

## Addendum: “Impact of polarization observables and $B_c \rightarrow \tau\nu$ on new physics explanations of the $b \rightarrow c\tau\nu$ anomaly”

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In this addendum to Ref. [1], we update our results including the recent measurement of  $\mathcal{R}(D)$  and  $\mathcal{R}(D^*)$  by the Belle collaboration [2]:  $\mathcal{R}(D)_{\text{Belle}} = 0.307 \pm 0.037 \pm 0.016$  and  $\mathcal{R}(D^*)_{\text{Belle}} = 0.283 \pm 0.018 \pm 0.014$ , resulting in the new HFLAV fit result  $\mathcal{R}(D) = 0.340 \pm 0.027 \pm 0.013$ ,  $\mathcal{R}(D^*) = 0.295 \pm 0.011 \pm 0.008$ , exhibiting a  $3.1\sigma$  tension with the Standard Model. We present the new fit results and update all figures, including the relevant new collider constraints. The updated prediction for  $\mathcal{R}(\Lambda_c)$  from our sum rule reads  $\mathcal{R}(\Lambda_c) = \mathcal{R}_{\text{SM}}(\Lambda_c)(1.15 \pm 0.04) = 0.38 \pm 0.01 \pm 0.01$ . We also comment on theory predictions for the fragmentation function  $f_c$  of  $b \rightarrow B_c$  and their implication on the constraint from  $B_{u/c} \rightarrow \tau\nu$  data.

In this addendum, we present an update of our article [1] in which we studied the impact of polarization observables and the bound on  $\text{BR}(B_c \rightarrow \tau\nu)$  on new physics explanations of the  $b \rightarrow c\tau\nu$  anomaly.

Our updated results incorporate the new experimental results for  $\mathcal{R}(D)$  and  $\mathcal{R}(D^*)$  measured by the Belle collaboration [2]:

$$\begin{aligned}\mathcal{R}(D)_{\text{Belle}} &= 0.307 \pm 0.037 \pm 0.016, \\ \mathcal{R}(D^*)_{\text{Belle}} &= 0.283 \pm 0.018 \pm 0.014.\end{aligned}\quad (1)$$

The first quoted error is statistical and the second one is systematic. The new measurement is consistent with the standard model (SM) predictions [3]

$$\begin{aligned}\mathcal{R}_{\text{SM}}(D) &= 0.299 \pm 0.003, \\ \mathcal{R}_{\text{SM}}(D^*) &= 0.258 \pm 0.005\end{aligned}\quad (2)$$

at the  $0.2\sigma$  and  $1.1\sigma$  level, respectively.

Combining this with the previous measurements presented by the BaBar, Belle, and LHCb collaborations in

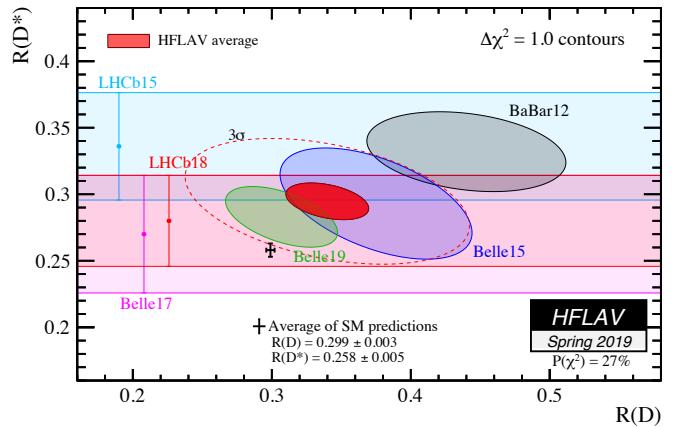


FIG. 1. The green ellipse shows the result of the new measurement by the Belle collaboration [2], while the red ellipse shows the new world average. The SM predictions are represented by the black bars. Figure taken from [3].

Refs. [4–12], the HFLAV collaboration [3] has determined the averages

$$\begin{aligned}\mathcal{R}(D) &= 0.340 \pm 0.027 \pm 0.013, \\ \mathcal{R}(D^*) &= 0.295 \pm 0.011 \pm 0.008,\end{aligned}\quad (3)$$

with an  $\mathcal{R}(D)$ – $\mathcal{R}(D^*)$  correlation of  $-0.38$ . The new world averages deviate from the SM at  $1.4\sigma$  [ $\mathcal{R}(D)$ ],  $2.5\sigma$  [ $\mathcal{R}(D^*)$ ], and  $3.1\sigma$  [ $\mathcal{R}(D)$ – $\mathcal{R}(D^*)$  combination] [3]. This situation is shown in Fig. 1.

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Including all four observables  $\mathcal{R}(D)$ ,  $\mathcal{R}(D^*)$ ,  $P_\tau(D^*)$  and  $F_L(D^*)$ , we find the new  $p$ -value of the two-sided test for the SM

$$p\text{-values}_{\text{SM}} \sim 0.1\%, \quad (4)$$

which corresponds to a  $3.3\sigma$  tension, where we neglect the SM uncertainty. Note that our choice of the form factors is explained in Ref. [1], and we obtain the following central values of the SM predictions:

$$\begin{aligned} \mathcal{R}_{\text{SM}}(D) &= 0.301, & \mathcal{R}_{\text{SM}}(D^*) &= 0.254, \\ P_{\tau,\text{SM}}(D) &= 0.32, & P_{\tau,\text{SM}}(D^*) &= -0.49, \\ F_{L,\text{SM}}(D^*) &= 0.46, & \mathcal{R}_{\text{SM}}(\Lambda_c) &= 0.33. \end{aligned} \quad (5)$$

All our fit results are based on these numbers.<sup>#1</sup>

The authors of Ref. [13] had deduced the stringent constraint  $\text{BR}(B_c \rightarrow \tau\nu) < 10\%$  from data on a mixed sample of  $B_c^- \rightarrow \tau\nu_\tau$  and  $B^- \rightarrow \tau\nu$  candidate events taken at the  $Z$  peak in the LEP experiment. To this end, the fragmentation function  $f_c$  of  $b \rightarrow B_c^-$  has been extracted from data accumulated at hadron colliders. For asymptotically large values of the transverse  $b$  momentum  $p_T$ , fragmentation functions are numbers which are independent of the kinematical variables and the  $b$  production mechanism. In Ref. [1], we have pointed out that hadron collider data exhibit a sizable  $p_T$  dependence and pointed to production mechanisms beyond fragmentation (see also Ref. [14]). In Fig. 1 of Ref. [13],  $f_c/f_u$  has been extracted from CMS and LHCb data. Using the world average of the  $b \rightarrow B^-$  fragmentation function  $f_u = 0.404(6)$  [15], we find that the result of Ref. [13] implies

$$2.1 \times 10^{-3} \lesssim f_c \lesssim 4.4 \times 10^{-3}. \quad (6)$$

If one instead uses a calculation of  $B_c^-$  production on the  $Z$  peak at  $e^+e^-$  colliders employing non-relativistic quantum chromodynamics (NRQCD) at next-to-leading order [16, 17] (see also Ref. [18]), one finds

$$f_c \sim 3 \times 10^{-4}, \quad (7)$$

with essentially the same estimate for  $b \rightarrow B_c^{*-}$  fragmentation. If one further assumes that  $B_c^{*-}$  decays into final states with  $B_c^-$  with a branching ratio of 1,<sup>#2</sup> then  $f_c$  effectively changes to

$$f_c \sim 6 \times 10^{-4}. \quad (8)$$

Therefore by comparing Eqs. (6) and (8), we conclude that the constraint on  $\text{BR}(B_c \rightarrow \tau\nu)$  derived in Ref. [13]

is too stringent by a factor of three to four. Taking into account the intrinsic uncertainties of the NRQCD calculation, the  $Z$  peak data cannot rule out our most conservative scenario which permits  $\text{BR}(B_c \rightarrow \tau\nu)$  to be as large as 60%.

Tables I and II update the respective tables in Ref. [1], showing the numerical results of the fit in the various one- and two-dimensional scenarios for the Wilson coefficients. The corresponding plots are shown in Figs. 2 and 3. In all cases, the best-fit points moved closer to the SM, with the biggest change in the one-dimensional scalar scenarios. In the  $C_S^R$  scenario, the best-fit point is hence no longer in tension with the aggressive  $\text{BR}(B_c \rightarrow \tau\nu) < 10\%$  bound.

The most general and powerful collider constraint to the  $b \rightarrow c\tau\nu$  operators comes from high- $p_T$  tails in mono- $\tau$  searches. Reference [22] had investigated the constraints on the effective field theory (EFT) operators mediating  $b \rightarrow c\tau\nu$ . This EFT analysis is valid for certain leptoquark models if the leptoquarks are sufficiently heavy.<sup>#3</sup> The resulting  $2\sigma$  upper bounds from the current collider data are [22]

$$|C_V^L| < 0.32, \quad |C_S^{L(R)}| < 0.57, \quad |C_T| < 0.16, \quad (9)$$

at the scale  $\mu = m_b$ . In Fig. 3, we apply these collider bounds on the four two-dimensional scenarios, where we assume that interference between two different operators is suppressed. Note that in contrast to our findings in Ref. [1], the best-fit points in the complex  $C_S^L = 4C_T$  scenario are no longer in tension with the collider constraints. Scenarios with color-singlet  $s$ -channel mediators, like a charged scalar, require model-dependent studies beyond the EFT framework, see e.g. [25, 26]. Hence, for the  $(C_S^R, C_S^L)$  scenario originating from the exchange of a charged Higgs boson, the collider bound is valid only in the heavy-mass limit, and we therefore indicate it by a dashed line.

Figure 4 shows the prediction for  $\mathcal{R}(\Lambda_c)$  in the four two-dimensional scenarios, as functions of  $\mathcal{R}(D)$  and  $\mathcal{R}(D^*)$ , respectively. In Ref. [1], we obtained a sum rule

$$\frac{\mathcal{R}(\Lambda_c)}{\mathcal{R}_{\text{SM}}(\Lambda_c)} \simeq 0.262 \frac{\mathcal{R}(D)}{\mathcal{R}_{\text{SM}}(D)} + 0.738 \frac{\mathcal{R}(D^*)}{\mathcal{R}_{\text{SM}}(D^*)}. \quad (10)$$

The decrease in  $\mathcal{R}(D^{(*)})$  implied by the new Belle measurement leads to a decreased prediction for  $\mathcal{R}(\Lambda_c)$  through our sum rule [1]

$$\begin{aligned} \mathcal{R}(\Lambda_c) &= \mathcal{R}_{\text{SM}}(\Lambda_c)(1.15 \pm 0.04) \\ &= 0.38 \pm 0.01 \pm 0.01, \end{aligned} \quad (11)$$

where the first error arises from the experimental uncertainty of  $\mathcal{R}(D^{(*)})$ , while the second error comes from

<sup>#1</sup> On the other hand, based on the SM predictions in Eq. (2), we obtain  $p\text{-values}_{\text{SM}} \sim 0.2\%$  corresponding to a  $3.1\sigma$  tension instead of Eq. (4).

<sup>#2</sup> While  $B_c(2S)^-$  and  $B_c^*(2S)^-$  have been observed through a transition of  $B_c^{(*)}(2S)^- \rightarrow B_c^{(*)-}\pi^+\pi^-$  [19–21], no  $B_c^{*-}$  has been detected yet.

<sup>#3</sup> Direct searches for leptoquarks coupled to third-generation quarks constrain their masses to roughly  $m_{\text{LQ}} > 1\text{ TeV}$  [23, 24]. These direct collider bounds significantly depend on branching fractions of the leptoquarks.

1D hyp.	best-fit	$1\sigma$ range	$2\sigma$ range	$p$ -value (%)	pull <sub>SM</sub>	$\mathcal{R}(D)$	$\mathcal{R}(D^*)$	$F_L(D^*)$	$P_\tau(D^*)$	$P_\tau(D)$	$\mathcal{R}(\Lambda_c)$
$C_V^L$	0.07	[0.05, 0.09]	[0.04, 0.11]	44	4.0	0.347 $+0.2\sigma$	0.292 $-0.2\sigma$	0.46 $-1.6\sigma$	-0.49 $-0.2\sigma$	0.32	0.38
$C_S^R$	0.09	[0.06, 0.11]	[0.03, 0.14]	2.7	3.1	0.380 $+1.4\sigma$	0.260 $-2.6\sigma$	0.47 $-1.5\sigma$	-0.46 $-0.1\sigma$	0.46	0.36
$C_S^L$	0.07	[0.04, 0.10]	[-0.00, 0.13]	0.26	2.1	0.364 $+0.8\sigma$	0.250 $-3.3\sigma$	0.45 $-1.7\sigma$	-0.51 $-0.2\sigma$	0.44	0.35
$C_S^L = 4C_T$	-0.03	[-0.07, 0.01]	[-0.11, 0.04]	0.04	0.7	0.278 $-2.1\sigma$	0.263 $-2.3\sigma$	0.46 $-1.6\sigma$	-0.47 $-0.2\sigma$	0.27	0.33

TABLE I. Updated fit results for the 1D hypotheses (hyp.) of Ref. [1], with the Wilson coefficients defined at the scale  $\mu = 1$  TeV.

2D hyp.	best-fit	$p$ -value (%)	pull <sub>SM</sub>	$\mathcal{R}(D)$	$\mathcal{R}(D^*)$	$F_L(D^*)$	$P_\tau(D^*)$	$P_\tau(D)$	$\mathcal{R}(\Lambda_c)$
$(C_V^L, C_S^L = -4C_T)$	(0.10, -0.04)	29.8	3.6	0.333 $-0.2\sigma$	0.297 $+0.2\sigma$	0.47 $-1.5\sigma$	-0.48 $-0.2\sigma$	0.25	0.38
$(C_S^R, C_S^L) _{60\%}$	(0.29, -0.25) (-0.16, -0.69)	75.7	3.9	0.338 $0.1\sigma$	0.297 $+0.1\sigma$	0.54 $-0.7\sigma$	-0.27 $+0.2\sigma$	0.39	0.38
$(C_S^R, C_S^L) _{30\%}$	(0.21, -0.15) (-0.26, -0.61)	30.9	3.6	0.353 $+0.4\sigma$	0.280 $-1.1\sigma$	0.51 $-1.0\sigma$	-0.35 $0.0\sigma$	0.42	0.37
$(C_S^R, C_S^L) _{10\%}$	(0.11, -0.04) (-0.37, -0.51)	2.6	2.9	0.366 $+0.9\sigma$	0.263 $-2.3\sigma$	0.48 $-1.4\sigma$	-0.44 $-0.1\sigma$	0.44	0.36
$(C_V^L, C_S^R)$	(0.08, -0.01)	26.6	3.6	0.343 $+0.1\sigma$	0.294 $-0.1\sigma$	0.46 $-1.6\sigma$	-0.49 $-0.2\sigma$	0.31	0.38
$(\text{Re}[C_S^L = 4C_T], \text{Im}[C_S^L = 4C_T]) _{60,30\%}$	(-0.06, $\pm 0.31$ )	25.0	3.6	0.339 $0.0\sigma$	0.295 $0.0\sigma$	0.45 $-1.7\sigma$	-0.41 $-0.1\sigma$	0.41	0.38
$(\text{Re}[C_S^L = 4C_T], \text{Im}[C_S^L = 4C_T]) _{10\%}$	(-0.03, $\pm 0.24$ )	5.9	3.2	0.330 $-0.3\sigma$	0.275 $-1.4\sigma$	0.46 $-1.6\sigma$	-0.45 $-0.1\sigma$	0.38	0.36

TABLE II. Updated fit results for the 2D hypotheses (hyp.) of Ref. [1], with the Wilson coefficients defined at the scale  $\mu = 1$  TeV.

the form factors. This model-independent relation between  $\mathcal{R}(D)$ ,  $\mathcal{R}(D^*)$ , and  $\mathcal{R}(\Lambda_c)$  originates from heavy-quark symmetry: in the heavy-quark limit the inclusive  $b \rightarrow c\tau\nu$  rate is saturated by the sum of  $B \rightarrow D\tau\nu$  and  $B \rightarrow D^*\tau\nu$  in the mesonic case, and by  $\Lambda_b \rightarrow \Lambda_c\tau\nu$  in the baryonic case [27]. We have checked that the sum rule in Eq. (10) also holds for new physics scenarios with right-handed neutrinos, although they are not considered in our analysis.

As shown in Fig. 5, the pairwise correlations between the polarisation observables  $P_\tau(D)$ ,  $P_\tau(D^*)$ , and  $F_L(D^*)$  are still distinct for the various two-dimensional scenarios. In order to fully exploit their potential, besides better measurements also more precise theoretical predictions for the  $B \rightarrow D$  and  $B \rightarrow D^*$  form factors are necessary.

Figures 6 and 7 show the contour lines of the polarization observables  $P_\tau(D)$ ,  $P_\tau(D^*)$ , and  $F_L(D^*)$  and the ratio  $\mathcal{R}(\Lambda_c)$  in the  $\mathcal{R}(D)$ – $\mathcal{R}(D^*)$  plane. In these plots only the position of the experimentally preferred region for  $\mathcal{R}(D)$  and  $\mathcal{R}(D^*)$  has been changed with respect to the version shown in Figs. 5 and 6 of Ref. [1].

In conclusion, we have updated our fit results for the  $b \rightarrow c\tau\nu$  anomaly including the recent data by the Belle collaboration [2]. The predictions for polarization observables from the fit significantly depend on the Wil-

son coefficient scenario. Therefore, by accurately probing their correlations at the ongoing Belle II experiment [28], one can in principle distinguish between different new physics models. To exploit their full discriminatory power, however, also more precise predictions of the relevant form factors are necessary. Furthermore we revisited the constraint on  $\text{BR}(B_c \rightarrow \tau\nu_\tau)$  from LEP data at the  $Z$ -peak, focusing on the theory predictions for the fragmentation of a  $b$  quark into a  $B_c$  meson, and concluding that our most conservative scenario  $\text{BR}(B_c \rightarrow \tau\nu_\tau) < 60\%$  is not excluded at present. Moreover, re-evaluating our sum rule connecting  $\mathcal{R}(\Lambda_c)$  with  $\mathcal{R}(D^{(*)})$ , we predict an enhancement of  $\mathcal{R}(\Lambda_c)$  of  $(15 \pm 4)\%$  with respect to its SM value model-independently, which serves as a good experimental crosscheck of the  $b \rightarrow c\tau\nu$  anomaly.

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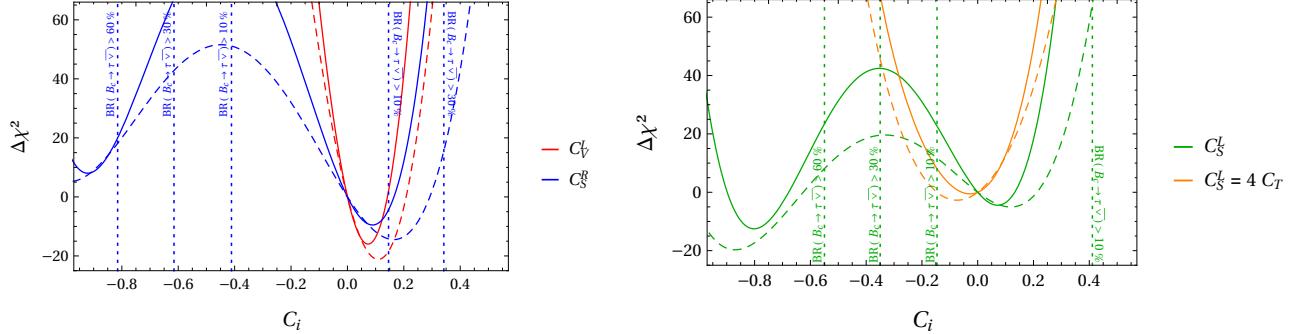


FIG. 2.  $\Delta\chi^2$  of  $\mathcal{R}(D)$ ,  $\mathcal{R}(D^*)$ ,  $P_\tau(D^*)$  and  $F_L(D^*)$  for the four one-dimensional (1D) scenarios where  $\mu = 1$  TeV. The dashed lines do not include the latest Belle results [2], while the solid lines include all data. The dotted vertical lines correspond to the limit on  $C_S^{L,R}$  from  $\text{BR}(B_c \rightarrow \tau\nu)$  assuming a maximal value of 10%, 30% and 60%. Best fit points are not constrained from the 10% limit.

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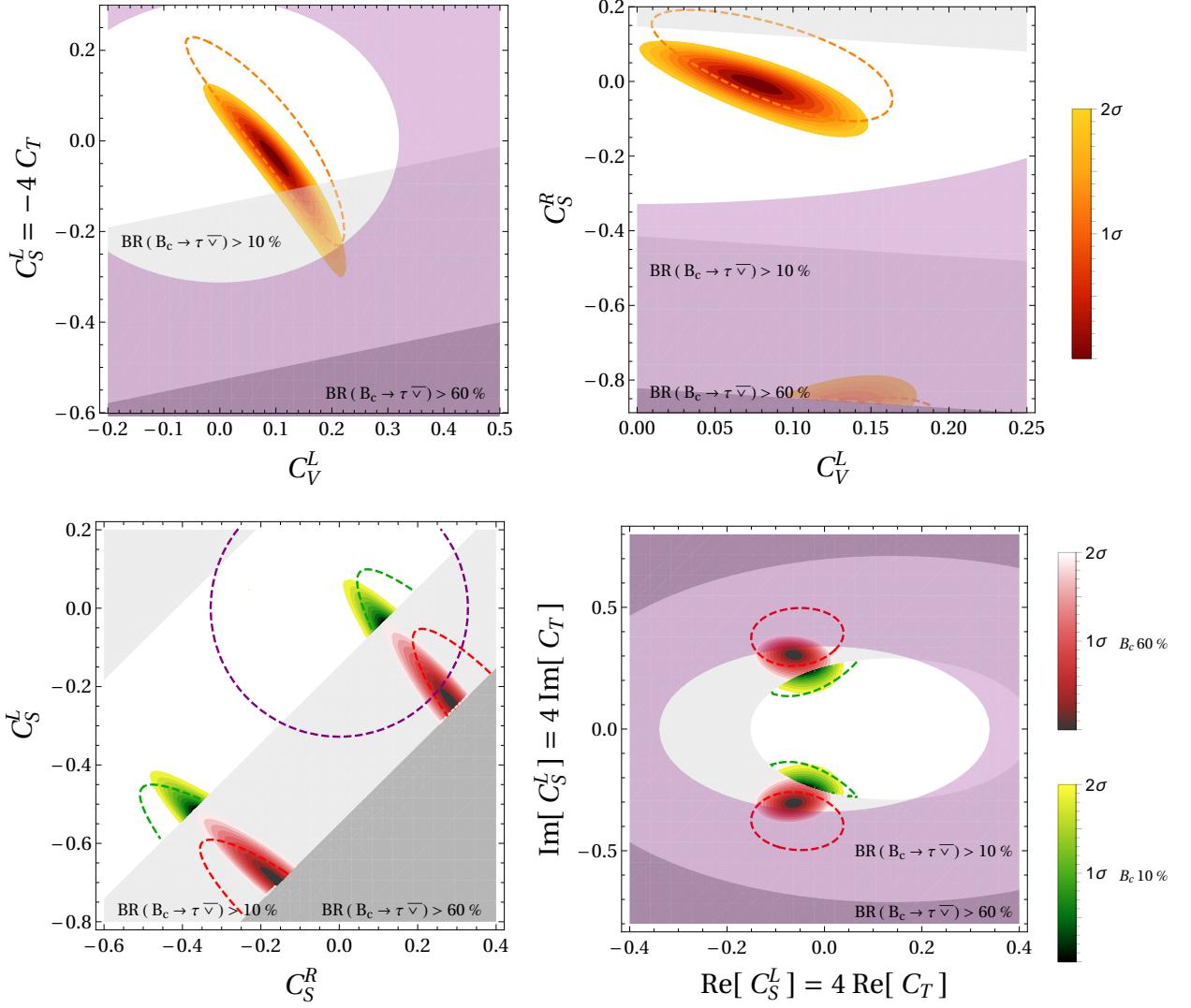


FIG. 3. Updated results of the fits for the  $2\sigma$  regions in the four 2D scenarios of Ref. [1], with Wilson coefficients given at the matching scale of 1 TeV. The dashed contours do not include the latest Belle results [2], while the shaded ellipses include all data. The current collider bounds in Eq. (9) exclude the purple shaded regions at  $2\sigma$  level. The dashed purple circle in the lower left plot indicates the collider constraint on the charged Higgs scenario (see text).

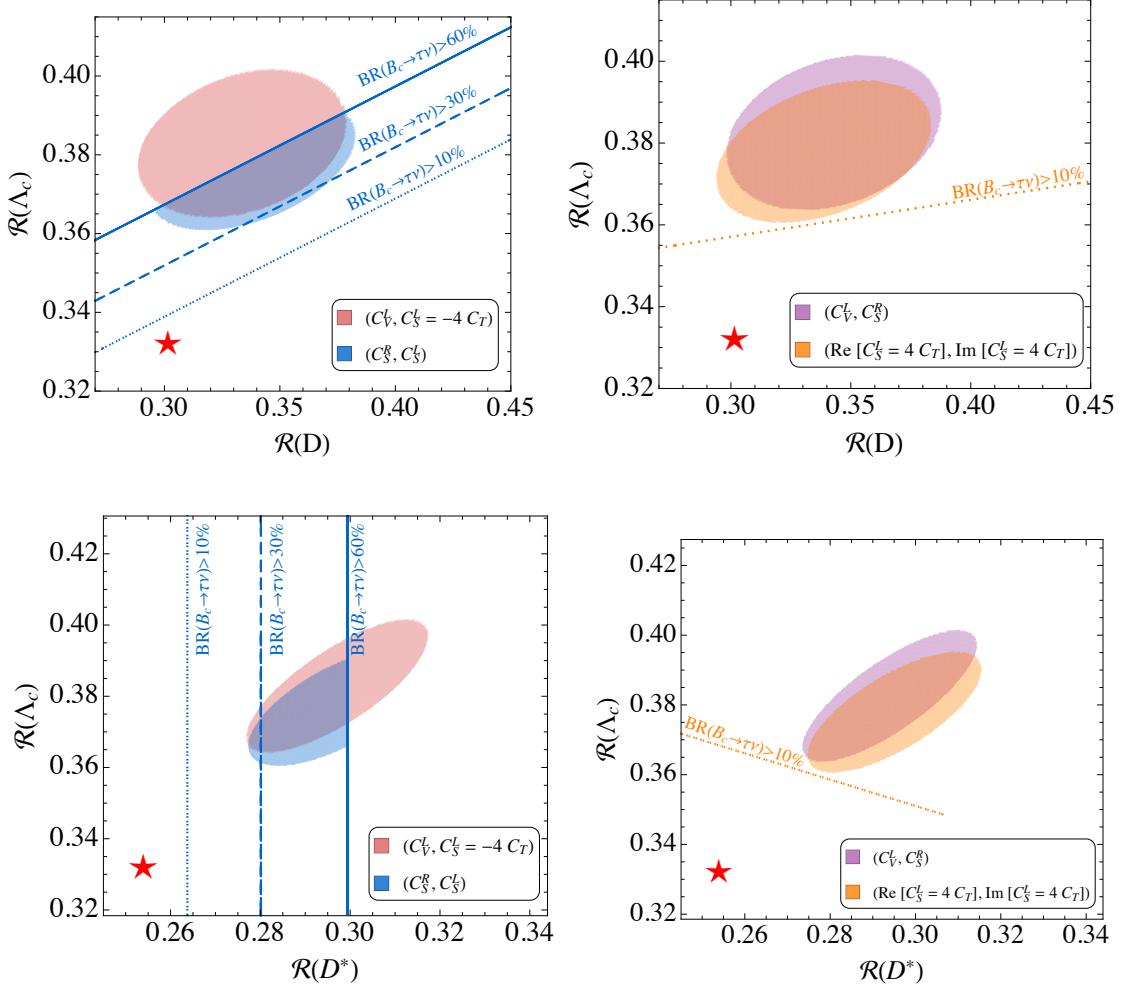


FIG. 4. Preferred  $1\sigma$  regions in the four two-dimensional scenarios in the  $\mathcal{R}(D^{(*)})$ - $\mathcal{R}(\Lambda_c)$  plane for  $\text{BR}(B_c \rightarrow \tau\nu_\tau) < 60\%$ , updating Fig. 3 of Ref. [1].

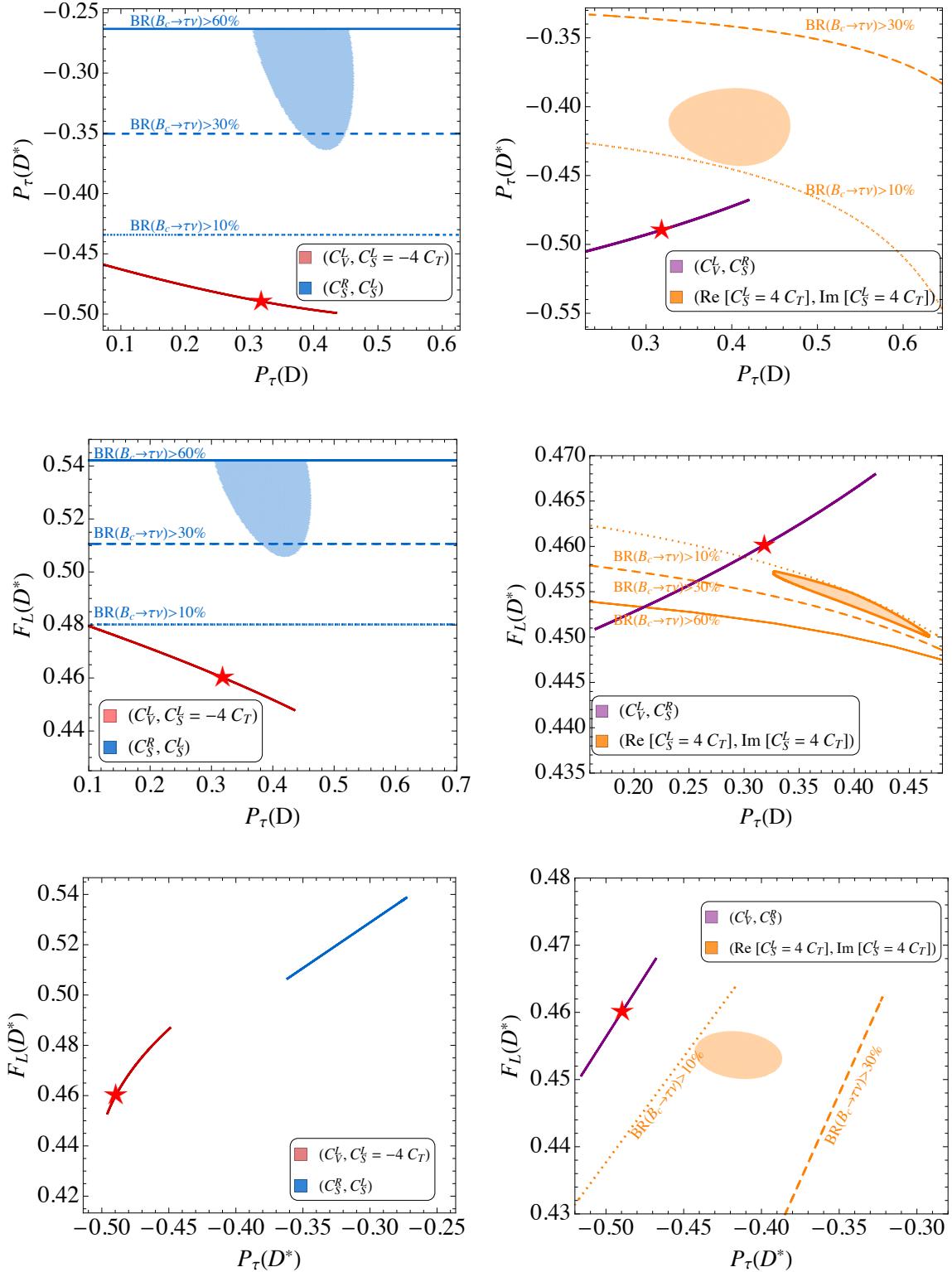


FIG. 5. Pairwise correlations between the observables  $P_\tau(D)$ ,  $P_\tau(D^*)$  and  $F_L(D^*)$ , updating Fig. 4 of Ref. [1].

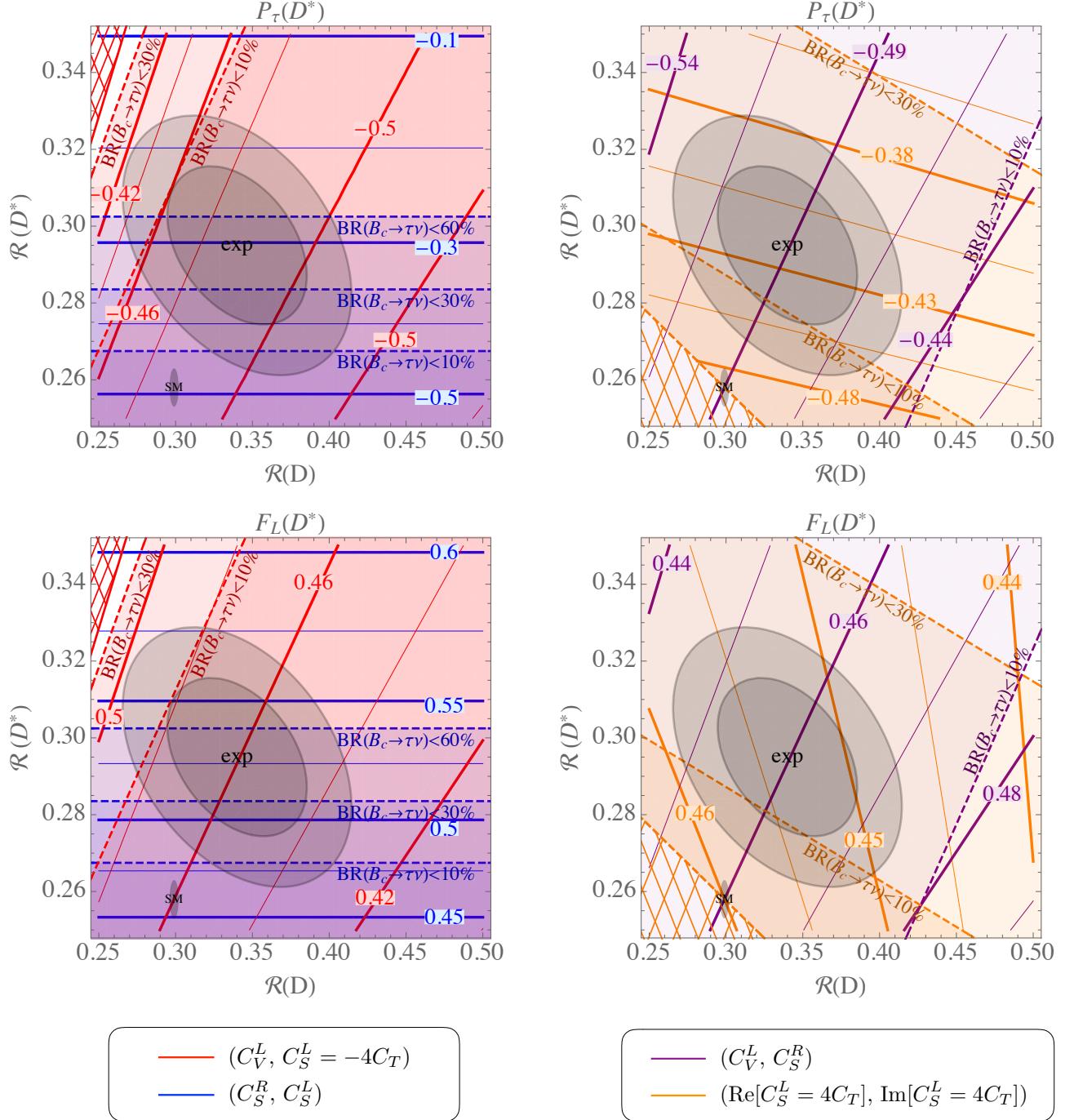


FIG. 6. Contour lines of the  $\tau$  polarization and the longitudinal  $D^*$  polarization for the two-dimensional scenarios in the  $\mathcal{R}(D)$ - $\mathcal{R}(D^*)$  plane, updating Fig. 5 of Ref. [1].

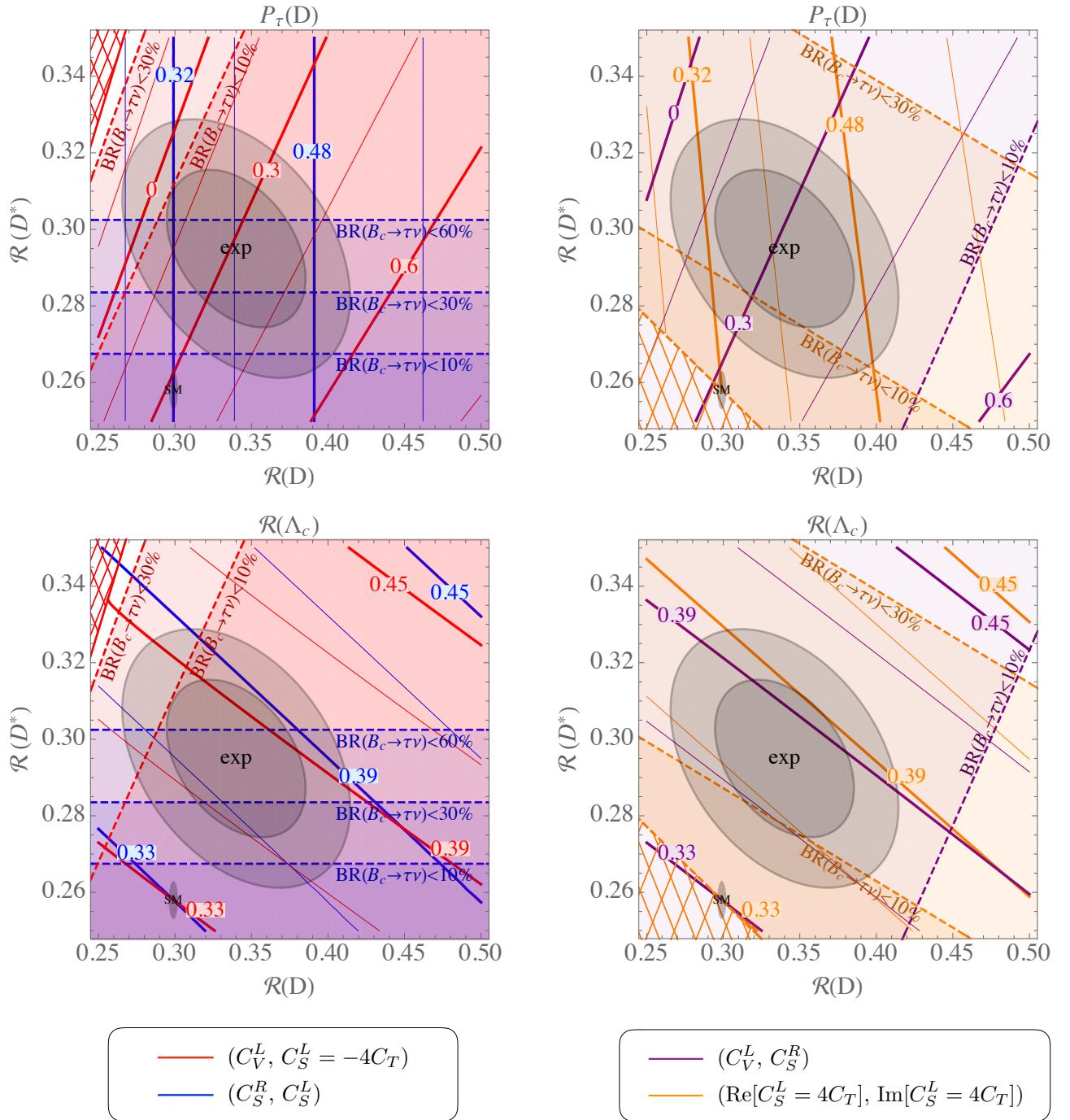


FIG. 7. Contour lines of  $P_\tau(D)$  and  $\mathcal{R}(\Lambda_c)$  for the two-dimensional scenarios in the  $\mathcal{R}(D)$ – $\mathcal{R}(D^*)$  plane, updating Fig. 6 of Ref. [1].