TTP19-016, P3H-19-02

Little hierarchies solve the little fine-tuning problem: a case study in supersymmetry with heavy guinos

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(Dated: August 3, 2019)

Radiative corrections with new heavy particles coupling to Higgs doublets destabilize the electroweak scale and require an ad-hoc counterterm cancelling the large loop contribution. If the mass scale m_1 of these new particles in in the TeV range, this feature constitutes the *little fine-tuning problem*. We consider the case that the new-physics spectrum has a little hierarchy with two particle mass scales $m_{1,2}$ and $m_2 = \mathcal{O}(10\, m_1)$ and no tree-level couplings of the heavier particles to Higgs doublets. As a concrete example we study the (next-to-)minimal supersymmetric standard model ((N)MSSM) for the case that the gluino mass M_3 is significantly larger than the stop mass parameters $m_{L,R}$ and show that the usual one-loop fine-tuning analysis breaks down. If $m_{L,R}$ is defined in the dimensional-reduction ($\overline{\text{DR}}$) or any other fundamental scheme, corrections enhanced by powers of $M_3^2/m_{L,R}^2$ occur in all higher loop orders. After resumming these terms we find the fine-tuning measure substantially improved compared to the usual analyses with $M_3 \lesssim m_{L,R}$. In our hierarchical scenario the stop self-energies grow like M_3^2, so that the stop masses $m_{L,R}^{\text{OS}}$ in the on-shell (OS) scheme are naturally much larger than their $\overline{\text{DR}}$ counterparts $m_{L,R}^{\overline{\text{DR}}}$. This feature permits a novel solution to the little fine-tuning problem: $\overline{\text{DR}}$ stop masses are close to the electroweak scale, but radiative corrections involving the heavy gluino push the OS masses, which are probed in collider searches, above their experimental lower limits. As a byproduct, we clarify which renormalization scheme must be used for squark masses in loop corrections to low-energy quantities such as the $B-\overline{B}$ mixing amplitude.

INTRODUCTION

Theoretical attempts to unify gauge forces necessarily lead to new particles with masses way above the electroweak scale $v = 174$ GeV defined by the vacuum expectation value (vev) of the Standard-Model (SM) Higgs boson. Such heavy particles generally lead to unduly large radiative corrections to v^2, in conflict with the naturalness principle which forbids fine-tuned cancellations between loop contribution and counterterm for any fundamental parameter in the lagrangian [1–4]. The observation that in supersymmetric field theories [5] corrections to the electroweak scale vanish exactly [6–8] made supersymmetric models the most popular framework for studies of beyond-Standard-Model (BSM) phenomenology.

Supersymmetry breaking introduces a mass splitting between the SM particles and their superpartners. Increasing lower bounds on the masses of the latter derived from unsuccessful searches at the LEP, Tevatron, and LHC colliders brought the fine-tuning problem back: Specifically, stops heavier than 1 TeV induce loop corrections to the Higgs potential which must be cancelled by tree-level parameters to two or more digits. Owing to this *little fine-tuning problem* low-energy supersymmetry has lost some of its appeal as a candidate for BSM physics. Nevertheless, analyses of naturalness in supersymmetric theories, which are under study since the pre-LEP era, still receive a lot of attention [9–42].

In this paper we study the little fine-tuning problem for the case of a hierarchical superpartner spectrum, with gluinos several times heavier than the stops. The gluino mass is less critical for fine-tuning, because gluinos couple to Higgs fields only at the two-loop level. In such a scenario the usual fine-tuning analyses based on fixed order perturbation theory break down. Denoting the left-chiral and right-chiral stop mass parameters by $m_{L,R}^2$ and the gluino mass by M_3 we identify n-loop corrections enhanced by $\left[M_3^2/m_{L,R}^2\right]^{n-1}$ and resum them. These terms are not captured by renormalization-group (RG) analyses of effective Lagrangians derived by successively integrating out heavy particles at their respective mass scales, which instead target large logarithms.

Our findings do not depend on details of the Higgs sector, and we exemplify our results for both the Minimal Supersymmetric Standard Model (MSSM) and its next-to-minimal variant NMSSM. The results also trivially generalise to non-supersymmetric theories with little hierarchies involving a heavy scalar field coupling to Higgs fields and a heavier fermion coupling to this scalar.

CORRECTIONS TO THE HIGGS MASS PARAMETERS IN THE (N)MSSM

We consider only small or moderate values of the ratio $\tan\beta \equiv v_2/v_1$ of the vacuum expectation values (vevs) of the two Higgs doublets $H_1 = (h_1^0, h_1^-)^T$, $H_2 = (h_2^+, h_2^0)^T$, so that all Yukawa couplings are small except for the coupling y_t of the (s)tops to H_2. Our (N)MSSM loop calculations involve the gluino-stop-top vertices as well

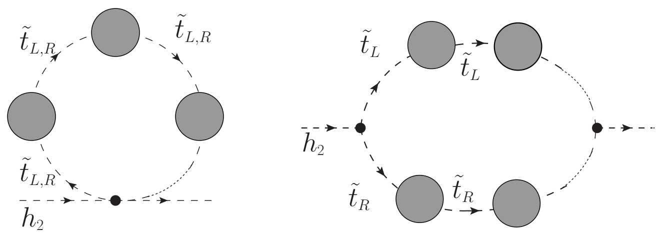

FIG. 1. Resummed contributions to m_{22}^2.

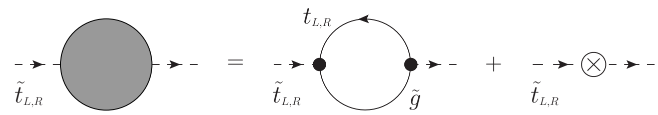

FIG. 2. Stop self-energies with gluino loop and counterterm.

as the couplings encoded in the superpotential

$$\mathcal{W} = y_t \left(\tilde t_R \tilde t_L\, h_2^0 - \tilde t_R \tilde b_L\, h_2^+\right) \tag{1}$$

and the supersymmetry-breaking Lagrangian

$$\begin{aligned}-\mathcal{L}_{\rm soft} = A_t &\left(\tilde t_R \tilde t_L\, h_2^0 - \tilde t_R \tilde b_L\, h_2^+\right) + \text{H.c.}\\ &+ m_L^2 \left(\tilde t_L^\star \tilde t_L + \tilde b_L^\star \tilde b_L\right) + m_{h_2}^2 \left(h_2^{0,\star} h_2^0 + h_2^{+,\star} h_2^+\right)\\ &+ m_{h_1}^2 \left(h_1^{0,\star} h_1^0 + h_1^{-,\star} h_1^-\right) + m_R^2\, t_R t_R^\star\\ &+ \frac{1}{2}\, M_3\, \overline{\psi_{\tilde g}} \psi_{\tilde g}\end{aligned} \tag{2}$$

with the stop, sbottom, and gluino fields $\tilde t_{L,R}$,$\tilde b_{L,R}$,$\psi_{\tilde g}$, respectively. In the notation of Ref. [43] the ($\mathbb{Z}_3$ symmetric) NMSSM Higgs potential reads

$$\begin{aligned}V_{higgs} =&\left|\kappa s^2 - \lambda h_1^0 h_2^0\right|^2 + (m_{h_1^2}^2 + \lambda^2 |s|^2)\left|h_1^0\right|^2\\ &+ (m_{h_2^2}^2 + \lambda^2 |s|^2)\left|h_2^0\right|^2 + \frac{g^2}{4}\left(\left|h_2^0\right|^2 - \left|h_1^2\right|\right)^2\\ &+ m_s^2 |s|^2 + \left(\frac{1}{3} A_\kappa s^3 - A_\lambda h_1^0 h_2^0 s + \text{H.c.}\right).\end{aligned} \tag{3}$$

Note that $g^2 \equiv (g_1^2 + g_2)^2/2$ and terms with charged fields are dropped. The singlet field s acquires the vev v_s. The electroweak scale is represented by the Z boson mass M_Z. Minimizing V_{higgs} gives

$$\frac{1}{2} M_Z^2 = \frac{m_{11}^2 \cos^2\beta - m_{22}^2 \sin^2\beta}{\sin^2\beta - \cos^2\beta} \tag{4}$$

with the tree-level contributions

$$m_{11}^{2\,(0)} = m_{h_1}^2 + \lambda^2 |v_s|^2, \quad m_{22}^{2\,(0)} = m_{h_2}^2 + \lambda^2 |v_s|^2. \tag{5}$$

In the MSSM Eqs. (3) and (5) hold with the replacements $\lambda s, \lambda v_s \to \mu_h$, $\lambda, \kappa, A_\kappa \to 0$, and $A_\lambda s, A_\lambda v_s \to B\mu_h$ with the higgsino mass term μ_h and the soft supersymmetry breaking term $B\mu_h$. In the following we identify $\mu_h \equiv \lambda v_s$ and $B\mu_h \equiv A_\lambda v_s$, which allows us to use the same notation for MSSM and NMSSM. Next we integrate out the heavy sparticles and thereby match the (N)MSSM onto an effective two-Higgs-doublet model. We parametrize the loop contributions as

$$m_{22}^2 = m_{22}^{2\,(0)} + m_{22}^{2\,(1)} + m_{22}^{2\,(2)} + m_{22}^{2\,(\geq 3)} \tag{6}$$

with the well-known one-loop term

$$\begin{aligned}m_{22}^{2,(1)} =& - \frac{3\,|y_t|^2}{16\,\pi^2}\left[m_L^2 \left(1 - \log\frac{m_L^2}{\mu^2}\right) + L \to R\right]\\ &- \frac{3\,|A_t|^2}{16\,\pi^2}\, \frac{m_R^2 - m_R^2 \log\frac{m_R^2}{\mu^2} - m_L^2 + m_L^2 \log\frac{m_L^2}{\mu^2}}{m_L^2 - m_R^2}\end{aligned} \tag{7}$$

in the modified dimensional reduction ($\overline{\text{DR}}$) scheme. $\mu = \mathcal{O}(m_{L,R})$ is the renormalization scale. The corrections to other mass parameters like m_{11}^2 are small as long as $|A_t|, |\mu_h|$ are not too large. At one-loop order the fine-tuning issue only concerns the first term in $m_{22}^{2,(1)}$, which requires sizable cancellations with $m_{22}^{(0)}$ to reproduce the correct M_Z in Eq. (4).

At n-loop level with $n \geq 2$ we only consider the contributions enhanced by $\left(M_3^2/m_{L,R}^2\right)^{n-1}$ with respect to $m_{22}^{2\,(1)}$ stemming solely from Feynman diagrams with $n-1$ stop self-energies shown in Fig. 1. Other multi-loop diagrams involve fewer stop propagators and do not contribute to the highest power of $M_3^2/m_{L,R}^2$. The self-energies involve a gluino-top loop and a stop mass counterterm, see Fig. 2. We decompose $m_{22}^{2\,(n)}$ as

$$m_{22}^{2\,(n)} = m_{22\,I}^{2\,(n)} + m_{22\,II}^{2\,(n)} \tag{8}$$

for the two sets of diagrams in Fig. 1. The left diagrams constituting $m_{22\,I}^{2\,(n)}$ have n stop propagators while the right ones summing to $m_{22\,II}^{2\,(n)}$ have $n{+}1$ stop propagators. Inspecting the UV behaviour of the stop loop shows that only $m_{22\,I}^{2\,(2)}$ contains a logarithm $\log(M_3/m_{L,R})$. Explicit calculation of the two-loop diagrams yields

$$\begin{aligned}m_{22}^{2\,(2)} =&\frac{\alpha_s(\mu)\,|y_t|^2\, M_3^2}{4\,\pi^3}\Bigg[\\ &- \left(1 + \log\frac{\mu^2}{M_3^2}\right)\left(1 + 2\log\frac{\mu\, M_3}{m_L\, m_R}\right)\\ &+ \frac{\pi^2}{3} + \mathcal{O}\left(\frac{m_{L,R}^2}{M_3^2}\right)\Bigg]\end{aligned} \tag{9}$$

in the ($\overline{\text{DR}}$) scheme. If one considers very large mass splitting between $m_{L,R}$ and M_3, one may choose to integrate out these sparticles at different scales and finds $\mu \sim M_3$ more appropriate than $\mu = \mathcal{O}(m_{L,R})$ in $\alpha_s(\mu)$ and the first logarithm in Eq. (9). $m_{22\,II}^{2\,(2)}$ has no $\log(M_3/m_{L,R})$

and amounts to only $\sim 10\%$ of $m^{2\,(2)}_{22\,I}$ for the numerical examples considered below.

For $M_3 \gg m_{L,R}$ we find for the resummed higher-order contributions:

$$\begin{aligned} m^{2\,(\geq 3)}_{22\,I} &= \frac{3\,|y_t|^2}{16\pi^2} m_L^2 \sum_{k=2}^{\infty} \frac{\xi_L^k}{k(k-1)} + L \to R \\ &= \frac{3\,|y_t|^2}{16\pi^2} m_L^2 \left[\xi_L + (1-\xi_L)\log(1-\xi_L)\right] + L \to R \end{aligned} \tag{10}$$

$$\begin{aligned} m^{2\,(2)}_{22\,II} + m^{2\,(\geq 3)}_{22\,II} &= -\frac{3\,|A_t|^2}{16\pi^2} \sum_{k=1}^{\infty} \frac{\xi^k_{L,R}}{k} \\ &= \frac{3\,|A_t|^2}{16\pi^2} \log(1-\xi_{L,R}) \end{aligned} \tag{11}$$

with

$$\xi_{L,R} \equiv \quad -\frac{4\alpha_s(\mu)}{3\pi} \frac{M_3^2}{m^2_{L,R}} \left[1 + \log\frac{\mu^2}{M_3^2}\right] + \Delta\xi_{L,R}. \tag{12}$$

$\Delta\xi_{L,R}$ controls the renormalization scheme of the stop masses, $\Delta\xi_{L,R} = 0$ for the $\overline{\rm DR}$ scheme. For simplicity we quote the numerically less important term in Eq. (11) for the special case $m_L = m_R$. For $M_3 \sim 5\, m_{L,R}$ one finds $\xi_{L,R} \sim -1$, so that $m^{2\,(\geq 3)}_{22\,I,II}$ is of similar size as $m^{2\,(1)}_{22\,I,II}$. The expressions above define $m^{2\,(n)}_{22,I,II}$ at the scale $\mu \sim m_{L,R}$. We minimize the Higgs potential at the lower scale m_t (denoting the top mass) where

$$m^2_{22}(m_t) = \left(1 - \frac{6\,|y_t|^2}{16\,\pi^2} \log\frac{\mu}{m_t}\right) m^2_{22}\,(\mu)\,, \tag{13}$$

while the running of m^2_{11} and $m^2_{12} \equiv B\mu_h$ is negligible.

Next we switch to the on-shell (OS) scheme for the stop masses. For clarity we consider the case of small $|A_t|$ and $|\mu_h|$, so that stop mixing is negligible and $m^{\rm OS}_{L,R}$ coincide with the two mass eigenstates. In the OS scheme the counterterm $\Delta\xi_{L,R}$ in Eq. (12) cancels the stop self-energies and renders $\xi_{L,R} = 0$. Thus $m^{2\,(\geq 3)}_{22} = m^{2\,(2)}_{22,II} = 0$, while $m^{2\,(2)}_{22,I}$ is non-zero due to the different UV behavior of the stop momentum loop:

$$\begin{aligned} m^{2,(2)\,\rm OS}_{22} = \frac{\alpha_s(\mu)\,|y_t|^2\,M_3^2}{4\,\pi^3} \Bigg[&-1 + \log\frac{\mu^2}{m_L\,m_R} \\ &+ \log^2\frac{\mu^2}{M_3^2} + \frac{\pi^2}{3} + \mathcal{O}\left(\frac{m^2_{L,R}}{M_3^2}\right)\Bigg] \end{aligned} \tag{14}$$

Thus with stop pole masses no $M_3^2/m^2_{L,R}$ enhanced terms appear beyond two loops and the resummation of the higher-order terms is implicitly contained in the shift $m_{L,R} \to m^{\rm OS}_{L,R}$, which absorbs the higher-order terms into $m^{(1)}_{22}$ and $m^{(2)}_{22}$. The μ dependence in Eq. (14) results from the stop loop integration, i.e. the superscript "OS" in Eq. (14) only refers to the definition of the stop mass, while m^2_{22} is still $\overline{\rm DR}$ renormalized.

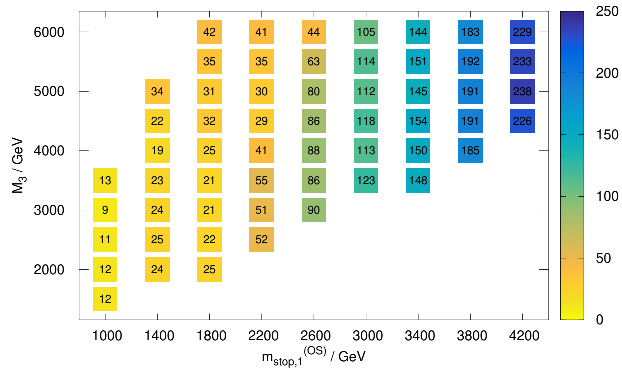

FIG. 3. Fine-tuning measure $\Delta(m_L)$ for different values of the lighter on-shell stop mass (essentially equal to $m_L^{\rm OS}$ in our analysis) and M_3. The number gives the mean of 100 sample points that correctly reproduce $M_Z = 91$ GeV and $m_h = 125$ GeV [44, 45].

For the fine-tuning issue there are several important lessons: Most importantly, $m^{2\,\rm OS}_{L,R}$ is *larger* than $m^2_{L,R}$ by terms $\propto \alpha_s M_3^2$, meaning that the LHC lower bound on $m^{\rm OS}_{L,R}$ permits a $\overline{\rm DR}$ mass $m_{L,R}$ closer to the electroweak scale complying with naturalness. That is, $m^{\rm OS}_{L,R}$ could well be dominated by the gluino-top self-energy. In the on-shell scheme we observe moderate fine-tuning in m^2_{22} if we vary $m_{L,R}$, partly because the large radiative piece of $m^{\rm OS}_{L,R}$ depends only logarithmically on $m_{L,R}$, and partly because the effects from $m^{2\,(1)}_{22}$ and $m^{2\,(2)}_{22}$ have opposite signs and tend to cancel out. This behavior can be better understood if we solely work in the $\overline{\rm DR}$ scheme: For $m_{L,R}$ close to the electroweak scale none of the infinite number of terms $m^{(n)}_{22}$ is individually so large that it calls for a fine-tuned $m^{(0)}_{22}$ in Eq. (6). We may instead be concerned about the fine-tuning related to a variation of M_3: In a perturbation series truncated at order n we see a powerlike growth with terms up to $\xi^n_{L,R}$ in the sum in Eq. (10), with the terms of different loop orders having similar magnitude and alternating signs. However, the resummation tempers this behaviour to $m^2_{L,R}\xi_{L,R} \sim M_3^2$. We have numerically checked that we obtain the same results for m^2_{22} in both approaches, i.e. by either employing the explicit resummation in the $\overline{\rm DR}$ scheme or converting the stop masses to the OS scheme.

NUMERICAL STUDY OF THE FINE-TUNING

We use the Ellis-Barbieri-Giudice fine-tuning measure

[10, 11]

$$\Delta(p) = \left| \frac{p}{M_Z(p)} \frac{\partial M_Z(p)}{\partial p} \right| , \tag{15}$$

where p stands for any Lagrangian parameter. Using $\overline{\text{DR}}$ stop masses as input we calculate the $\overline{\text{OS}}$ masses which enter the loop-corrected Higgs potential through Eqs. (7) and (14). For the latter we determine all two-loop contributions to m_{11}^2, m_{12}^2, and m_{22}^2 involving α_s, y_t, A_t exactly. E.g. we go beyond the large-M_3 limit of the previous section and calculate 205 two-loop diagrams in total. For this we have used the Mathematica packages `FeynArts` [46] (with the Feynman rules of Ref. [47]) and `Medusa` [48, 49], which performs asymptotic expansions in small external momenta and large masses. The analytic methods involved are based on Refs. [50–55].

We start with the discussion of the NMSSM: With two of the three minimization conditions we trade the parameters m_s^2 and A_λ for $\mu_h \equiv \lambda v_s$ and $\tan\beta$. (The third minimization condition is Eq. (4) yielding M_Z.) For the illustrative example in Fig. 3 we fix the parameters $\tan\beta = 3$, $\lambda = 0.64$, $\kappa = 0.25$, $\mu_h = 200\,\text{GeV}$, and $m_{11}^{(0)} = 600\,\text{GeV}$. Then we choose $m_{22}^{2,(0)}$, A_t, $m_L^{\overline{\text{DR}}}$, $m_R^{\overline{\text{DR}}}$, A_κ randomly subject to the contraints that the correct values of M_Z and the lightest Higgs mass $m_h = 125$ GeV as well as the smaller stop mass $m_{\tilde{t},1}^{\text{OS}}$ displayed in Fig. 3 are reproduced for a given value of M_3. We calculate $\Delta(m_L)$ for over 100 different parameter points corresponding to a given point $(m_{\tilde{t},1}^{\text{OS}}, M_3)$; the number in the colored square is the average $\Delta(m_L)$ found for these points. For most of our parameter points $m_{\tilde{t},1}^{\text{OS}} \approx m_L$, but this feature is irrelevant because the formulae are symmetric under $m_L \leftrightarrow m_R$. By quoting the average rather than the minimum of $\Delta(m_L)$ we make sure that a good fine-tuning measure is not due to accidental cancellations.

To illustrate the result of Fig. 3 with an example we consider the parameter point with

$$\begin{aligned} m_{11}^{(0)} &= 600\,\text{GeV} \quad m_{22}^{(0)} = 94\,\text{GeV} \quad M_3 = 3\ \text{TeV} \\ A_\kappa &= -6.5\,\text{GeV} \quad A_t = 453\,\text{GeV} \\ m_L^{\overline{\text{DR}}} &= 611\,\text{GeV} \quad m_R^{\overline{\text{DR}}} = 902\,\text{GeV} \end{aligned} \tag{16}$$

which yields $m_{\tilde{t},1}^{\text{OS}} = 1$ TeV, lying substantially above m_L. Note that $M_3/m_L \approx 5$, while the hierarchy in the physical masses is moderate, $M_3/m_{\tilde{t}_1}^{\text{OS}} = 3$. The fine-tuning measures for this benchmark point are $\Delta(m_L) = 6.0$, $\Delta(m_R) = 10.8$, $\Delta(M_3) = 6.3$, $\Delta(A_t) = 0.2$, and all other $\Delta(p)$ are negligibly small.

Next we briefly discuss the MSSM. A recent analysis has found values of $\Delta \equiv \max_p \Delta(p) \geq 63$ for special versions of the MSSM in scans over the parameter spaces [42]. Compared to the NMSSM one needs larger stop masses to accomodate $m_h = 125$ GeV, which then leads to larger values of Δ. Yet also for the MSSM the hierarchy $M_3 \gg m_{L,R}$ with proper resummation of higher-order terms improves Δ. We exemplify this with the parameter point

$$\begin{aligned} m_{11}^{(0)} &= 1583\ \text{GeV} & m_{22}^{(0)} &= 124\ \text{GeV} \\ \mu_h &= 400\ \text{GeV} & \tan\beta &= 5 \\ M_3 &= 4500\ \text{GeV} & A_t &= 3370\ \text{GeV} \\ m_L &= 2787\ \text{GeV} & m_R &= 1435\ \text{GeV} \end{aligned}$$

The on-shell stop masses for this point are $m_{\tilde{t}_1}^{\text{OS}} = 2168$ GeV and $m_{\tilde{t}_1}^{\text{OS}} = 3012$ GeV. Despite these large masses the fine-tuning measures $\Delta(m_L) = 13$, $\Delta(m_R) = 25$, $\Delta(M_3) = 8$ have moderate values while a fine-tuning measure $\Delta(A_t) = 41$ reflects the large A_t needed to accomodate $m_h = 125$ GeV.

Finally we remark that also low-energy observables like the $B-\overline{B}$ mixing amplitude or the branching ratios of rare meson decays (such as $b \to s\gamma$, $K \to \pi\nu\overline{\nu}$) involve higher-order corrections enhanced by a relative factor of $M_3^2/m_{L,R}^2$, if the stop masses are renormalised in a mass-independent scheme like $\overline{\text{DR}}$. This remark applies to supersymmetric theories with minimal flavor violation (MFV) in which the leading contribution is dominated by a chargino-stop loop and the gluino is relevant only at next-to-leading order and beyond. The resummation of the gluino-stop self-energies on the internal stop lines is trivially achieved by using the on-shell stop masses in the leading-order prediction, because the flavor-changing loop is UV-finite; i.e. we face the same situation as with $m_{22}^{2\,(\geq 3)}$. Thus low-energy observables effectively probe the same stop masses as the collider searches at high p_T.

CONCLUSIONS

We have investigated the fine-tuning of the electroweak scale in models of new physics with a heavy and hierarchical mass spectrum. Studying supersymmetric models with $M_Z < m_{L,R} < M_3$ we have demonstrated that the usual fine-tuning analysis employing fixed-order perturbation theory breaks down for $M_3 \sim 5\, m_{L,R}$. Resumming terms enhanced by $M_3^2/m_{L,R}^2$ tempers the fine-tuning. This behavior is transparent if the stop masses are renormalized on-shell: The resummation is then encoded in the shift from the $\overline{\text{DR}}$ masses to the larger on-shell masses and new allowed parameter ranges with small values of $m_{L,R}^2$ emerge, because large radiative corrections proportional to $\alpha_s M_3^2$ push the physical on-shell masses over the experimental lower bounds. In these scenarios the heavy stops are *natural*, as their masses are larger than the –parametrically large– self-energies. As a byproduct we have found that low-energy observables probe the on-shell stop masses.

Acknowledgements. We thank Stefan de Boer for checking expressions Eqs. (10) and (11) and several helpful discussions and acknowledge the support of *Deutsche Forschungsgemeinschaft* (DFG, German Research Foundation) through RTG 1694 and grant 396021762 - TRR 257 "Particle Physics Phenomenology after the Higgs Discovery".

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