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Neutral $D \to KK^*$ decays as discovery channels for charm CP violation

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We point out that the CP asymmetries in the decays $D^0 \to K_S K^{*0}$ and $D^0 \to K_S \overline{K}^{*0}$ are potential discovery channels for charm CP violation in the Standard Model. We stress that no flavor tagging is necessary, the untagged CP asymmetry $a_{CP}^{\rm dir}(\overset{(-)}{D} \to K_S K^{*0})$ is essentially equal to the tagged one, so that the untagged measurement comes with a significant statistical gain. Depending on the relevant strong phase, $|a_{CP}^{\rm dir,\ untag}|$ can be as large as 0.003. The CP asymmetry is dominantly generated by exchange diagrams and does not require non-vanishing penguin amplitudes. While the CP asymmetry is smaller than in the case of $D^0 \to K_S K_S$, the experimental analysis is more efficient due to the prompt decay $K^{*0} \to K^+\pi^-$. One may further search for favourable strong phases in the Dalitz plot in the vicinity of the K^{*0} peak.

I. INTRODUCTION

Charm CP violation has not been discovered yet. Within the Standard Model (SM) all CP asymmetries involve the combination $\lambda_b \equiv V_{cb}^* V_{ub}$ of elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The smallness of $|\lambda_b|$ had nurtured the hope that new physics would manifest itself in orders-of-magnitude enhancements of CP asymmetries. However, this scenario is seemingly not realized in nature, so that the scientific goals to discover charm CP violation and to establish new physics involve distinct strategies. In this paper we address the first topic and discuss how charm CP violation can be discovered best, assuming that there is only the SM contributions governed by λ_b.

Singly Cabibbo-suppressed (SCS) decay amplitudes of D mesons involve the CKM elements $\lambda_q \equiv V_{cq}^* V_{uq}$ with $q = d, s$ or b. Using $\lambda_d + \lambda_s + \lambda_b = 0$ one may express the amplitude of some decay d as

$$\mathcal{A}(d) \equiv \lambda_{sd}\mathcal{A}_{sd}(d) - \frac{\lambda_b}{2}\mathcal{A}_b(d)\,, \tag{1}$$

with $\lambda_{sd} = (\lambda_s - \lambda_d)/2$. Branching ratios are completely dominated by the first term $\lambda_{sd}\mathcal{A}_{sd}(d)$. The direct CP asymmetry reads

$$a_{CP}^{\rm dir}(d) \equiv \frac{|\mathcal{A}(d)|^2 - |\overline{\mathcal{A}}(d)|^2}{|\mathcal{A}(d)|^2 + |\overline{\mathcal{A}}(d)|^2} \tag{2}$$

$$= \mathrm{Im}\,\frac{\lambda_b}{\lambda_{sd}}\mathrm{Im}\,\frac{\mathcal{A}_b(d)}{\mathcal{A}_{sd}(d)}. \tag{3}$$

$\mathcal{A}_{sd}(d)$ and $\mathcal{A}_b(d)$ can be written as the sum of different topological amplitudes; in the limit of exact flavor-SU(3) symmetry these are the *tree (T)*, *color-suppressed tree (C)*, *exchange (E)*, *annihilation (A)*, *penguin (P_q)*, and *penguin annihilation (PA_q)* amplitudes. The latter two topologies involve a loop with the indicated internal quark $q = d, s, b$. In essentially all commonly studied decays (including the popular modes $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$) $\mathcal{A}_b(d)/\mathcal{A}_{sd}(d)$ is proportional to $P \equiv P_s + P_d - 2P_b$.

Now

$$\mathrm{Im}\,\frac{\lambda_b}{\lambda_{sd}} = -6\cdot 10^{-4} \tag{4}$$

defines the typical size of $|a_{CP}^{\rm dir}(d)|$. In Ref. [1] we have found that $|a_{CP}^{\rm dir}(D^0 \to K_S K_S)|$ can be as large as $1.1 \cdot 10^{-2}$ and proposed $D^0 \to K_S K_S$ as a discovery channel for charm CP violation. Experiments start to probe this region [2–4]. The reason for this enhancement compared to the expectation in Eq. (4) is two-fold:

(i) $|\mathcal{A}_{sd}(D^0 \to K_S K_S)|$ is suppressed, because it vanishes in the $\mathrm{SU}(3)_F$ symmetry limit, see also Refs. [5–7].

(ii) $|\mathcal{A}_b(D^0 \to K_S K_S)|$ is enhanced, because it involves the large topological amplitude E. Contrary to P, this amplitude involves no loop (see Fig. 1) and a global fit to measured branching ratios supports a large value of $|E|$ [8], comparable to $|T|$. This feature is easily understood, because the color suppression of E is offset by a large Wilson coefficient $2C_2 \sim 2.4$ [9].

In this paper we extend the analysis of Ref. [1] to the decays $D^0 \to \overline{K}^0 K^{*0}$ and $D^0 \to K^0 \overline{K}^{*0}$. The $K^{*0} = K^{*0}(892)$ is understood to be observed as $K^{*0} \to K^+\pi^-$, *i.e.* in a flavor-specific decay distinguishing K^{*0} from $\overline{K}^{*0}$ decaying as $\overline{K}^{*0} \to K^-\pi^+$. For the corresponding amplitudes we write

$$\mathcal{A}(\overline{K}^{*0}) \equiv \mathcal{A}(D^0 \to \overline{K}^{*0} K^0) \tag{5}$$

$$\mathcal{A}(K^{*0}) \equiv \mathcal{A}(D^0 \to K^{*0} \overline{K}^0). \tag{6}$$

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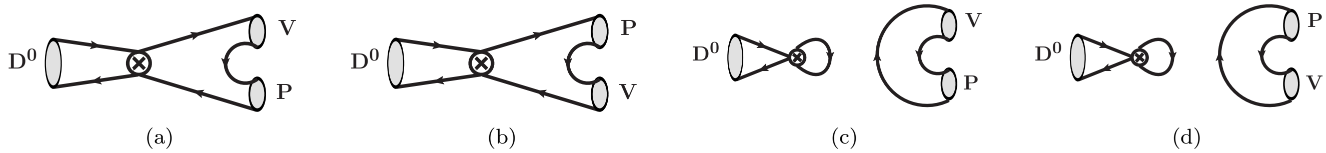

FIG. 1. $SU(3)_F$-limit topological amplitudes E_P (a), E_V (b), PA_{Pq} (c), and PA_{Vq} (d) entering $D^0 \to \overline{K}^0 K^{*0}$ and $D^0 \to K^0 \overline{K}^{*0}$. "$V$" and "$P$" stand for "vector" and "pseudoscalar", respectively, and label the two different positions of $\overline{K}^0$ and K^{*0} in the diagrams. The q in PA_{Pq} and PA_{Vq} labels the quark running in the loop at the weak vertex. We define $PA_P \equiv PA_{Ps} + PA_{Pd} - 2PA_{Pb}$ and analogous for PA_V. Note that the contributions from PA_P and PA_V cannot be distinguished from each other. We use therefore the notation $PA_{PV} \equiv PA_P + PA_V$.

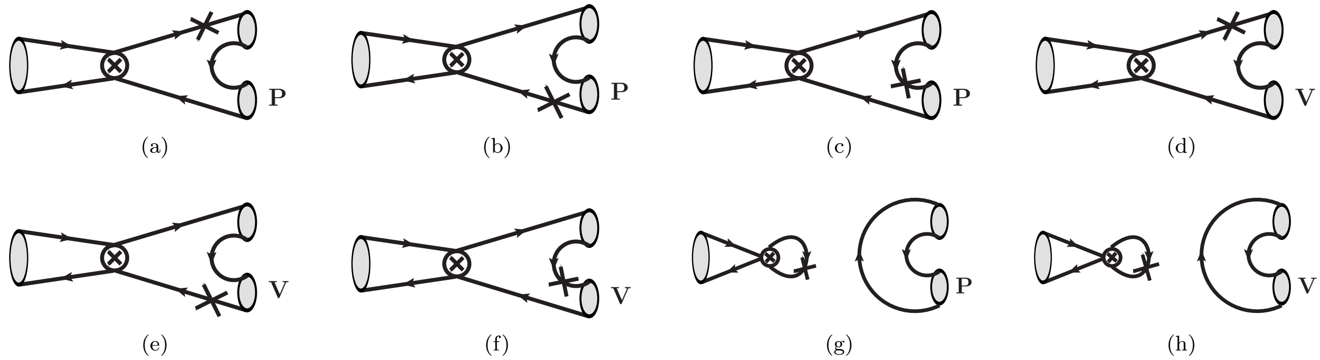

FIG. 2. $SU(3)_F$-breaking topological amplitudes E_{P1} (a), E_{P2} (b), E_{P3} (c), E_{V1} (d) , E_{V2} (e), E_{V3} (f), $PA_P^{\rm break} \equiv PA_{Ps} - PA_{Pd}$ (g) and $PA_V^{\rm break} \equiv PA_{Vs} - PA_{Vd}$ (h) contributing to $D^0 \to \overline{K}^0 K^{*0}$ and $D^0 \to K^0 \overline{K}^{*0}$. Note that the contributions from $PA_P^{\rm break}$ and $PA_V^{\rm break}$ cannot be distinguished from each other. We use therefore the notation $PA_{PV}^{\rm break} \equiv PA_P^{\rm break} + PA_V^{\rm break}$.

At present, these modes are compatible with CP conservation [10], however with large errors. The modes $D^0 \to \overline{K}^0 K^{*0}, K^0 \overline{K}^{*0}$ share the properties (i) and (ii) with $D^0 \to K_S K_S$, except that the suppression of $|\mathcal{A}_{sd}|$ cannot be inferred from symmetry arguments. Instead, the smallness of $|\mathcal{A}_{sd}|$ is only found empirically, from the branching ratios that we extract from the literature [10, 11] as

$$\mathcal{B}^{\rm exp}(D^0 \to K^{*0} K_S) = (1.1 \pm 0.2) \cdot 10^{-4}\,, \tag{7}$$

$$\mathcal{B}^{\rm exp}(D^0 \to \overline{K}^{*0} K_S) = (0.9 \pm 0.2) \cdot 10^{-4}\,. \tag{8}$$

Note that to the given precision, Eqs. (7), (8) do not depend on the choice of the GLASS or LASS scheme in Ref. [10]. GLASS and LASS are two models for the $K\pi$ S-wave contributions, see Ref. [10] for details. The topological amplitudes contributing to these decays are shown in Fig. 1. Eqs. (7) and (8) entail $E_V \sim E_P$ for the two exchange amplitudes, while global fits to the branching ratios of D decays into a pseudoscalar and a vector meson show that $|E_V|$ and $|E_P|$ are individually large, with ratios of exchange over tree diagrams between 0.2 and 0.5 [12–14]. For a dedicated discussion of the rates and phases of $D \to KK^*$ as well as comparisons to BaBar [15] and Belle [16] Dalitz plot data see Refs. [13, 17]. In addition to (i) and (ii) there are more features making $D^0 \to \overline{K}^0 K^{*0}, K^0 \overline{K}^{*0}$ interesting for the hunt for charm CP violation:

(iii) The prompt decay $K^{*0} \to K^+ \pi^-$ produces charged tracks pointing directly to the D^0 decay vertex and the problem with the sizable K_S lifetime in $D^0 \to K_S K_S$ is alleviated. Unlike the phase-space suppressed decay $D^0 \to \overline{K}^{*0} K^{*0}$ the proposed modes require no angular analysis.

(iv) Direct CP asymmetries vanish if $\mathcal{A}_b/\mathcal{A}_{sd}$ is real, *i.e.* if the relative strong phase of the interfering amplitudes equals zero or π. Thus to discover CP violation one must be lucky with the uncalculable strong phases. However, in the analysis of the (K^+, π^-, K_S) Dalitz plot one can relax the requirement $M(K^+, \pi^-) = M_{K^{0*}} = 892\,\text{MeV}$ and scan over invariant masses $M(K^+, \pi^-)$ in the vicinity of the K^{0*} mass, exploiting that strong phases strongly vary in the vicinity of resonances.

(v) The CP asymmetry does not vanish in the untagged D^0 decay, *i.e.* the decay rates of $\overset{(-)}{D} \to \overline{K}^0 K^{*0}$ and $\overset{(-)}{D} \to K^0 \overline{K}^{*0}$, differ from each other. Thus no flavor tagging is needed.

We define the untagged rates $\Gamma(\overset{(-)}{D} \to f) \equiv \Gamma(D^0 \to$

$f) + \Gamma(\overline{D}^0 \to f)$ and obtain the direct CP asymmetry of the untagged D^0 decay as

$$a_{CP}^{\text{dir,untag}}(K^{*0}) \equiv \frac{\Gamma(\overset{(-)}{D} \to \overline{K}^0 K^{*0}) - \Gamma(\overset{(-)}{D} \to K^0 \overline{K}^{*0})}{\Gamma(\overset{(-)}{D} \to \overline{K}^0 K^{*0}) + \Gamma(\overset{(-)}{D} \to K^0 \overline{K}^{*0})} \tag{9}$$

$$= \mathrm{Im}\frac{\lambda_b}{\lambda_{sd}} \frac{\mathrm{Im}\left(\mathcal{A}_{sd}^*(K^{*0})\mathcal{A}_b(K^{*0}) - \mathcal{A}_{sd}^*(\overline{K}^{*0})\mathcal{A}_b(\overline{K}^{*0})\right)}{|\mathcal{A}_{sd}(K^{*0})|^2 + |\mathcal{A}_{sd}(\overline{K}^{*0})|^2} \tag{10}$$

$$= -a_{CP}^{\text{dir,untag}}(\overline{K}^{*0}) \,. \tag{11}$$

Below, we give the topological decompositions of $\mathcal{A}(K^{*0})$ and $\mathcal{A}(\overline{K}^{*0})$, respectively. Subsequently, we insert these into the expressions for the CP asymmetries. We analyze the phenomenological implications of the results and conclude.

II. TOPOLOGICAL DECOMPOSITION

Similar in this respect to $\mathcal{A}(D^0 \to K_S K_S)$, the topological decompositions of $\mathcal{A}(\overline{K}^{*0})$ and $\mathcal{A}(K^{*0})$, see Eqs. (5) and (6), depend on exchange and penguin annihilation topologies only:

$$\begin{aligned}
\mathcal{A}_{sd}(K^{*0}) &= E_P - E_V \\
&\quad + E_{P3} - E_{V1} - E_{V2} - PA_{PV}^{\text{break}} \,, && (12) \\
\mathcal{A}_b(K^{*0}) &= -E_P - E_V \\
&\quad - E_{P3} - E_{V1} - E_{V2} - PA_{PV} && (13) \\
&= \mathcal{A}_{sd}(K^{*0}) - 2E_P - 2E_{P3} \\
&\quad - PA_{PV} + PA_{PV}^{\text{break}} \,, && (14) \\
\mathcal{A}_{sd}(\overline{K}^{*0}) &= -E_P + E_V \\
&\quad - E_{P1} - E_{P2} + E_{V3} - PA_{PV}^{\text{break}} \,, && (15) \\
\mathcal{A}_b(\overline{K}^{*0}) &= -E_P - E_V \\
&\quad - E_{P1} - E_{P2} - E_{V3} - PA_{PV} && (16) \\
&= \mathcal{A}_{sd}(\overline{K}^{*0}) - 2E_V - 2E_{V3} \\
&\quad - PA_{PV} + PA_{PV}^{\text{break}} \,. && (17)
\end{aligned}$$

Note that we express $\mathcal{A}_b$ by $\mathcal{A}_{sd}$ in order to make the subsequent topological dependences of the CP asymmetry more transparent, analogous to Refs. [1, 18]. Furthermore, we differentiate exchange and penguin annihilation diagrams where the antiquark from the weak vertex goes into the pseudoscalar meson (E_P, PA_P) or into the vector meson (E_V, PA_V). The exact naming scheme for the topologies is defined in Figs. 1 and 2. The $\mathrm{SU}(3)_F$ limit of Eqs. (12)–(17) agrees with Ref. [19], the CKM-leading $\mathrm{SU}(3)_F$ limit also with Ref. [12]. We use the amplitude normalization [13]

$$|\mathcal{A}(D \to VP)| = \sqrt{\frac{8\pi m_D^2\, \mathcal{B}(D \to VP)}{\tau_D (p^*)^3}} \,, \tag{18}$$

with the D^0 lifetime τ_D and p^* the magnitude of the K_S, $\overset{(-)}{K}{}^*$ 3-momentum. For the kaon states we use the conventions $K_S = \frac{1}{\sqrt{2}}(K^0 - \overline{K}^0)$ and $K_L = \frac{1}{\sqrt{2}}(K^0 + \overline{K}^0)$.[1] For the amplitudes it follows

$$\mathcal{A}(D^0 \to K^{*0} K_{S,L}) = \mp \frac{1}{\sqrt{2}} \mathcal{A}(D^0 \to K^{*0} \overline{K}^0) \,, \tag{19}$$

$$\mathcal{A}(D^0 \to \overline{K}^{*0} K_{S,L}) = \frac{1}{\sqrt{2}} \mathcal{A}(D^0 \to \overline{K}^{*0} K^0) \,, \tag{20}$$

so that we have for the direct CP asymmetries with tagged charm flavor

$$\begin{aligned}
a_{CP}^{\text{dir}}(D^0 \to K^{*0} K_S) &= a_{CP}^{\text{dir}}(D^0 \to K^{*0} K_L) && (21) \\
&= a_{CP}^{\text{dir}}(D^0 \to K^{*0} \overline{K}^0) \,, && (22) \\
a_{CP}^{\text{dir}}(D^0 \to \overline{K}^{*0} K_S) &= a_{CP}^{\text{dir}}(D^0 \to \overline{K}^{*0} K_L) && (23) \\
&= a_{CP}^{\text{dir}}(D^0 \to \overline{K}^{*0} K^0) \,. && (24)
\end{aligned}$$

We write therefore shortly

$$a_{CP}^{\text{dir}}(K^{*0}) \equiv a_{CP}^{\text{dir}}(D^0 \to K^{*0} K_S) \,, \tag{25}$$

$$a_{CP}^{\text{dir}}(\overline{K}^{*0}) \equiv a_{CP}^{\text{dir}}(D^0 \to \overline{K}^{*0} K_S) \,. \tag{26}$$

Inserting the topological parametrizations Eqs. (12)–(17) into Eq. (3) we arrive at

$$a_{CP}^{\text{dir}}(K^{*0}) = -R(K^{*0}) \sin\delta(K^{*0}) \,, \tag{27}$$

$$a_{CP}^{\text{dir}}(\overline{K}^{*0}) = -R(\overline{K}^{*0}) \sin\delta(\overline{K}^{*0}) \,, \tag{28}$$

with the magnitudes

$$\begin{aligned}
R(K^{*0}) &\equiv -\mathrm{Im}(\lambda_b)/|\mathcal{A}(K^{*0})| \times \\
&\quad |-2(E_P + E_{P3}) - PA_{PV} + PA_{PV}^{\text{break}}| \,, && (29) \\
R(\overline{K}^{*0}) &\equiv -\mathrm{Im}(\lambda_b)/|\mathcal{A}(\overline{K}^{*0})| \times \\
&\quad |-2(E_V + E_{V3}) - PA_{PV} + PA_{PV}^{\text{break}}| \,, && (30)
\end{aligned}$$

and the phases

$$\delta(K^{*0}) = \arg\left(\frac{-2(E_P + E_{P3}) - PA_{PV} + PA_{PV}^{\text{break}}}{\mathcal{A}_{sd}(K^{*0})}\right) \,, \tag{31}$$

$$\delta(\overline{K}^{*0}) = \arg\left(\frac{-2(E_V + E_{V3}) - PA_{PV} + PA_{PV}^{\text{break}}}{\mathcal{A}_{sd}(\overline{K}^{*0})}\right) \,. \tag{32}$$

It is instructive to study the $\mathrm{SU}(3)_F$ limit of the above expressions. To begin with, in the $\mathrm{SU}(3)_F$ limit Eqs. (12)-(17) imply

$$\mathcal{A}_{sd}(K^{*0}) = -\mathcal{A}_{sd}(\overline{K}^{*0}) \,, \tag{33}$$

$$\mathcal{A}_b(K^{*0}) = \mathcal{A}_b(\overline{K}^{*0}) \,. \tag{34}$$

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[1] We assume that effects of kaon CP violation are eliminated with the formula of Ref. [20].

Eq. (33) agrees with Refs. [13, 17]. Although in Eqs. (12), (15) several $\mathrm{SU}(3)_F$-breaking topologies are present, which in principle could affect Eq. (33) considerably, the latest LHCb data entail [10]

$$\left|\frac{\mathcal{A}(D^0 \to K_S K^{*0})}{\mathcal{A}(D^0 \to K_S \overline{K}^{*0})}\right| = \begin{cases} 1.12 \pm 0.05 \pm 0.11 & \text{(GLASS)} \\ 1.17 \pm 0.04 \pm 0.05 & \text{(LASS)} \end{cases}, \tag{35}$$

meaning small $\mathrm{SU}(3)_F$ breaking. In the $\mathrm{SU}(3)_F$ limit we have

$$a_{CP}^{\mathrm{dir}}(K^{*0}) = \frac{\mathrm{Im}(\lambda_b)}{\lambda_{sd}} \mathrm{Im}\left(\frac{-2E_P - PA_{PV}}{E_P - E_V}\right) \tag{36}$$

$$= -\frac{\mathrm{Im}(\lambda_b)}{\lambda_{sd}} \mathrm{Im}\left(\frac{E_P + E_V + PA_{PV}}{E_P - E_V}\right), \tag{37}$$

and analogously

$$a_{CP}^{\mathrm{dir}}(\overline{K}^{*0}) = \frac{\mathrm{Im}(\lambda_b)}{\lambda_{sd}} \mathrm{Im}\left(\frac{E_P + E_V + PA_{PV}}{E_P - E_V}\right), \tag{38}$$

showing that a_{CP}^{dir} is enhanced for $E_P \sim E_V$. In the step to Eq. (37) we added $(E_P - E_V)/(E_P - E_V)$ to the term in brackets. Eqs. (37), (38) imply the sum rule

$$a_{CP}^{\mathrm{dir}}(K^{*0}) + a_{CP}^{\mathrm{dir}}(\overline{K}^{*0}) = 0, \tag{39}$$

found in Refs. [21, 22], which also complies with the numerical results of Ref. [19]. Eq. (39) is a test of $\mathrm{SU}(3)_F$ breaking in the CP asymmetries, sensitive to other topological amplitudes than Eq. (33).

For the untagged CP asymmetry we arrive at

$$a_{CP}^{\mathrm{dir,untag}}(K^{*0}) = a_{CP}^{\mathrm{dir}}(K^{*0}) \tag{40}$$

$$= -a_{CP}^{\mathrm{dir,untag}}(\overline{K}^{*0}) = -a_{CP}^{\mathrm{dir}}(\overline{K}^{*0}) \tag{41}$$

in the $\mathrm{SU}(3)_F$ limit, *i.e.* there is no dilution of the untagged CP asymmetry with respect to the tagged one. Barring the possibility of accidentally vanishing strong phases, $a_{CP}^{\mathrm{dir}}(K^{*0})$ and $a_{CP}^{\mathrm{dir}}(\overline{K}^{*0})$ neither vanish in the $\mathrm{SU}(3)_F$ limit nor in the limit of vanishing penguin annihilation. On the contrary, following the above discussion one can expect that the main contribution to the CP asymmetry stems in fact from the $\mathrm{SU}(3)_F$-limit exchange diagrams E_P, E_V.

III. PHENOMENOLOGY

From the LHCb measurements Eqs. (7) and (8) we extract the absolute value of the difference of the exchange topologies as:

$$|E_P - E_V| = (1.6 \pm 0.2) \cdot 10^{-6}. \tag{42}$$

We use this bound together with the solution for the absolute values of E_P and E_V in Table 1 of Ref. [13],

$$|E_P| = (2.94 \pm 0.09) \cdot 10^{-6}, \tag{43}$$

$$|E_V| = (2.37 \pm 0.19) \cdot 10^{-6}. \tag{44}$$

For a rough estimate of $a_{CP}^{\mathrm{dir,\,untag}}$ near the K^* peak we use Eq. (37) where we vary $|E_P|$ and $|E_V|$ flat inside the 2σ ranges of Eqs. (43)–(44), while imposing the branching ratio constraint Eq. (42) to be also fulfilled at 2σ. Furthermore, we use $0 \leq |PA_{PV}| \leq 0.2 \times (E_P + E_V)/2$ with the central values of E_P, E_V in Eqs. (43, 44). All relative strong phases are varied freely in the interval $[-\pi, +\pi]$. We find the relative phase between E_P and E_V in the range $[-0.24\pi, +0.24\pi]$ by combining Eqs. (42)–(44). The maximum value of $a_{CP}^{\mathrm{dir,\,untag}}$ near the peak of the K^* resonance is then

$$|a_{CP}^{\mathrm{dir,\,untag}}| \lesssim 0.003, \tag{45}$$

with the maximum found for $\arg(E_V/E_P) = 0.14\,\pi$. In the experimental analysis one can scan the Dalitz plot around the K^* resonance to look for favourable strong phases which maximize $|a_{CP}^{\mathrm{dir,\,untag}}|$.

In order to inspect the dependence of this result on the size of penguin annihilation diagrams we also look at the case $PA_{PV} = 0$. As the dominant piece of the CP asymmetry stems from the exchange topologies, we find the result in Eq. (45) unchanged.

IV. CONCLUSIONS

CP asymmetries in neutral $D \to KK^*$ decays are driven by exchange topologies and persist in the limit of vanishing penguins. In the $\mathrm{SU}(3)_F$ limit the untagged CP asymmetry is equal to the tagged one, *i.e.* there is no dilution, which enables the search for charm CP violation with high statistics in untagged samples. Therefore $D \to KK^*$ decays are promising discovery channels for charm CP violation. Our estimate for the maximum possible CP asymmetry is given in Eq. (45).

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