# Direct CP Violation in $K \to \mu^+\mu^-$

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A rare decay $K_L \to \mu^+\mu^-$ has been measured precisely, while a rare decay $K_S \to \mu^+\mu^-$ will be observed by an upgrade of the LHCb experiment. Although both processes are almost CP-conserving decays, we point out that an interference contribution between $K_L$ and $K_S$ in the kaon beam emerges from a genuine direct CP violation. It is found that the interference contribution can change $K_S \to \mu^+\mu^-$ standard-model predictions at $\mathcal{O}(60\%)$. We also stress that an unknown sign of $\mathcal{A}(K_L \to \gamma\gamma)$ can be determined by a measurement of the interference, which can much reduce a theoretical uncertainty of $\mathcal{B}(K_L \to \mu^+\mu^-)$. We also investigate the interference in a new physics model, where the $\epsilon'_K/\epsilon_K$ tension is explained by an additional $Z$-penguin contribution.

Keywords: rare kaon decay, direct CP violation



## INTRODUCTION

Rare kaon decays have played a crucial role in flavor physics; now this physics program is even more exciting due to NA62 experiment at CERN, aiming to reach a precision of 10 % in $\mathcal{B}(K^+ \to \pi^+\nu\overline{\nu})$ compared to the SM already in 2018 [1, 2], and KOTO experiment at J-PARC aiming in a first step at measuring $\mathcal{B}(K_L \to \pi^0\nu\overline{\nu})$ around the SM sensitivity [3–5]; also LHCb experiment has an impressive kaon physics program [6]. New physics motivated from the $\epsilon'_K/\epsilon_K$ tension [7–9] or $B$-physics anomalies may be tested in rare kaon decays too. Experimentally kaons in two muons in the final state can be considered gold channels and this motivates theoretical studies.

Within the Standard Model (SM), the branching ratios are predicted to be [10–12]

$$\mathcal{B}(K_L \to \mu^+\mu^-)_{\rm SM} = \begin{cases} (6.85 \pm 0.80 \pm 0.06) \times 10^{-9} (+), \\ (8.11 \pm 1.49 \pm 0.13) \times 10^{-9} (-), \end{cases} \tag{1}$$

$$\begin{aligned}\mathcal{B}(K_S \to \mu^+\mu^-)_{\rm SM} &= (4.99\,({\rm LD}) + 0.19\,({\rm SD})) \times 10^{-12} \\ &= (5.18 \pm 1.50 \pm 0.02) \times 10^{-12}, \end{aligned} \tag{2}$$

where the first uncertainty comes from long-distance contributions and the second one denotes remaining theoretical uncertainties including the Cabibbo-Kobayashi-Maskawa (CKM) parameters'. The long-distance (short-distance) contribution to $\mathcal{B}(K_S \to \mu^+\mu^-)_{\rm SM}$ is indicated by LD (SD). Here, ${\rm sgn}(G_8) < 0$, where $G_8$ represents a leading coupling of the $|\Delta S| = 1$ non-leptonic weak Lagrangian [13], is chosen. That is predicted under reasonable assumptions [11, 14, 15]. The values of Eqs. (1), (2) are based on the best-fit result for the CKM parameters in Ref. [16]. One should note that $\mathcal{B}(K_L \to \mu^+\mu^-)_{\rm SM}$ depends on an unknown sign of $\mathcal{A}(K_L \to \gamma\gamma)$. When ${\rm sgn}(\mathcal{A}(K_L \to \gamma\gamma)) = \pm{\rm sgn}(\mathcal{A}(K_L \to (\pi^0)^* \to \gamma\gamma))$, we represent + or − in Eq. (1). The choice of + (−) gives a destructive (constructive) interference between short- and long-distance contributions to $\mathcal{B}(K_L \to \mu^+\mu^-)$ in the SM [14, 15] .

On the other hand, experimental results are [17]

$$\mathcal{B}(K_L \to \mu^+\mu^-)_{\rm exp} = (6.84 \pm 0.11) \times 10^{-9}, \tag{3}$$

and the 90 % C.L. upper bound [18]

$$\mathcal{B}(K_S \to \mu^+\mu^-)_{\rm exp} < 0.8 \times 10^{-9}. \tag{4}$$

Although a current bound of $\mathcal{B}(K_S \to \mu^+\mu^-)$ is weaker than the SM prediction by two orders of magnitude, an upgrade of the LHCb experiment is aiming to reach the SM sensitivity, specifically the LHC Run 3 (from 2021) [19]. Note that the branching ratios into the electron mode are suppressed by $m_e^2/m_\mu^2$, and the detector sensitivity to the electron mode in the LHCb is weaker than the muonic mode.

Equations (1) and (2) are predictions of pure $K_L$ and $K_S$ initial states, respectively. In this Letter, we focus on interference between $K_L$ and $K_S$ states:

$$\Gamma(K \to f)_{\rm int} \propto \mathcal{A}(K_S \to f)^* \mathcal{A}(K_L \to f), \tag{5}$$

where the initial state is the same $K^0$ (or $\overline{K}^0$), and a *lifetime* of this contribution is $2\tau_S$. Such an interference contribution is first discussed in Refs. [20, 21], and has been observed and utilized in many processes: e.g., $K \to \pi\pi$ [22], $K \to 3\pi^0$ [23, 24], $K \to \pi^+\pi^-\pi^0$ [25], and $K \to \pi^0 e^+e^-$ [26].

## INTERFERENCE BETWEEN $K_L$ AND $K_S$

We first review the interference contribution briefly, then we investigate the numerical impact in the mode of $\mu^+\mu^-$ in the SM. A state of $K^0$ (or $\overline{K}^0$) at $t = 0$,

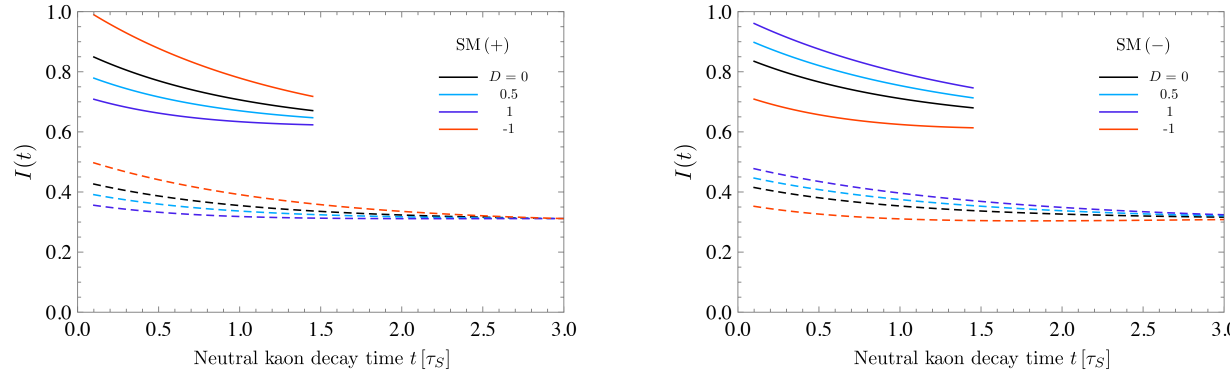


FIG. 1. The time distributions of $K \to \mu^+\mu^-$ ($I(t)$) are shown within the SM with several choices of $D$, which are normalized by the decay intensity from $0.1\tau_S$ to $1.45\tau_S$ (solid lines) and to $3\tau_S$ (dashed lines) with $D = 0$. The left and right panels correspond to the positive and negative signs of $A^{\mu}_{L\gamma\gamma}$ in Eq. (11), respectively.

which is produced by, e.g., $pp \to K^0K^-\pi^+$, evolves into a mixture of $K_1$ (CP-even) and $K_2$ (CP-odd) states,

$$|\overset{(-)}{K}{}^0(t)\rangle = \frac{1}{\sqrt{2}(1\pm\bar{\epsilon})}\left[e^{-iH_St}\left(|K_1\rangle + \bar{\epsilon}|K_2\rangle\right) \pm e^{-iH_Lt}\left(|K_2\rangle + \bar{\epsilon}|K_1\rangle\right)\right], \quad (6)$$

where $H_{L,S} = M_{L,S} - (i/2)\Gamma_{L,S}$, $|K_{1,2}\rangle = (1/\sqrt{2})(|K^0\rangle \pm |\overline{K}^0\rangle)$, and $\mathrm{CP}|K_{1,2}\rangle = \pm|K_{1,2}\rangle$. The CP impurity parameter $\bar{\epsilon}$ is related to $\epsilon_K$ as $\epsilon_K = (\bar{\epsilon} + i\mathrm{Im}\,A_0/\mathrm{Re}\,A_0)/(1 + i\bar{\epsilon}\mathrm{Im}\,A_0/\mathrm{Re}\,A_0)$ with $\mathcal{A}(K^0 \to (\pi\pi)_{I=0}) \equiv A_0 e^{i\delta_0}$ and $\delta_0$ is a strong phase for $I = 0$ two pion state.

The decay intensity of a neutral kaon beam into $f$ is

$$I(t) = \frac{1+D}{2}\left|\langle f| - \mathcal{H}^{|\Delta S|=1}_{\mathrm{eff}}|K^0(t)\rangle\right|^2 + \frac{1-D}{2}\left|\langle f| - \mathcal{H}^{|\Delta S|=1}_{\mathrm{eff}}|\overline{K}^0(t)\rangle\right|^2 \quad (7)$$

$$= \frac{1}{2}\Big[\left\{(1-2D\mathrm{Re}\,[\bar{\epsilon}])\,|\mathcal{A}(K_1)|^2 + 2\mathrm{Re}\,[\bar{\epsilon}\mathcal{A}(K_1)^*\mathcal{A}(K_2)]\right\}e^{-\Gamma_S t} + \left\{(1-2D\mathrm{Re}\,[\bar{\epsilon}])\,|\mathcal{A}(K_2)|^2 + 2\mathrm{Re}\,[\bar{\epsilon}\mathcal{A}(K_1)\mathcal{A}(K_2)^*]\right\}e^{-\Gamma_L t}$$
$$+ \left\{2D\mathrm{Re}\,\left[e^{-i\Delta M_K t}\left(\mathcal{A}(K_1)^*\mathcal{A}(K_2) + \bar{\epsilon}|\mathcal{A}(K_1)|^2 + \bar{\epsilon}^*|\mathcal{A}(K_2)|^2\right)\right] - 4\mathrm{Re}\,[\bar{\epsilon}]\mathrm{Re}\,\left[e^{-i\Delta M_K t}\mathcal{A}(K_1)^*\mathcal{A}(K_2)\right]\right\}e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\Big]$$
$$+ \mathcal{O}(\bar{\epsilon}^2), \quad (8)$$

where $M_L - M_S \equiv \Delta M_K > 0$, $\mathcal{A}(K_{1,2}) \equiv \mathcal{A}(K_{1,2} \to f)$, and a dilution factor $D$ is a measure of the initial ($t = 0$) asymmetry of the number of $K^0$ and $\overline{K}^0$:

$$D = \frac{K^0 - \overline{K}^0}{K^0 + \overline{K}^0}. \quad (9)$$

The term proportional to $\exp(-\Gamma_S t)$ (or $\exp(-\Gamma_L t)$) arises from $K_S$ (or $K_L$) decay in the mode $f$, while the term proportional to $\exp(-(\Gamma_S+\Gamma_L)t/2)$ represents the interference between $K_L$ and $K_S$, whose lifetime is $2/(\Gamma_S+\Gamma_L) \simeq 2\tau_S$.

## INTERFERENCE EFFECT ON $K \to \mu^+\mu^-$ IN THE SM

When $f = \mu^+\mu^-$ case, all $\mathcal{O}(\bar{\epsilon})$ terms are numerically negligible, which is certainly different situation from $K \to 2\pi$ and $K \to 3\pi$. Then, a term of Eq. (5) is relevant, which is the first term in the second line of Eq. (8). The $|\mathcal{A}(K_{1,2})|^2$ term provides the SM prediction of $\mathcal{B}(K_{S,L} \to \mu^+\mu^-)_{\mathrm{SM}}$ in Eqs. (1), (2) [10–12], which is significantly dominated by a CP conserving long-distance contribution. Within the SM, regarding the interference term, we obtain

$$\sum_{\mathrm{spin}}\mathcal{A}(K_1 \to \mu^+\mu^-)^*\mathcal{A}(K_2 \to \mu^+\mu^-)$$
$$= \frac{16iG_F^4M_W^4F_K^2M_K^2m_\mu^2\sin^2\theta_W}{\pi^3}\mathrm{Im}\,[\lambda_t]y'_{7A} \times\left\{A^{\mu}_{L\gamma\gamma} - 2\pi\sin^2\theta_W\left(\mathrm{Re}\,[\lambda_t]y'_{7A} + \mathrm{Re}\,[\lambda_c]y_c\right)\right\}, \quad (10)$$

where the spin of the muons is summed up, $\lambda_q \equiv V^*_{qs}V_{qd}$, $\sin^2\theta_W \equiv \sin^2\hat{\theta}^{\overline{\mathrm{MS}}}_W(M_Z) = 0.23129(5)$ [17], $f_K = \sqrt{2}F_K = 0.1556(4)\mathrm{GeV}$ [17], the top-quark contribution in next-to-leading order of QCD is $y'_{7A} = -0.654(34)$ [12, 27] (which is defined in next section), the charm-quark contribution in next-to-next-to-leading order of

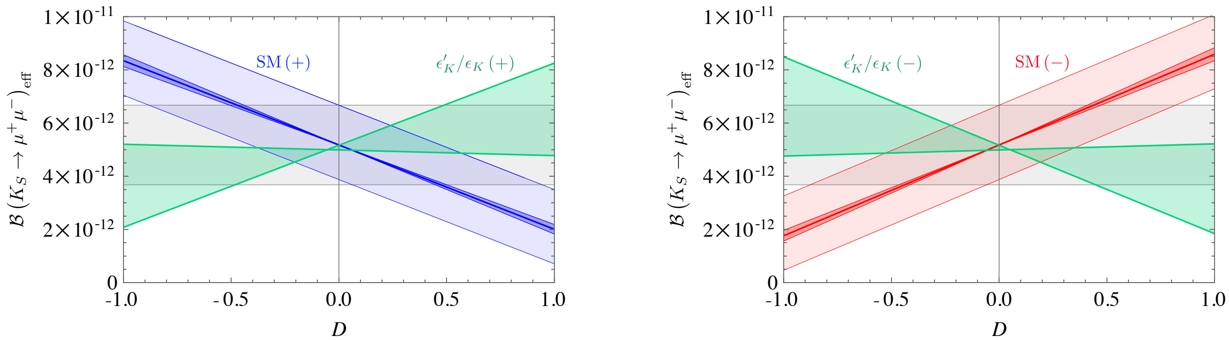


FIG. 2. The effective branching ratio into $\mu^+\mu^-$ in Eq. (12) as a function of the dilution factor. The left and right panels correspond to the positive and negative signs of $A^\mu_{L\gamma\gamma}$ in Eq. (11), respectively. The SM predictions are represented by blue and red lines, where the darker bands stand for uncertainty from the interference in Eq. (10) and the lighter bands denote uncertainty from $A^\mu_{S\gamma\gamma}$ in Eq. (13). Gray bands represent $\mathcal{B}(K_S \to \mu^+\mu^-)_{\rm SM}$ in Eq. (2). The $\epsilon'_K/\epsilon_K$ anomaly can be explained at $1\,\sigma$ in the green regions within the modified $Z$-coupling model.

QCD is $y_c = -2.03(32)\cdot 10^{-4}$ [12], and an amplitude of the CP conserving long-distance contributions for $K_2$ is [11, 28]

$$\begin{aligned} A^\mu_{L\gamma\gamma} &= \frac{\pm 2\pi\alpha_0}{G_F^2 M_W^2 F_K M_K}\sqrt{\frac{\pi}{M_K}\Gamma(K_L\to\gamma\gamma)_{\rm exp}} \\ &\quad \times(\chi_{\rm disp}+i\chi_{\rm abs}) \\ &= \pm 2.01(1)\cdot 10^{-4}\cdot(0.71(101)-i\,5.21)\,, \end{aligned} \tag{11}$$

with $\mathcal{B}(K_L\to\gamma\gamma)_{\rm exp} = 5.47(4)\cdot 10^{-4}$ [17] and $\alpha_0 = 1/137.04$. Here, the sign ambiguity in $A^\mu_{L\gamma\gamma}$ comes from the unknown sign of $\mathcal{A}(K_L\to\gamma\gamma)$, and this $\pm$ corresponds to $\mathrm{sgn}(\mathcal{A}(K_L\to\gamma\gamma)) = \pm\mathrm{sgn}(\mathcal{A}(K_L\to(\pi^0)^*\to\gamma\gamma))$ and Eq. (1). Obviously, the interference in Eq. (10) is proportional to the direct CP violating contribution.

Figure 1 shows a time distribution of $K\to\mu^+\mu^-$ in Eq. (8) with several choices of $D$ and the sign of $A^\mu_{L\gamma\gamma}$, which are normalized by an integrated decay intensity from $0.1\tau_S$ to $1.45\tau_S$ (solid lines) and to $3\tau_S$ (dashed lines) with $D=0$. It is shown that the interference effect emerges prominently around $t\simeq 0$, which can give $\mathcal{O}(10\%)$ difference. Besides, another important point found here is that one can probe the unknown sign of $A^\mu_{L\gamma\gamma}$ by precise measurement of the interference correction.

Using the result of Eq. (8), let us define an *effective* branching ratio into $\mu^+\mu^-$, which includes the interference correction and would correspond to event numbers in experiments after a removal of the $K_L$ background:

$$\begin{aligned} &\mathcal{B}(K_S\to\mu^+\mu^-)_{\rm eff} \\ &= \tau_S\left[\int_{t_{min}}^{t_{max}} dt\left(\Gamma(K_1)e^{-\Gamma_S t} + \frac{D}{8\pi M_K}\sqrt{1-\frac{4m_\mu^2}{M_K^2}}\sum_{\rm spin}\mathrm{Re}\left[e^{-i\Delta M_K t}\mathcal{A}(K_1)^*\mathcal{A}(K_2)\right]e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\right)\varepsilon(t)\right] \\ &\times\left(\int_{t_{min}}^{t_{max}} dt\, e^{-\Gamma_S t}\,\varepsilon(t)\right)^{-1}, \end{aligned} \tag{12}$$

where $\Gamma(K_1) = \Gamma(K_1\to\mu^+\mu^-)$, $t_{min}$ to $t_{max}$ corresponds to a range of detector for $K_S$ tagging, and $\varepsilon(t)$ is a decay-time acceptance of the detector. Note that $\mathcal{B}(K_S\to\mu^+\mu^-)_{\rm eff} = \mathcal{B}(K_S\to\mu^+\mu^-)_{\rm SM}$ in Eq. (2) is obtained when $D=0$ is chosen.

We investigate the effective branching ratio in Eq. (12) as a function of $D$ in Fig. 2. Here, the experimental setup of the LHCb detector is adopted: The decay-time acceptance is $\varepsilon(t) = \exp(-\beta t)$ where $\beta\simeq 86\,(\mathrm{ns})^{-1}$ [29]. The range of the detector for selecting $K\to\mu^+\mu^-$ is $t_{min} = 8.95\,\mathrm{ps} = 0.1\tau_S$ and $t_{max} = 130\,\mathrm{ps} = 1.45\tau_S$ [29]. Gray bands represent $\mathcal{B}(K_S\to\mu^+\mu^-)_{\rm SM}$ in Eq. (2). The blue and red lines are the SM predictions, where the lighter (darker) bands stand for uncertainty from $A^\mu_{S\gamma\gamma}$ (from the interference term in Eq. (10)), which is an amplitude of the CP conserving long-distance contributions for $K_1$ [10, 11, 28]

$$\begin{aligned} A^\mu_{S\gamma\gamma} &= \frac{\pi\alpha_0}{G_F^2 M_W^2 F_K M_K |H(0)|}\sqrt{\frac{\pi}{M_K}\Gamma(K_S\to\gamma\gamma)_{\rm exp}} \\ &\quad\times(\mathcal{I}_{\rm disp}+i\mathcal{I}_{\rm abs}) \\ &= 2.48(35)\cdot 10^{-4}\cdot(-2.83+i\,1.22), \end{aligned} \tag{13}$$

where $\mathcal{B}(K_S \to \gamma\gamma)_{\rm exp} = 2.63(17) \cdot 10^{-6}$ [17] and the pion one-loop function $H(0) = 0.331 + i0.583$ [10] are used. Since this evaluation includes a 17% enhancement of the amplitude by a final state interaction of the pions and it is reasonable for not off-shell but on-shell photons emission, a 30% uncertainty to the branching ratio is taken [11].

It is found that the interference affects the branching ratio at $\mathcal{O}(60\%)$ and the unknown sign of $A^\mu_{L\gamma\gamma}$ can be uncovered if $D = \mathcal{O}(1)$ can be used. Note that the error of $A^\mu_{S\gamma\gamma}$ dominates uncertainties of all lines. Since the dispersive treatment [30] will sharpen $A^\mu_{S\gamma\gamma}$, hence the interference and $\mathcal{B}(K_S \to \mu^+\mu^-)_{\rm SM}$ will be transparent in these figures.

Note that since $\sigma(pp \to K^0X) \simeq \sigma(pp \to \overline{K}^0X)$, $D$ would be 0 as a standard of the LHCb experiment. We propose two methods how to generate $K^0$–$\overline{K}^0$ asymmetry in the neutral kaon signals. The first one is a tagging of a charged kaon which accompanies the neutral kaon beam. An $\mathcal{O}(30\%)$ of prompt $K^0$ accompanies $K^-$ through $pp \to K^0K^-X$ [29]. Such a charged kaon track with $K \to \mu^+\mu^-$ signal can be tagged by using the RICH detectors. This charged kaon tagging has been utilized to tag $B^0_s$ in the LHCb [31]. A similar tagging would be possible for $\Lambda^0$ through $pp \to K^0\Lambda^0X$ with $\Lambda^0 \to p\pi^-$ [32]. Another proposal is a charged pion tagging using $pp \to K^{*+}X \to K^0\pi^+X$. A similar charged pion tagging for $D^0$ ($D^{*+} \to D^0\pi^+$) has been achieved in the LHCb experiment [33].

## PROBING NEW PHYSICS

In this section, we investigate new physics influence on the interference. In general new physics, only three operators can contribute to $K \to \mu^+\mu^-$, then the interference term in Eq. (10) can be extended to

$$\begin{aligned}
&\sum_{\rm spin} \mathcal{A}(K_1 \to \mu^+\mu^-)^* \mathcal{A}(K_2 \to \mu^+\mu^-) \\
&= \frac{8G_F^4 M_W^4 F_K^2 M_K^2 m_\mu^2}{\pi^4} \\
&\times \left[ \left(1 - \frac{4m_\mu^2}{M_K^2}\right) \left\{ \left(A^\mu_{S\gamma\gamma}\right)^* + \frac{M_K^2}{M_W^2} {\rm Re}\, \tilde{y}'_S \right\} i \frac{M_K^2}{M_W^2} {\rm Im}\, \tilde{y}'_S \right. \\
&+ \left\{ 2i\pi \sin^2\theta_W \left({\rm Im}\, [\lambda_t]\, y'_{7A} + {\rm Im}\, \tilde{y}'_{7A}\right) - i\frac{M_K^2}{M_W^2} {\rm Im}\, \tilde{y}'_P \right\} \\
&\quad \times \left\{ -2\pi \sin^2\theta_W \left({\rm Re}\, [\lambda_t]\, y'_{7A} + {\rm Re}\, [\lambda_c]\, y_c + {\rm Re}\, \tilde{y}'_{7A}\right) \right. \\
&\quad \left.\left. + A^\mu_{L\gamma\gamma} + \frac{M_K^2}{M_W^2} {\rm Re}\, \tilde{y}'_P \right\} \right], \qquad (14)
\end{aligned}$$

where the Wilson coefficients are defined in [28]

$$\begin{aligned}
\mathcal{H}_{\rm eff}^{|\Delta S|=1} &= \frac{G_F^2 m_s m_\mu}{\pi^2} \left\{ \tilde{y}'_S \,(\overline{s}\gamma_5 d)\,(\overline{\mu}\mu) + \tilde{y}'_P \,(\overline{s}\gamma_5 d)\,(\overline{\mu}\gamma_5\mu) \right\} \\
&+ \frac{G_F\alpha}{\sqrt{2}} \left(\lambda_t y'_{7A} + \tilde{y}'_{7A}\right) (\overline{s}\gamma_\mu\gamma_5 d)\,(\overline{\mu}\gamma^\mu\gamma_5\mu) + {\rm H.c.}, \quad (15)
\end{aligned}$$

here new physics contributions are represented by $\tilde{y}'$, and $\alpha \equiv \alpha_{\overline{\rm MS}}(M_Z) = 1/127.95$ [17]. We find that the interference in Eq. (14) is still a genuine direct CP violating contribution. The new physics contributions ($\tilde{y}'_S$, $\tilde{y}'_P$, and $\tilde{y}'_{7A}$) to $\Gamma(K_{1,2} \to \mu^+\mu^-)$ are given in Ref. [28].

The following is a specific example of new physics: we focus on a modified $Z$-coupling model [34–38], which can easily explain a 2.8-2.9 $\sigma$ discrepancy in $\epsilon'_K/\epsilon_K$ between the measured values and the predicted one at next-to-leading order [7–9]. In this model, after the electroweak symmetry breaking, the following flavor-changing $Z$ interactions emerge

$$\mathcal{H}_{\rm eff}^{|\Delta S|=1} = -\Delta_L^{\rm NP} \overline{s}\gamma_\mu P_L d Z^\mu + (L \leftrightarrow R) + {\rm H.c.}. \quad (16)$$

In our analysis, we assume that the new physics is only left-handed and it is pure imaginary for simplicity: $\Delta_R^{\rm NP} = {\rm Re}\,\Delta_L^{\rm NP} = 0$. According to Ref. [37], the $\epsilon'_K/\epsilon_K$ discrepancy is explained at $1\,\sigma$ level by the range of $-1.05 \cdot 10^{-6} < {\rm Im}\,\Delta_L^{\rm NP} < -0.50 \cdot 10^{-6}$ without conflict with $\epsilon_K$ and $\mathcal{B}(K_L \to \mu^+\mu^-)$. This range corresponds to

$$0.86 \cdot 10^{-4} < {\rm Im}\,\tilde{y}'_{7A} < 1.82 \cdot 10^{-4}, \quad \tilde{y}'_S = \tilde{y}'_P = 0. \quad (17)$$

Green bands in Fig. 2 show that the effective branching ratio into $\mu^+\mu^-$ in Eq. (12), which can explain the $\epsilon'_K/\epsilon_K$ discrepancy at $1\,\sigma$. It is observed that the interference vanishes or flips the sign compared to the SM predictions. It is because the interference is proportional to the direct CP violation (${\rm Im}\,[\lambda_t]\, y'_{7A} + {\rm Im}\,\tilde{y}'_{7A}$) and ${\rm Im}\,[\lambda_t] y'_{7A} = -0.92 \cdot 10^{-4}$.

The other new physics scenario which can explain the $\epsilon'_K/\epsilon_K$ discrepancy [39] will be presented in a forthcoming article [40].

## DISCUSSION AND CONCLUSIONS

In this Letter, we have demonstrated the interference between $K_L$ and $K_S$ in $K \to \mu^+\mu^-$ within the SM and the modified $Z$-coupling model, which could be probed by future upgrade LHCb experiment. We have pointed out that the interference is a genuine direct CP violation, so that one can investigate the direct CP violation by precise measurement of $K \to \mu^+\mu^-$. It is found that within the SM the interference can amplify the effective branching ratio of $K_S \to \mu^+\mu^-$ in Eq. (12) by $\mathcal{O}(60\%)$ with distinguishing the unknown sign of $\mathcal{A}(K_L \to \gamma\gamma)$, which can much reduce the theoretical uncertainty of $\mathcal{B}(K_L \to \mu^+\mu^-)$. It is also shown that in the modified $Z$-coupling model accounting for the $\epsilon'_K/\epsilon_K$ anomaly, the interference is predicted to vanish or flip the sign.

Such an investigation of the direct CP violation of kaon decay is important for, of course, $\epsilon'_K/\epsilon_K$ and, a cross-check of the KOTO experiment, which is probing a CP violating $K_L \to \pi^0\nu\overline{\nu}$ decay and will reach the SM sensitivity at 2021 [5, 41].

The similar study would be possible for $K_S \to \pi^+\pi^-\pi^0$ using the interference in the LHCb. Although there are significant background events from $K_L \to \pi^+\pi^-\pi^0$, the Dalitz analysis of the three pions momenta can cut the background [25, 42].

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[1] F. Newson *et al.*, arXiv:1411.0109 [hep-ex].
[2] A. Romano, arXiv:1411.6546 [hep-ex].
[3] T. K. Komatsubara, Prog. Part. Nucl. Phys. **67** (2012) 995 [arXiv:1203.6437 [hep-ex]].
[4] K. Shiomi [KOTO Collaboration], arXiv:1411.4250 [hep-ex].
[5] Talk by H. Nanjo on "KOTO and KOTO step2 to search for the rare kaon decay, $K_L \to \pi^0\nu\overline{\nu}$" at *International workshop on physics at the extended hadron experimental facility of J-PARC*, KEK Tokai Campus, 5-6 March 2016.
[6] Talk by M. R. Pernas on "Rare strange decays at LHCb" at *EPS Conference on High Energy Physics*, Venice, Italy 5-12 July 2017.
[7] Z. Bai *et al.* [RBC and UKQCD Collaborations], Phys. Rev. Lett. **115**, no. 21, 212001 (2015) [arXiv:1505.07863 [hep-lat]].
[8] A. J. Buras, M. Gorbahn, S. Jäger and M. Jamin, JHEP **1511**, 202 (2015) [arXiv:1507.06345 [hep-ph]].
[9] T. Kitahara, U. Nierste and P. Tremper, JHEP **1612**, 078 (2016) [arXiv:1607.06727 [hep-ph]].
[10] G. Ecker and A. Pich, Nucl. Phys. B **366**, 189 (1991).
[11] G. Isidori and R. Unterdorfer, JHEP **0401**, 009 (2004) [hep-ph/0311084].
[12] M. Gorbahn and U. Haisch, Phys. Rev. Lett. **97**, 122002 (2006) [hep-ph/0605203].
[13] G. D'Ambrosio, G. Ecker, G. Isidori and H. Neufeld, 2nd DAPHNE Physics Handbook:265-313 [hep-ph/9411439].
[14] A. Pich and E. de Rafael, Phys. Lett. B **374**, 186 (1996) [hep-ph/9511465].
[15] J. M. Gerard, C. Smith and S. Trine, Nucl. Phys. B **730**, 1 (2005) [hep-ph/0508189].
[16] Updated result of J. Charles *et al.* [CKMfitter Group], Eur. Phys. J. C **41**, 1 (2005) [arXiv:hep-ph/0406184] for the conference ICHEP 2016, see http://ckmfitter.in2p3.fr/www/results/plots_ichep16/num/ckmEval_results_ichep16.html.
[17] C. Patrignani *et al.* [Particle Data Group], Chin. Phys. C **40**, no. 10, 100001 (2016).
[18] R. Aaij *et al.* [LHCb Collaboration], arXiv:1706.00758 [hep-ex].
[19] Talk by D. M. Santos on "Physics of LHCb Upgrade(s)" at *FPCP 2017 - Flavor Physics & CP Violation*, Prague, Czech Republic 5-9 June 2017.
[20] J. S. Bell and J. Steinberger, *Proceedings of the Oxford International Conference on Elementary Particles*, 1965, In Wolfenstein, L. (ed.): CP violation 42-57.
[21] L. M. Sehgal and L. Wolfenstein, Phys. Rev. **162**, 1362 (1967).
[22] R. Carosi *et al.* [NA31 Collaboration], Phys. Lett. B **237**, 303 (1990).
[23] N. Cartiglia, Eur. Phys. J. C **33**, S337 (2004) [hep-ph/0310152].
[24] F. Ambrosino *et al.* [KLOE Collaboration], Phys. Lett. B **619**, 61 (2005) [hep-ex/0505012].
[25] J. R. Batley *et al.* [NA48 Collaboration], Phys. Lett. B **630**, 31 (2005) [hep-ex/0510008].
[26] J. F. Donoghue and F. Gabbiani, Phys. Rev. D **51**, 2187 (1995) [hep-ph/9408390]. G. O. Kohler and E. A. Paschos, Phys. Rev. D **52**, 175 (1995) [hep-ph/9411335].
[27] G. Buchalla and A. J. Buras, Nucl. Phys. B **400**, 225 (1993); M. Misiak and J. Urban, Phys. Lett. B **451**, 161 (1999) [hep-ph/9901278].
[28] F. Mescia, C. Smith and S. Trine, JHEP **0608**, 088 (2006) [hep-ph/0606081].
[29] D. M. Santos, private communication.
[30] G. Colangelo, R. Stucki and L. C. Tunstall, Eur. Phys. J. C **76**, no. 11, 604 (2016) [arXiv:1609.03574 [hep-ph]].
[31] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **108**, 101803 (2012) [arXiv:1112.3183 [hep-ex]].
[32] M. Adinolfi *et al.* [LHCb RICH Group], Eur. Phys. J. C **73**, 2431 (2013) [arXiv:1211.6759 [physics.ins-det]].
[33] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **108**, 111602 (2012) [arXiv:1112.0938 [hep-ex]].
[34] A. J. Buras, F. De Fazio and J. Girrbach, JHEP **1302**, 116 (2013) [arXiv:1211.1896 [hep-ph]]; Eur. Phys. J. C **74**, no. 7, 2950 (2014) [arXiv:1404.3824 [hep-ph]].
[35] A. J. Buras, D. Buttazzo and R. Knegjens, JHEP **1511**, 166 (2015) [arXiv:1507.08672 [hep-ph]].
[36] A. J. Buras, JHEP **1604**, 071 (2016) [arXiv:1601.00005 [hep-ph]].
[37] M. Endo, T. Kitahara, S. Mishima and K. Yamamoto, Phys. Lett. B **771**, 37 (2017) [arXiv:1612.08839 [hep-ph]].
[38] C. Bobeth, A. J. Buras, A. Celis and M. Jung, arXiv:1703.04753 [hep-ph].
[39] T. Kitahara, U. Nierste and P. Tremper, Phys. Rev. Lett. **117**, no. 9, 091802 (2016) [arXiv:1604.07400 [hep-ph]]; A. Crivellin, G. D'Ambrosio, T. Kitahara and U. Nierste, arXiv:1703.05786 [hep-ph].
[40] V. Chobanova, G. D'Ambrosio, T. Kitahara, M. L. Martinez, D. M. Santos, and K. Yamamoto, in preparation.
[41] Talk by G. Ruggiero on "Recent results from kaon physics" at *EPS Conference on High Energy Physics*, Venice, Italy 5-12 July 2017.
[42] G. D'Ambrosio, G. Isidori, A. Pugliese and N. Paver, Phys. Rev. D **50**, 5767 (1994) Erratum: [ Phys. Rev. D **51**, 3975 (1995)] [hep-ph/9403235].