49** (1994) 5925 doi:10.1103/PhysRevD.49.5925 [hep-ph/9307316].

[11] P. Bamert, C. P. Burgess and R. N. Mohapatra, Nucl. Phys. B **449** (1995) 25 doi:10.1016/0550-3213(95)00273-U [hep-ph/9412365].

[12] V. Brdar, M. Lindner, S. Vogl and X. J. Xu, arXiv:2003.05339 [hep-ph].

[13] U. Nierste and K. Riesselmann, Phys. Rev. D **53** (1996), 6638-6652 doi:10.1103/PhysRevD.53.6638 [arXiv:hep-ph/9511407 [hep-ph]].

[14] V. Cacchio, D. Chowdhury, O. Eberhardt and C. W. Murphy, JHEP **11** (2016), 026 doi:10.1007/JHEP11(2016)026 [arXiv:1609.01290 [hep-ph]].

[15] C. W. Murphy, Phys. Rev. D **96** (2017) no.3, 036006 doi:10.1103/PhysRevD.96.036006 [arXiv:1702.08511 [hep-ph]].