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The width difference in the $B_s - \overline{B}_s$ system: towards NNLO

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The width difference $\Delta\Gamma$ among the two mass eigenstates of the $B_s - \overline{B}_s$ system is measured with a precision of 7%. The theory prediction has a larger uncertainty which mainly stems from unknown perturbative higher-order QCD corrections. I discuss the subset of next-to-next-to-leading order diagrams proportional to $\alpha_s^2 N_f$, where $N_f = 5$ is the number of quark flavours. The results are published in [1].

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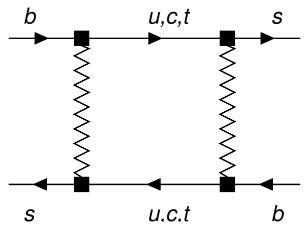

Figure 1: Box diagram describing B_s–$\overline{B}_s$ mixing. A second diagram is obtained by a 90° rotation.

1 B_s–$\overline{B}_s$ mixing and $\Delta\Gamma$

The box diagram of Fig. 1 describes B_s–$\overline{B}_s$ mixing, which is a transition changing the beauty quantum number B by two units. As a consequence of B_s–$\overline{B}_s$ mixing, the flavour eigenstates B_s and $\bar{B}_s$ are not equal to the mass eigenstates B_H and B_L which obey simple exponential decay laws. Denoting masses and decay widths of $B_{H,L}$ by $M_{H,L}$ and $\Gamma_{H,L}$ (with the subscripts denoting "heavy" and "light"), the mixing problem involves five observables:

$$M = \frac{M_L + M_H}{2}, \qquad \Gamma = \frac{\Gamma_L + \Gamma_H}{2}, \qquad \Delta m = M_H - M_L, \qquad \Delta\Gamma = \Gamma_L - \Gamma_H, \tag{1}$$

and the CP asymmetry in flavour-specific decays, a_{fs}, which quantifies CP violation in mixing and is typically measured in semileptonic decays. The mass difference $\Delta m = (17.757 \pm 0.021)\,\text{ps}^{-1}$ [2] has been determined very precisely from the B_s–$\overline{B}_s$ oscillation frequency [3,4]. The experimental value of the width difference [2],

$$\Delta\Gamma^{\text{exp}} = (0.088 \pm 0.006)\,\text{ps}^{-1}, \tag{2}$$

is an average of measurements by LHCb [5,6], ATLAS [7], CMS [8], and CDF [9].

$\Delta\Gamma$ is calculated from the absorptive part of the box diagram in Fig. 1, which is the piece of this diagram involving the imaginary part of the loop integral. Only the contributions with light u and c quarks contribute to $\Delta\Gamma$. In order to include strong-interaction effects one exploits that the bottom mass m_b is much larger than the fundamental scale of QCD, Λ_{QCD}, and employs an operator product expansion, the *heavy quark expansion (HQE)* [10–13]. This procedure results in a systematic expansion of $\Delta\Gamma$ in powers of $\Lambda_{QCD}/m_b \approx 0.1$ and $\alpha_s(m_b) \approx 0.2$. At the energy scale m_b, relevant for B_s decays, W exchange can be described by point-like interactions. The corresponding effective $|\Delta B| = 1$ hamiltonian for the $b \to s$ transitions of our interest reads

$$H_{eff}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{cb} \left\{ \sum_{i=1}^{6} C_i O_i + C_8 O_8 \right\} \;+\; \text{H.c.}, \tag{3}$$

with

$$\begin{aligned} O_1 &= (\bar{s}_i c_j)_{V-A}\,(\bar{c}_j b_i)_{V-A}, \qquad O_2 = (\bar{s}_i c_i)_{V-A}\,(\bar{c}_j b_j)_{V-A}, \\ O_8 &= \frac{g_s}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu}(1-\gamma_5) T^a_{ij} b_j G^a_{\mu\nu}. \end{aligned} \tag{4}$$

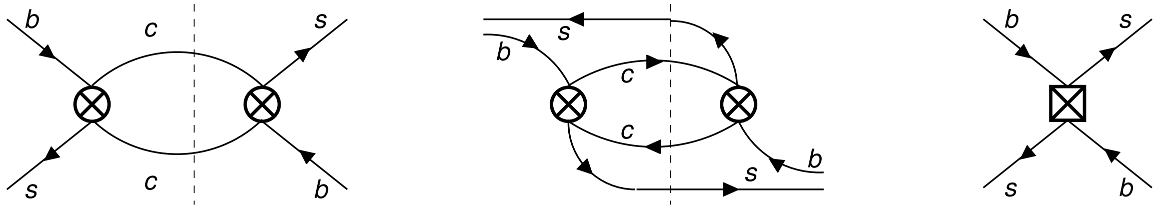

Figure 2: Left and middle: LO diagrams for $\Delta\Gamma$ corresponding to the box diagrams of Fig. 1. The crosses represent the operators O_1 or O_2 from $H_{eff}^{\Delta B=1}$ in Eq. (3). Right: Effective $|\Delta B| = 2$ operator.

The numerically less important four-quark penguin operators Q_{3-6} are not shown. G_F is the Fermi constant, i, j are colour indices, $V \pm A = \gamma_\mu(1 \pm \gamma_5)$, and V_{cs} and V_{cb} are elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Note that the doubly Cabibbo-suppressed contributions with u quarks have been neglected. The chromagnetic operator O_8 encodes a b-s-gluon and a b-s-gluon-gluon coupling. The Wilson coefficients C_j in Eq. (3) comprise the short-distance QCD effects of the energy scale of the W mass and above. They are known to next-to-next-to-leading order (NNLO) of QCD [14, 15]. The leading-order (LO) contribution to $\Delta\Gamma$ in both expansion parameters $\alpha_s(m_b)$ and Λ_{QCD}/m_b is shown in the left two diagrams of Fig. 2. $\Delta\Gamma$ can be understood to come from the interference of all $B_s \to f$ and $\bar{B}_s \to f$ decays, where f is any final state common to B_s and $\bar{B}_s$ decays. The Cabibbo-favoured contribution to $\Delta\Gamma$ stems from $b \to c\bar{c}s$ decays. The final state is indicated by the dashed line in Fig. 2. The results of the left and middle diagrams in Fig. 2 determine the LO coefficients of the effective $|\Delta B| = 2$ operators, depicted at right in Fig. 2. At leading order of Λ_{QCD}/m_b ("leading power") one needs two such operators:

$$Q = (\bar{s}_i b_i)_{V-A}\,(\bar{s}_j b_j)_{V-A}, \qquad \tilde{Q}_S = (\bar{s}_i b_j)_{S-P}\,(\bar{s}_j b_i)_{S-P}.$$

Here $S - P = 1 - \gamma_5$. Higher-order QCD corrections are calculated from diagrams involving gluons added to the diagrams of Fig. 2, penguin diagrams, and diagrams involving Q_8. Finally, non-perturbative QCD effects are contained in the $|\Delta B| = 2$ matrix elements:

$$\begin{aligned} \langle B_s|Q(\mu_2)|\overline{B}_s\rangle &= \frac{8}{3} M_{B_s}^2\, f_{B_s}^2 B(\mu_2) \\ \langle B_s|\widetilde{Q}_S(\mu_2)|\overline{B}_s\rangle &= \frac{1}{3} M_{B_s}^2\, f_{B_s}^2 \widetilde{B}'_S(\mu_2). \end{aligned} \tag{5}$$

Here M_{B_s} and f_{B_s} are mass and decay constant of the B_s meson, respectively, and $\mu_2 = \mathcal{O}(m_b)$ is the renormalisation scale at which the matrix elements are calculated. The dimensionless quantities $B(\mu_2)$ and $\widetilde{B}'_S(\mu_2)$ parametrise the matrix elements. The leading-power result can be written as

$$\Delta\Gamma = \frac{G_F^2 m_b^2}{12\pi\, M_{B_s}} |V_{cs}^* V_{cb}|^2 \left| G' \langle B_s|Q|\bar{B}_s\rangle \;+\; \widetilde{G}_S \langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle \right| \tag{6}$$

with perturbative coefficients G',$\widetilde{G}_S$. These coefficients are bilinear in the C_j's of $H_{eff}^{\Delta B=1}$ and are known to next-to-leading order (NLO) in $\alpha_s(m_b)$ [17–20]. The Wilson coefficients C_j depend on an unphysical renormalisation scale $\mu_1 = \mathcal{O}(m_b)$. The dependence of G',$\widetilde{G}_S$ on μ_1 diminishes order-by-order in α_s and serves as an estimate of the accuracy of the perturbative calculation. Also the dependence on the chosen renormalisation scheme decreases with higher orders of α_s. For instance, we can trade the pole mass m_b in Eq. (6) for the $\overline{\rm MS}$ mass $\bar{m}_b$ and replace e.g. $\widetilde{G}_S$ by

$$\widetilde{G}_S^{\overline{\rm MS}} \equiv \frac{m_b^{\rm pole\,2}}{\bar{m}_b^2} \widetilde{G}_S,$$

expanded in α_s to the order in which $\Delta\Gamma$ is calculated. Corrections to $\Delta\Gamma$ of order Λ_{QCD}/m_b involve additional operators and have been calculated in Ref. [16].

Including all known corrections one has

$$\begin{aligned} \Delta\Gamma &= \Big(0.091 \pm 0.020_{\rm scale} \pm 0.006_{B,\widetilde{B}_S} \pm 0.017_{\Lambda_{QCD}/m_b}\Big)\ \text{GeV} \qquad \text{(pole)} \\ \Delta\Gamma &= \Big(0.104 \pm 0.008_{\rm scale} \pm 0.007_{B,\widetilde{B}_S} \pm 0.015_{\Lambda_{QCD}/m_b}\Big)\ \text{GeV} \qquad (\overline{\rm MS}) \end{aligned} \tag{7}$$

These numbers are found from the expressions in Ref. [20] with present-day lattice-QCD results for the matrix elements in Eq. (5) taken from Ref. [21]. The uncertainties from different sources are indicated in Eq. (7). The size of the missing α_s^2 corrections to the diagrams in Fig. 2 can be estimated from the μ_1-dependence, denoted with "scale", or from the difference between the central values in the two schemes. This perturbative error is larger than the uncertainty stemming from the lattice-QCD calculation denoted with "$B, \widetilde{B}_S$" and also exceeds the experimental error in Eq. (2). Also the last error related to the power corrections originates mostly from the unknown NLO corrections to coefficients of the subleading-power operators. The matrix elements of these subleading operators have been estimated with QCD sum rules [22] and lattice-QCD calculations are making progress [23]. Thus perturbative uncertainties are dominant and call for the calculation of the NNLO corrections to the leading power contribution. Also NLO corrections to the Λ_{QCD}/m_b piece are needed. The phenomenology of $\Delta\Gamma$ within and beyond the Standard Model is dicussed in Refs. [20, 24, 25].

2 Towards NNLO

The NNLO calculation involves three-loop diagrams with loop integrals depending on one external momentum p with $p^2 = m_b^2$, i.e. these are propagator-type integrals. m_b and the charm mass m_c appear on internal lines. The calculation in Ref. [1] has addressed the subset of diagrams with a closed quark loop, shown in Fig. 3. These diagrams are a gauge-invariant subset of all NNLO diagrams and grow with the number N_f of active quark flavours. (In b decays one has $N_f = 5$.) Note that diagrams involving O_8 have less than three loops, because the definition of O_8 in Eq. (4) involves one power of the strong coupling g_s. We have also calculated the contributions with penguin operators O_{3-6} [1], counting their small Wilson coefficients as $\mathcal{O}(\alpha_s)$ [17], so that also here only one-loop and two-loop diagrams are needed.

In the calculation one can neglect the charm mass on the lines attached to a $O_{1,2}$ vertex, because the associated error is of order m_c^2/m_b^2, i.e. 5% of the expected $\mathcal{O}(15\%)$ NNLO correction. A charm quark running in the closed quark loop in the gluon propagator, however, leads to a term linear in m_c/m_b, so that we have kept a non-zero charm mass there.

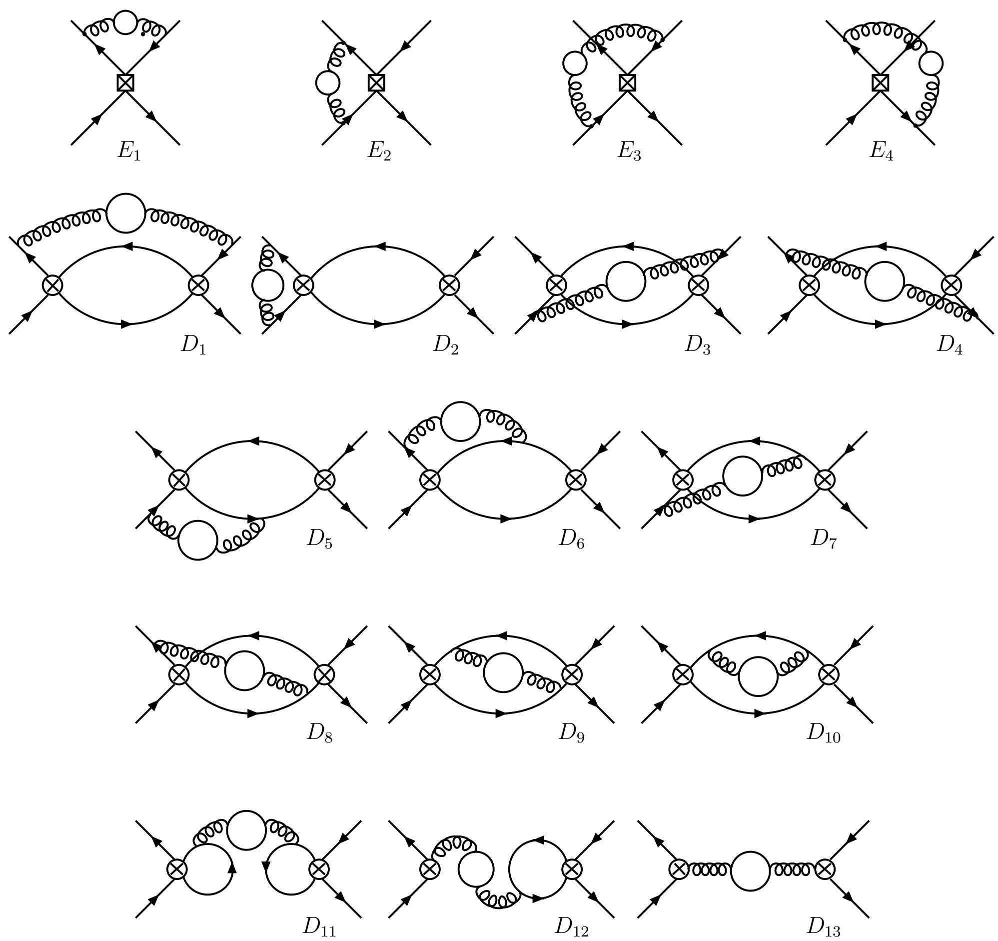

Figure 3: Diagrams with $O_{1,2}$ or O_8 contributing to the $\mathcal{O}(\alpha_s^2 N_f)$ corrections to $\Delta\Gamma$.

For illustration we show the charm-loop contribution to the coefficient multiplying C_2^2 in the NNLO correction to $\widetilde{G}_S$:

$$F_{S,22}^{(2),N_V}(z) = -9.01785 \log\frac{\mu_1}{m_b} - 11.8519 \log\frac{\mu_2}{m_b} - 14.2222 \log\frac{\mu_1}{m_b}\log\frac{\mu_2}{m_b} + 10.6667 \log^2\frac{\mu_1}{m_b} + 7.11111 \log^2\frac{\mu_2}{m_b} - 42.0084 + 105.276\frac{m_c}{m_b} + \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right)$$

Displaying only the error from the μ_1 dependence, our NLO and large-N_f NNLO results read

$$\begin{aligned}\Delta\Gamma^{NLO} &= (0.091 \pm 0.020_{\rm scale})\ \text{GeV} \qquad \text{(pole)}\\ \Delta\Gamma^{NLO} &= (0.104 \pm 0.015_{\rm scale})\ \text{GeV} \qquad (\overline{\rm MS})\end{aligned} \tag{8}$$

$$\begin{aligned}\Delta\Gamma^{NNLO} &= (0.108 \pm 0.021_{\rm scale})\ \text{GeV} \qquad \text{(pole)}\\ \Delta\Gamma^{NNLO} &= (0.103 \pm 0.015_{\rm scale})\ \text{GeV} \qquad (\overline{\rm MS})\end{aligned} \tag{9}$$

Note that we have used a different implementation of the $\overline{\rm MS}$ scheme here: In Eq. (7) the prefactor in Eq. (6) is chosen with $\bar{m}_b^2(\mu_1)$ and the μ_1 dependence of this factor nicely cancels with the one in G' and $\widetilde{G}_S$, and this feature seems to be accidental. If one chooses $\bar{m}_b^2(\bar{m}_b)$ instead (with properly adjusted μ_1 terms in G' and $\widetilde{G}_S$), one finds the larger μ_1 dependence of Eq. (8). Our partial NNLO correction is sizable in the pole scheme and lifts the result closer to the $\overline{\rm MS}$ result. In the $\overline{\rm MS}$ scheme instead our large-N_f correction is very small and unlikely to be the dominant piece of the full NNLO result.

The large N_f limit of QCD spoils asymptotic freedom, because the β function changes sign for sufficiently large values of N_f. One may remedy this by "naive non-Abelianisation (NNA)", which means to trade N_f for the leading coefficient $\beta_0 = 11 - 2/3 N_f$ of the QCD β function [26, 27]. This procedure flips the sign of the NNLO correction leading to

$$\begin{aligned}\Delta\Gamma^{NNA} &= (0.071 \pm 0.020_{\rm scale})\ \text{GeV} \qquad \text{(pole)}\\ \Delta\Gamma^{NNA} &= (0.099 \pm 0.012_{\rm scale})\ \text{GeV} \qquad (\overline{\rm MS}).\end{aligned} \tag{10}$$

In applications like ours, in which the size of the NNLO correction depends on the chosen renormalisation scheme for the Wilson coefficients, it is not clear whether NNA improves the result. E.g. in Ref. [28] it has been found that the $\alpha_s^2\beta_0$ term is not a good approximation to the full NNLO result to the calculated quantity.

3 Conclusions

The calculation of the $\alpha_s^2 N_f$ terms of the NNLO correction to $\Delta\Gamma$ has reduced the renormalisation scheme dependence of the theory prediction and has moved the pole scheme result close to the $\overline{\rm MS}$ result. But there is no progress in the reduction of the dependence on the renormalisation scale and the correction found in the $\overline{\rm MS}$ scheme is too small to be the dominant part of the full NNLO result. Therefore a complete NNLO calculation is needed. In the meantime, we advocate for the use of the $\overline{\rm MS}$ result with a conservative perturbative error [1]:

$$\Delta\Gamma = \left(0.104 \pm 0.015_{\rm scale} \pm 0.007_{B,\widetilde{B}_S} \pm 0.015_{\Lambda_{QCD}/m_b}\right)\ \text{GeV} \qquad (\overline{\rm MS}). \tag{11}$$

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