Massless four-loop form factors *

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In this contribution we briefly discuss the status of the calculation of the four-loop massless form factors.

1. Introduction

In QCD, form factors are building blocks for many physical quantities, most prominently for Higgs boson production and the Drell Yan process. Furthermore, they are the simplest infra-red divergent objects and are thus used to study general properties of QCD and to develop a deeper insight into the infra-red properties.

In this contribution we consider the massless photon quark vertex, Γ_q^μ and QCD corrections up to four loops to the corresponding form factor which is obtained via

$$F_q(q^2) \;=\; -\frac{1}{4(1-\epsilon)q^2}\mathrm{Tr}\left(\not{p}_2\,\Gamma_q^\mu \not{p}_1\,\gamma_\mu\right)\,. \tag{1}$$

In this formula, $D = 4 - 2\epsilon$ is the space-time dimension, p_1 and p_2 are the incoming momenta of the quark and anti-quark, respectively, and $q = p_1+p_2$ is the photon momentum.

Three-loop corrections to $F_q(q^2)$ have been computed for the first time in Ref. [1] and afterwards confirmed in [2]. In the final expression presented in these references the highest ϵ coefficients of the three most complicated master integrals were only known numerically. Their analytic expressions have been provided in [3].

First results at four-loop order have been obtained in Refs. [4, 5] where all planar contributions to F_q have been computed. This provides a complete result in the large-N_c limit. Note that the corrections with three closed quark loops have been computed in Ref. [6]. Let us also mention the work [7]

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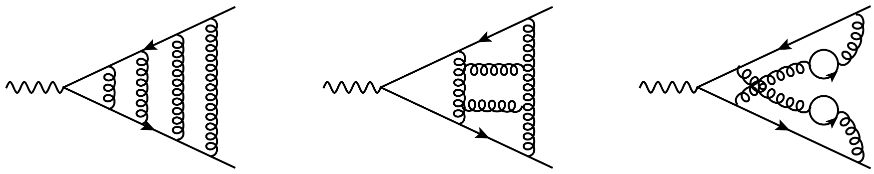

Fig. 1. Sample Feynman diagrams. For the n_f^2 contribution also non-planar contributions are considered.

where the $1/\epsilon^2$ pole of the four-loop form factor within $\mathcal{N} = 4$ super Yang-Mills theory has been computed using numerical methods.

In this contribution we concentrate on the fermionic terms to F_q with two closed fermion lines. This part receives planar and non-planar contributions which have been computed in Ref. [8].

In the next Section we briefly comment on the individual steps of the calculation and present analytic results in Section 3.

2. Calculation

Sample Feynman diagrams which contribute to the massless four-loop form factor are shown in Fig. 1. The first two diagrams represent purely gluonic planar contributions, which have been computed in Refs. [4, 5] and the last one represents one of the most complicated non-planar diagrams with two (massless) fermion insertions. A crucial step in the course to compute these diagrams is the identification of a (scalar) integral family for each Feynman amplitude. For the planar contributions there are 38 families which have been classified in Refs. [4, 5]. In general the non-planar families are more complicated and have not yet been identified by our group. However, in the case of the n_f^2 terms there are only two non-planar families which are shown in Fig. 2. Note that for the n_f^2 amplitudes many lines of the families in Fig. 2 are absent. However, in Ref. [8] all master integrals which belong to the two families have been computed. It is expected that all of them are needed for the n_f-independent four-loop form factor.

Once the integral families are classified the input files for `FIRE` [9, 10] and `LiteRed` [11, 12] can be generated which perform the reduction to master integrals. Then the main task is the analytic evaluation of the latter. In the following we briefly summarize the basic ideas which enter this part of the calculation.

- Differential equations are a powerful tool to compute master integrals [13, 14, 15, 16]. Since we have a one-scale problem at hand and thus no dimensionless ratio can be formed, a further mass scale has

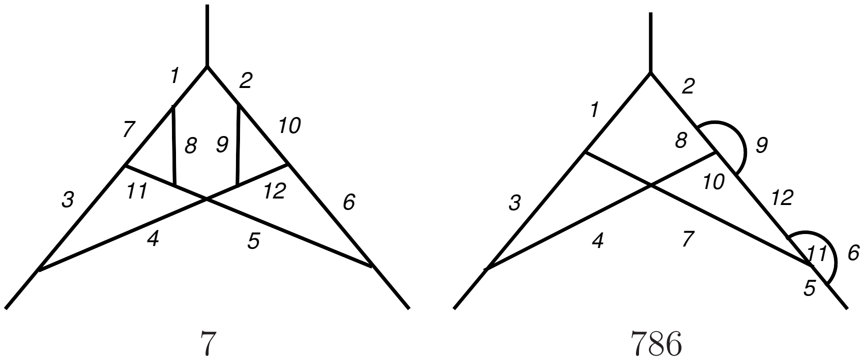

Fig. 2. Non-planar families needed for the n_f^2 contribution.

to be introduced in order to be able to use the differential equation method. One possible choice is $p_2^2 = xq^2 \neq 0$. Note that for $x = 1$ the integrals become massless two-point integrals which are available in the literature [17, 18, 19]. We will use this point for fixing the initial conditions. For the form factor we need the master integrals for $x = 0$.

A non-vanishing q_2^2 increases the number of master integrals. Furthermore, a more complicated reduction problem (with two dimensionful scales) has to be solved.

- A further crucial ingredient is the choice of basis. It has been suggested in [16] (see also Ref. [20]) to choose a so-called canonical basis where the differential equation for the master integrals has the form

$$g'(x, \epsilon) = \epsilon \, A(x) \cdot g(x, \epsilon) \,, \tag{2}$$

where the matrix $A(x)$ is ϵ-independent and only has simple poles of the form $1/(x - x_i)$. In our case we only have two such terms with $x_0 = 0$ and $x_1 = 1$.

We transform our primary basis (chosen by `FIRE`) to a canonical basis with the help of the algorithm presented in Ref. [21].

- It is simple to obtain the general solution of Eq. (2) in terms of Harmonic polylogarithms (HPL) [22, 23]. Afterwards we fix the boundary conditions for $x = 1$ using the analytic results from [18] and obtain analytic results for the (two-scale) master integrals. Next, the limit $x \to 0$ has to be considered. This is more complicated than one might expect since this limit has to be taken in the "naive" sense. This means that terms like $x^{k\epsilon}$ should be kept unexpanded and then terms with $k = 0$ have to be selected. Thus, simply expanding the analytic result does not work. Instead, we go back to the differential equation in Eq. (2). For $x \to 0$ it takes the form

$$g'(x, \epsilon) = \epsilon \, \frac{a}{x} \cdot g(x, \epsilon) \tag{3}$$

where all matrix elements of a are just numbers. The solution of this equation reads

$$g_{x\to 0} = x^{\epsilon a} h(\epsilon) \,, \tag{4}$$

where $x^{\epsilon a}$ is a matrix where each element is a linear combination of $x^{k\epsilon}$ with $k \leq 0$. The column vector $h(\epsilon)$ is obtained by comparing the original analytic results and $g_{x\to 0}$ after a Taylor expansion for $x \to 0$.

- The sought-after master integrals in the primary basis are obtained by substituting $g_{x\to 0}|_{x^{0\epsilon}}$ and taking the limit $x \to 0$.

3. Some selected results

After inserting the analytic results for the master integrals into the amplitude we expand in ϵ and obtain results for the form factor up to the finite term. From the $1/\epsilon^2$ pole it is possible to extract four-loop results for the cusp anomalous dimension

$$\gamma_{\rm cusp} \;=\; \sum_{n\geq 0} \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^n \gamma^n_{\rm cusp} \,, \tag{5}$$

which reads

$$\begin{aligned}
\gamma^3_{\rm cusp} \;=\; & \left(\frac{128\pi^2\zeta_3}{9} + 224\zeta_5 - \frac{44\pi^4}{27} - \frac{16252\zeta_3}{27} + \frac{13346\pi^2}{243} - \frac{39883}{81}\right) N_c^2 n_f \\
& + \Bigg(- 32\zeta_3^2 - \frac{176\pi^2\zeta_3}{9} + \frac{20992\zeta_3}{27} - 352\zeta_5 - \frac{292\pi^6}{315} + \frac{902\pi^4}{45} \\
& - \frac{44416\pi^2}{243} + \frac{84278}{81}\Bigg) N_c^3 + n_f^2 \Bigg[C_A \left(\frac{2240\zeta_3}{27} - \frac{56\pi^4}{135} - \frac{304\pi^2}{243} \right.\\
& \left. + \frac{923}{81}\right) + C_F \left(-\frac{640\zeta_3}{9} + \frac{16\pi^4}{45} + \frac{2392}{81}\right)\Bigg] + \left(\frac{64\zeta_3}{27} - \frac{32}{81}\right) n_f^3 \,.
\end{aligned} \tag{6}$$

Note that in this expression the n_f^3 and n_f^2 terms are complete and contain the colour factors C_A and C_F. On the other hand, for the linear n_f and the

n_f-independent terms only the leading N_c terms are shown. The n_f^3 terms have already been computed in Refs. [24, 25] and all other contributions in Eq. (6) have been obtained independently in Refs. [26, 27]. Numerical results for all other colour coefficients can be found in Ref. [27].

Analytic results for the collinear anomalous dimension and the finite part of the form factor can be found in Refs. [4, 5, 8].

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REFERENCES

[1] P. A. Baikov, K. G. Chetyrkin, A. V. Smirnov, V. A. Smirnov and M. Steinhauser, Phys. Rev. Lett. **102** (2009) 212002 [arXiv:0902.3519 [hep-ph]].

[2] T. Gehrmann, E. W. N. Glover, T. Huber, N. Ikizlerli and C. Studerus, JHEP **1006** (2010) 094 [arXiv:1004.3653 [hep-ph]].

[3] R. N. Lee and V. A. Smirnov, JHEP **1102** (2011) 102 [arXiv:1010.1334 [hep-ph]].

[4] J. M. Henn, A. V. Smirnov, V. A. Smirnov and M. Steinhauser, JHEP **1605** (2016) 066 [arXiv:1604.03126 [hep-ph]].

[5] J. Henn, A. V. Smirnov, V. A. Smirnov, M. Steinhauser and R. N. Lee, arXiv:1612.04389 [hep-ph].

[6] A. von Manteuffel and R. M. Schabinger, Phys. Rev. D **95** (2017) no.3, 034030 [arXiv:1611.00795 [hep-ph]].

[7] R. H. Boels, T. Huber and G. Yang, arXiv:1705.03444 [hep-th].

[8] R. N. Lee, A. V. Smirnov, V. A. Smirnov and M. Steinhauser, Phys. Rev. D **96** (2017) no.1, 014008 [arXiv:1705.06862 [hep-ph]].

[9] A. V. Smirnov and V. A. Smirnov, Comput. Phys. Commun. **184** (2013) 2820 [arXiv:1302.5885 [hep-ph]].

[10] A. V. Smirnov, Comput. Phys. Commun. **189** (2015) 182 [arXiv:1408.2372 [hep-ph]].

[11] R. N. Lee, arXiv:1212.2685 [hep-ph].

[12] R. N. Lee, J. Phys. Conf. Ser. **523** (2014) 012059 [arXiv:1310.1145 [hep-ph]].

[13] A. V. Kotikov, Phys. Lett. B **254** (1991) 158.

[14] E. Remiddi, Nuovo Cim. A **110** (1997) 1435 [hep-th/9711188].

[15] T. Gehrmann and E. Remiddi, Nucl. Phys. B **580** (2000) 485 [hep-ph/9912329].

[16] J. M. Henn, Phys. Rev. Lett. **110** (2013) 251601 [arXiv:1304.1806 [hep-th]].

[17] P. A. Baikov and K. G. Chetyrkin, Nucl. Phys. B **837** (2010) 186 [arXiv:1004.1153 [hep-ph]].

[18] A. V. Smirnov and M. Tentyukov, Nucl. Phys. B **837** (2010) 40 [arXiv:1004.1149 [hep-ph]].

[19] R. N. Lee, A. V. Smirnov and V. A. Smirnov, Nucl. Phys. B **856** (2012) 95 [arXiv:1108.0732 [hep-th]].

[20] J. M. Henn, J. Phys. A **48** (2015) 153001 [arXiv:1412.2296 [hep-ph]].

[21] R. N. Lee, JHEP **1504** (2015) 108 [arXiv:1411.0911 [hep-ph]].

[22] E. Remiddi and J. A. M. Vermaseren, Int. J. Mod. Phys. A **15** (2000) 725 [hep-ph/9905237].

[23] D. Maitre, Comput. Phys. Commun. **174** (2006) 222 [arXiv:hep-ph/0507152].

[24] J. A. Gracey, Phys. Lett. B **322** (1994) 141 [hep-ph/9401214].

[25] M. Beneke and V. M. Braun, Nucl. Phys. B **454** (1995) 253 [hep-ph/9506452].

[26] J. Davies, A. Vogt, B. Ruijl, T. Ueda and J. A. M. Vermaseren, Nucl. Phys. B **915** (2017) 335 [arXiv:1610.07477 [hep-ph]].

[27] S. Moch, B. Ruijl, T. Ueda, J. A. M. Vermaseren and A. Vogt, JHEP **1710** (2017) 041 [arXiv:1707.08315 [hep-ph]].