$\epsilon'_{\mathbf{K}}/\epsilon_{\mathbf{K}}$: Standard Model and Supersymmetry

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I give a pedagogical introduction into flavour-changing neutral current interactions of kaons and their role to reveal or constrain physics beyond the Standard Model (SM). Then I discuss the measure ϵ'_K of direct CP violation in $K \to \pi\pi$ decays, which deviates from the SM prediction by 2.8σ. A supersymmetric scenario with flavour mixing among left-handed squarks can accomodate the measured value of ϵ'_K even for very heavy sparticles, outside the reach of the LHC. The considered scenario employs mass splittings among the right-handed up and down squarks (to enhance ϵ'_K) and a gluino which is heavier than the left-handed strange-down mixed squarks by at least a factor of 1.5 (to suppress excessive contribution to ϵ_K, the measure of indirect CP violation). The branching ratios of the rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, to be measured by the NA62 and KOTO-step2 experiments, respectively, are only moderately affected. These measurements have the capability to either falsify the model or to constrain the CP phase associated with strange-down squark mixing accurately.

1 Basics

The gauge interactions of the Standard Model (SM) are *flavour blind*, meaning that they treat all three fermion generations equally. The Yukawa interaction between fermions and Higgs field, however, involves different coupling strengths of the Higgs field to fermions of different generations, encoded in the complex 3×3 Yukawa matrices. The products of these matrices and the vacuum expectation value of the Higgs field constitute the fermion mass matrices. The diagonalisation of the mass matrices involves unitary rotation of the fermion fields which cancel everywhere except in the couplings of the W boson to fermions. The unitary matrix appearing in the charged-current W couplings to quarks of different generations is the Cabibbo-Kobayashi-Maskawa (CKM) matrix V.

W couplings necessarily change the fermion electric charge by one unit. Thus flavour-changing neutral current (FCNC) transitions are forbidden at tree-level in the SM and only proceed through loops. Decays of charged ($K^+ \sim \bar{s}u$) and neutral ($K^0 \sim \bar{s}d$) kaons can be used to study the $s \to d$ FCNC transitions. Here the FCNC transition of interest is either the decay amplitude ($|\Delta S| = 1$ transition where S means *strangeness*) or the $K-\overline{K}$ mixing amplitude, a

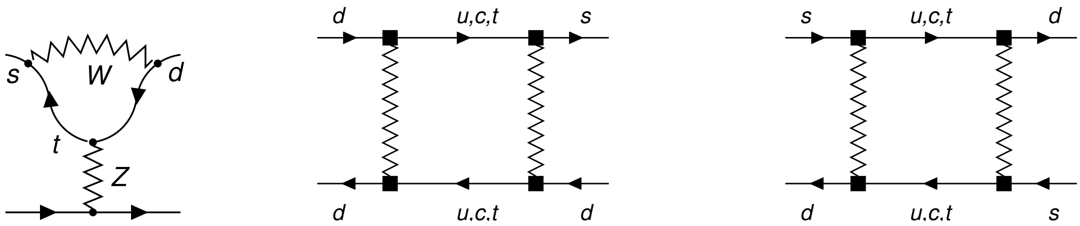

Figure 1 – Left and middle: sample diagrams of electroweak $\Delta S = 1$ penguins and boxes which contribute to the low-scale value of the Wilson coefficient of Q_8 (through renormalisation group evolution from other electroweak coefficients). Right: $\Delta S = 2$ box diagram of $K-\overline{K}$ mixing.

$|\Delta S| = 2$ process (see Fig. 1 for sample diagrams). $K-\overline{K}$ mixing implies that $K^0 \sim \bar{s}d$ and $\overline{K}^0 \sim s\bar{d}$ are not the physical eigenstates of neutral kaons. Instead, these are K_L and K_S, the long-lived and short-lived neutral kaons, which are linear combinations of the flavour eigenstates K^0 and $\overline{K}^0$.

Within the SM, FCNC transitions of kaons are not only loop suppressed: The CKM-favoured $s \to d$ transitions proportional to $V_{us}V_{ud}^*$ and $V_{cs}V_{cd}^*$ involve an up and a charm quark on the internal line, respectively. These two contributions almost cancel each other perfectly, owing to $V_{us}V_{ud}^* \approx -V_{cs}V_{cd}^*$ and the smallness of $m_c^2 - m_u^2$ compared to M_W^2, rendering the up and charm loop integral almost equal in size. This feature, the *Glashow-Iliopoulos-Maiani mechanism*, makes kaon FCNC processes sensitive to virtual effects of the heavy top quark and of potential new particles predicted by theories beyond the SM. To this end charge-parity (CP) violating observables and branching ratios of rare decays are of key importance.

If CP were a good symmetry, the neutral kaon mass eigenstates $|K_L\rangle$ and $|K_S\rangle$ should coincide with the CP eigenstates $(|K^0\rangle \pm |\overline{K}^0\rangle)/\sqrt{2}$. The two-pion states $|\pi^+\pi^-\rangle$ and $|\pi^0\pi^0\rangle$ are CP even and indeed K_S mesons predominantly decay into two pions, while K_L prefers three-pion decays, suggesting that K_S and K_L are CP-even and CP-odd, respectively. But the 1964 discovery of $K_L \to \pi\pi$ decays has shown that K_L is not a CP eigenstate and therefore established CP violation [1]. Today we know that the discovered effect is related to the top quark in the box diagram in Fig. 1. That is, the decay of particle with a mass of 0.5 GeV revealed the virtual effect of a particle which is roughly 350 times heavier, which testifies to the tremendous discovery potential of kaon FCNCs!

The strong interaction poses the main challenge for theoretical predictions of kaon decays. Short-distance QCD effects can be calculated in perturbation theory, from e.g. Feynman diagrams with gluons dressing the diagrams of Fig. 1. To separate these from the non-perturbative long-distance QCD effects, one sets up an effective hamiltonian which reads

$$\mathcal{H}_{\text{eff}}^{|\Delta S|=1} = \frac{G_F}{\sqrt{2}} \lambda_u \sum_{i=1}^{10} Q_i(\mu)\,(z_i(\mu) + \tau y_i(\mu)) + \text{H.c.} \tag{1}$$

for the case of $K \to \pi\pi$ decays. Here $\lambda_u = V_{us}^* V_{ud}$ and $\tau = -V_{ts}^* V_{td}/\,(V_{us}^* V_{ud})$ and G_F is the Fermi constant. Pictorially, the operators Q_i are found by contracting the W lines and loops in the Feynman diagrams to a point; they describe effective four-quark interactions. The operator basis comprises ten operators which are defined in Ref. [2]. In the following we will discuss

$$Q_6 = \bar{s}^j \gamma_\mu (1-\gamma_5) d^k \sum_q \bar{q}^k \gamma^\mu (1+\gamma_5) q^j \quad \text{and} \quad Q_8 = \frac{3}{2} \bar{s}^j \gamma_\mu (1-\gamma_5) d^k \sum_q e_q \bar{q}^k \gamma^\mu (1+\gamma_5) q^j \tag{2}$$

with colour indices j, k and e_q being the charge of quark q. The Wilson coefficients z_i and y_i in Eq. (1) are calculated from the Feynman diagrams and comprise the short-distance physics, in particular they depend on the masses of the heavy W, top and potential new-physics particles. The coefficients depend on the renormalisation scale, the diagrams in Fig. 1 determine the values of the coefficients at the high scale of order of the W or top mass. Their values at the low scale around 1 GeV relevant for kaon physics are found by solving the renormalisation group equations. Fig. 1 shows diagrams contributing to $y_{7,9}$ at the high scale. As a result of the renormalisation group evolution, the low-scale values of the y_i are linear combinations of the high-scale values, which renders the coeffient of interest, y_8, non-zero and important for ϵ'_K. The Wilson coefficients are known to next-to-leading order in the strong coupling constant α_s [3].

The effective $\Delta S = 2$ hamiltonian needed to describe $K-\overline{K}$ mixing is much simpler and contains only one operator $\bar{s}\gamma_\mu(1-\gamma_5)d\,\bar{s}\gamma^\mu(1-\gamma_5)d$.

In order to predict $K \to \pi\pi$ decay amplitudes one must calculate the hadronic matrix elements $\langle\pi\pi|Q_i|K\rangle$ with non-perturbative methods such as lattice gauge theory. To this end it is advantageous to switch to eigenstates of the strong isospin I:

$$|\pi^0\pi^0\rangle = \sqrt{\frac{1}{3}}\,|\,(\pi\pi)_{I=0}\rangle - \sqrt{\frac{2}{3}}\,|\,(\pi\pi)_{I=2}\rangle, \qquad |\pi^+\pi^-\rangle = \sqrt{\frac{2}{3}}\,|\,(\pi\pi)_{I=0}\rangle + \sqrt{\frac{1}{3}}\,|\,(\pi\pi)_{I=2}\rangle.$$

(Bose symmetry forbids the $I=1$ state.) Strong isospin is an excellent symmetry of QCD and was exact, if up and down quark had the same mass.

In this talk I present new analyses of ϵ'_K in the SM and its minimal supersymmetric extension (MSSM) from the papers [4,5,6]. This research was stimulated by a breakthrough of the RBC and UKQCD collaborations in the calculation of the matrix elements $\langle(\pi\pi)_{I=0}\,|Q_i|K^0\rangle$ with lattice gauge theory [7]. The results of Ref. [6] are further covered by Teppei Kitahara's talk at this conference [8].

2 Indirect and direct CP violation

We express the CP-violating quantities of interest in terms of the decay amplitudes $A(K_{L,S} \to (\pi\pi)_{I=0,2})$. Indirect CP violation (stemming from the $\Delta S = 2$ box diagrams) is quantified by

$$\epsilon_K \equiv \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} = (2.228 \pm 0.011)\cdot 10^{-3} \cdot e^{i(0.97\pm0.02)\pi/4}. \tag{3}$$

The measure of direct CP violation, which originates from the $\Delta S = 1$ kaon decay amplitude and is calculated from $\mathcal{H}^{|\Delta S|=1}_{\text{eff}}$ in Eq. (1) is[a]

$$\epsilon'_K \simeq \frac{\epsilon_K}{\sqrt{2}}\left[\frac{A(K_L \to (\pi\pi)_{I=2})}{A(K_L \to (\pi\pi)_{I=0})} - \frac{A(K_S \to (\pi\pi)_{I=2})}{A(K_S \to (\pi\pi)_{I=0})}\right] = (16.6 \pm 2.3)\cdot 10^{-4}\cdot \epsilon_K. \tag{4}$$

This experimental result was established in 1999 and constituted the first measurement of direct CP violation in any decay [9]. In kaon physics one usually adopts the standard phase convention of the CKM matrix, so that the dominant tree-level $\bar{s} \to \bar{d}u\bar{u}$ amplitude proportional to $V_{ud}V^*_{us}$ is real. CP violation requires the interference of this amplitude with another amplitude having a different weak phase, which in the SM stems from another combination of CKM elements. Diagrams with internal top quark like the penguin diagram shown in Fig. 1 are proportional to $V_{td}V^*_{ts}$ and charm loops contribute to both CKM structures, owing to CKM unitarity, $V_{cd}V^*_{cs} = -V_{td}V^*_{ts} - V_{ud}V^*_{us}$. Thus SM predictions for CP-violating quantities in $K \to \pi\pi$ decays all involve

$$\tau = -\frac{V_{td}V^*_{ts}}{V_{ud}V^*_{us}} \sim (1.5 - i0.6)\cdot 10^{-3}. \tag{5}$$

But ϵ'_K is not only suppressed by the smallness of $\operatorname{Im}\tau$, strong interaction effects also render the amplitudes in the numerators in Eq. (4) much smaller than the denominators. Experimentally one finds for $A_I \equiv A(K^0 \to (\pi\pi)_I)$:[b]

$$\operatorname{Re}A_0 = (3.3201 \pm 0.0018)\times 10^{-7}\ \text{GeV}, \qquad \operatorname{Re}A_2 = (1.4787 \pm 0.0031)\times 10^{-8}\ \text{GeV}, \tag{6}$$

[a] Accidentally, ϵ'_K/ϵ_K is essentially real.

[b] By convention the CP-conserving, strong phase δ_I is factored out, so that $\langle(\pi\pi)_I|\mathcal{H}^{|\Delta S|=1}_{\text{eff}}|K^0\rangle \equiv A_I \exp(i\delta_I)$. The phase δ_I is generated by final-state rescattering of the pions. A non-zero $\operatorname{Im}A_I$ therefore only arises from the non-zero CP phase $\arg\tau$.

with a ratio $\mathrm{Re}\,A_0/\mathrm{Re}\,A_2 \approx 22$! This feature is called $\Delta I = 1/2$ *rule*, because I changes by half a unit in $K_{L,S} \to (\pi\pi)_{I=0}$.

The master equation for ϵ'_K/ϵ_K (see e.g. Ref. [10]) reads:

$$\frac{\epsilon'_K}{\epsilon_K} = \frac{\omega_+}{\sqrt{2}|\epsilon_K^{\rm exp}|\mathrm{Re}\,A_0^{\rm exp}} \left\{ \frac{\mathrm{Im}\,A_2}{\omega_+} - \left(1 - \hat{\Omega}_{\rm eff}\right) \mathrm{Im}\,A_0 \right\}. \tag{7}$$

Here $\omega_+ \simeq \frac{\mathrm{Re}\,A_2}{\mathrm{Re}\,A_0} = (4.53 \pm 0.02) \cdot 10^{-2}$ is determined from the charged counterparts of $\mathrm{Re}\,A_{0,2}$ and $\hat{\Omega}_{\rm eff} = (14.8 \pm 8.0) \cdot 10^{-2}$ quantifies isospin breaking. One also takes $|\epsilon_K^{\rm exp}|$ and $\mathrm{Re}\,A_0^{\rm exp}$ from experiment, as indicated. The theoretical challenge is the calculation of $\mathrm{Im}\,A_{0,2}$ with non-perturbative methods. Within the SM $\mathrm{Im}\,A_0$ is dominated by gluon penguins, with roughly 2/3 stemming from the matrix element $\langle(\pi\pi)_{I=0}|Q_6|K^0\rangle$ (with the operator Q_6 of Eq. (2)), while about 3/4 of the contribution to $\mathrm{Im}\,A_2$ stems from $\langle(\pi\pi)_{I=2}|Q_8|K^0\rangle$. Lattice-gauge theory has $\langle(\pi\pi)_{I=2}|Q_8|K^0\rangle$ (and thereby $\mathrm{Im}\,A_2$) under good control for some time [11], while reliable lattice calculations of $\langle(\pi\pi)_{I=0}|Q_6|K^0\rangle$ have become possible only recently [7]. Using these matrix elements from lattice QCD we find [4]

$$\frac{\epsilon'_K}{\epsilon_K} = (1.06 \pm 4.66_{\rm Lattice} \pm 1.91_{\rm NNLO} \pm 0.59_{\rm IV} \pm 0.23_{m_t}) \times 10^{-4}, \tag{8}$$

a value which is $2.8\,\sigma$ below the experimental result in Eq. (4). The various sources of errors are indicated by the subscripts: The largest uncertainty stems from the hadronic matrix elements calculated with lattice QCD. The next error is the perturbative uncertainty from the unknown next-to-next-to-leading (NNLO) QCD corrections. "IV" denotes strong-isospin violation (stemming e.g. from $m_u \neq m_d$) and the last error comes from the error in m_t.

This result, obtained with a novel compact solution of the renormalization group equations, agrees with the one in Ref. [10]. The quoted lattice results are consistent with earlier analytic calculations in the large-N_c "dual QCD" approach [12]. Thus lattice gauge theory is currently starting to resolve a long-standing controversy about $\mathrm{Im}\,A_0$ between the large-N_c [12] and chiral perturbation theory [13] communities. While the latter method can reproduce the large-N_c values, it can likewise easily accomodate the experimental range in Eq. (4).

3 $\epsilon'_{\mathbf{K}}$ in the MSSM

The large factor $1/\omega_+$ multiplying $\mathrm{Im}\,A_2$ in Eq. (7) renders ϵ'_K/ϵ_K especially sensitive to new physics in the $\Delta I = 3/2$ decay $K \to (\pi\pi)_{I=2}$. This feature makes ϵ'_K/ϵ_K special among all FCNC processes. However, it is difficult to place a large effect into ϵ'_K without overshooting ϵ_K: The SM contributions to both quantities depend on the CKM combination τ in Eq. (5) as

$$\epsilon_K'^{\rm SM} \propto \mathrm{Im}\,\tau \qquad \text{and} \qquad \epsilon_K^{\rm NP} \propto \mathrm{Im}\,\tau^2. \tag{9}$$

In new-physics scenarios τ is replaced by some new $\Delta S = 1$ parameter δ and the new-physics contributions scale as

$$\epsilon_K'^{\rm NP} \propto \mathrm{Im}\,\delta \qquad \text{and} \qquad \epsilon_K^{\rm NP} \propto \mathrm{Im}\,\delta^2. \tag{10}$$

If new-physics enters through a loop with super-heavy particles, the only chance to have a detectable effect in ϵ'_K is a scenario with $|\delta| \gg |\tau|$. Thus if $\epsilon_K'^{\rm NP} \sim \epsilon_K'^{\rm SM}$ one expects $\epsilon_K^{\rm NP} \gg \epsilon_K^{\rm SM}$, in contradiction with the experimental value. Thus large effects in ϵ'_K from loop-induced new physics are seemingly forbidden. Many studies of ϵ'_K indeed involve new-physics scenarios with tree-level contributions to ϵ'_K [14], in which the requirement $|\delta| \gg |\tau|$ can be relaxed.

The MSSM has the required ingredients to explain ϵ'_K in Eq. (4) without conflict with ϵ_K despite $\delta \gg \tau$. Moreover, this is possible with squark and gluino masses in the range 3–7 TeV, far above the reach of the LHC. The enhancement of ϵ'_K is achieved with "Trojan penguin"

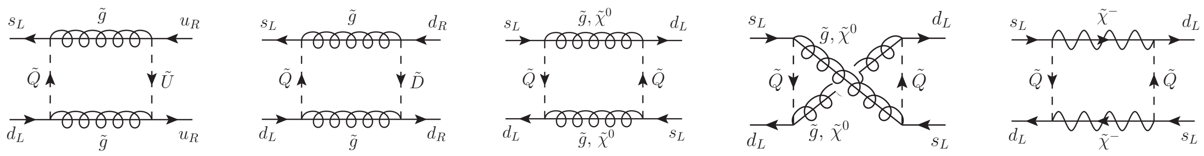

Figure 2 – Left two diagram: “Trojan penguin” box diagrams [15]. The difference of the two boxes contributes to the $\mathrm{Im}\, A_2$ and increases with the mass difference between right-handed up-type ($\tilde{U}$) and down-type ($\tilde{D}$) squark. $\tilde{Q}$ denotes a left-handed squark, which is a strange-down mixture. Right three diagrams: MSSM contribution to ϵ_K. The (second to last) crossed-box gluino diagram cancels the middle gluino diagram very efficiently for $m_{\tilde{g}} \geq 1.5 m_{\tilde{Q}}$ [16] and the chargino diagram to the right and the boxes with one or two neutralinos ($\tilde{\chi}^0$) become important.

box diagrams [15], which contribute to $\mathrm{Im}\, A_2$ through the strong interaction, as shown in Fig. 2. This mechanism involves a mass splitting among the right-handed up- and down squark and flavour mixing among the left-handed down and strange squark. The FCNC parameter is the (1,2) element Δ_{ds}^{LL} of the left-handed squark mass matrix, the CP phase is $\theta \equiv \arg(\Delta_{sd}^{LL})$. The suppression of ϵ_K in this MSSM scenario exploits the mechanism of Ref. [16]: For $m_{\tilde{g}} \simeq 1.5 m_{\tilde{Q}}$ the two gluino boxes in Fig. 2 cancel and for $m_{\tilde{g}} > 1.5 m_{\tilde{Q}}$ decoupling sets in quickly, so that the MSSM contribution to ϵ_K stays small. In Fig. 3 the region of sparticle masses capable to explain ϵ'_K is shown for the choice $\theta = -45^\circ$. This phase maximises the MSSM contribution to ϵ_K (proportional to $\sin(2\theta)$), so that it is clear that the suppression of $|\epsilon_K|$ is not caused by a tuning of θ.

4 $\mathbf{K} \to \pi\nu\bar{\nu}$

The two decay modes $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ are remarkable in two respects: On one hand their branching ratios can be predicted with high precision, because all hadronic effects are under good control. On the other hand the two branching ratios are highly sensitive to new physics. The SM predictions of the branching ratios are[17]:

$$\begin{aligned} \mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\rm SM} &= (2.9 \pm 0.2 \pm 0.0) \times 10^{-11}\,, \\ \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} &= (8.3 \pm 0.3 \pm 0.3) \times 10^{-11}, \end{aligned} \qquad (11)$$

where the first error stems from the CKM elements and the second error summarises the remaining uncertainties. The experiment NA62 at CERN will probe $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ at the 10% level already in 2018 [18]. KOTO at J-PARC will, in a first step, probe $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ at the level around the SM sensitivity [19]. Later KOTO-step2 aims at a measurement with an error of 10% as well [20].

In the SM the decay $s \to d\nu\bar{\nu}$ triggering $K \to \pi\nu\bar{\nu}$ proceeds through Z penguin and box diagrams similar to those constituing ϵ'_K. It is therefore natural to ask, whether the new physics which may contribute to ϵ'_K in Eq. (4) will also affect $\mathcal{B}(K \to \pi\nu\bar{\nu})$ and whether the measurements of these branching fractions will help to distinguish among different new-physics models. Such correlations typically appear in models with Z' bosons or modified Z couplings [21]. In our MSSM scenario there is also a strong correlation between ϵ'_K and $\mathcal{B}(K \to \pi\nu\bar{\nu})$. However, this correlation does not involve $Z^{(\prime)}$ penguins but instead the box diagrams of Figs. 2 and 4. This diagram is further correlated to the neutralino (middle and second-to-right) and chargino (right) diagrams in Fig. 2. This correlation now links sizable enhancements of the $K \to \pi\nu\bar{\nu}$ branching ratios to excessive effects on ϵ_K, unless one either fine-tunes $m_{\tilde{g}}$ to cancel the sum of the two gluino boxes with the chargino box in Fig. 2 or tunes the CP phase θ to values close to $\pm 90^\circ$ [c].

[c] The MSSM contribution to ϵ_K also vanishes for $\theta \approx 0, 180^\circ$, but then ϵ'_K cannot be explained.

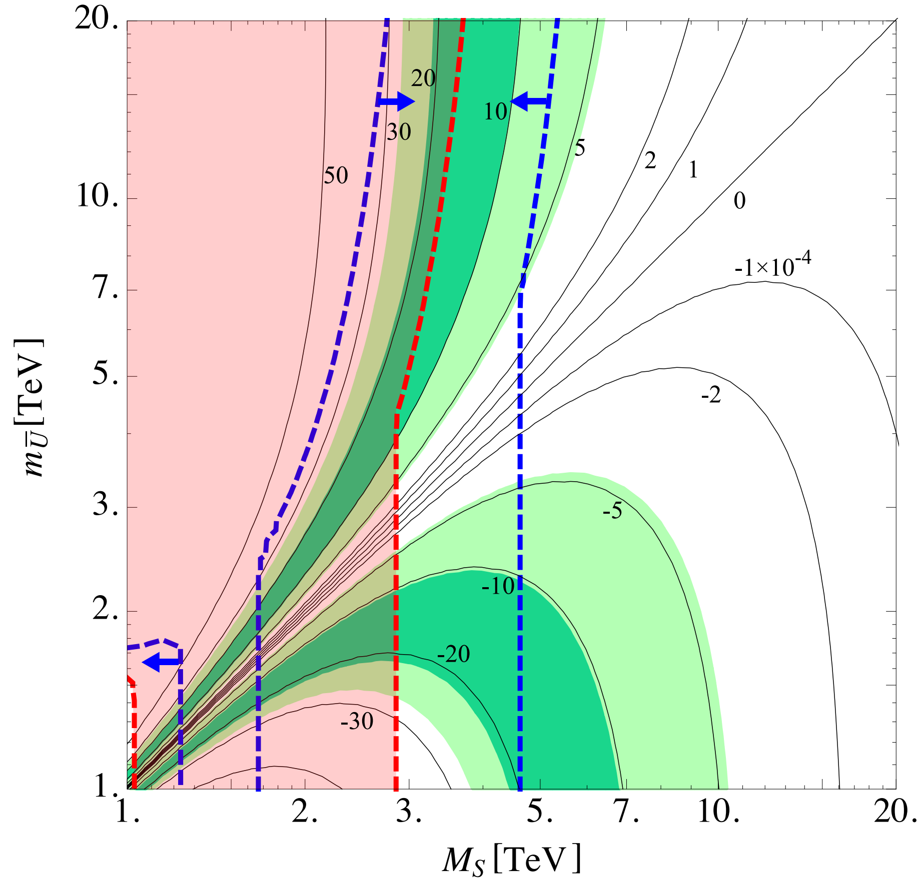

Figure 3 – Parameter region explaining ϵ'_K/ϵ_K while complying with the measured ϵ_K for the point $m_{\tilde{g}} = 1.5 M_S$ and $M_S = m_{\tilde{Q}} = m_{\tilde{D}}$. M_S is a common mass of all superpartners except the gluino and right-handed up squark, labeling the y-axis. The lines labeled with negative values of the MSSM contribution $\epsilon'^{SUSY}_K/\epsilon_K$ correspond to correct (positive) solutions if the CP phase θ is changed from $-45°$ to $135°$. The SM prediction for ϵ_K strongly depends on $|V_{cb}|$. The blue (red) lines in both plots delimit the region which complies with ϵ_K if $|V_{cb}|$ is determined from exclusive (inclusive) $b \to c\ell\nu$ decays. If the exclusive determination is correct, some new physics in ϵ_K is welcome. In the inclusive case the forbidden region is marked with the red shading. For more details see Ref. [5], from which the plots are taken.

If both (fine-)tunings are restricted to levels below 10%, one finds

$$\frac{\mathcal{B}(K_L \to \pi^0 \nu\overline{\nu})}{\mathcal{B}^{\rm SM}(K_L \to \pi^0 \nu\overline{\nu})} \lesssim 1.2 \qquad \text{and} \qquad \frac{\mathcal{B}(K^+ \to \pi^+ \nu\overline{\nu})}{\mathcal{B}^{\rm SM}(K^+ \to \pi^+ \nu\overline{\nu})} \lesssim 1.1, \tag{12}$$

if GUT relations between the gaugino masses are assumed. Thus the considered MSSM scenario is very predictive and forbids large effects on $\mathcal{B}(K \to \pi\nu\overline{\nu})$, although KOTO-step2 may detect the deviation from the SM prediction. While it is unlikely that Nature has fine-tuned the gluino mass to minimise the impact on ϵ_K, the possibility that accidentally θ is close to $\pm 90°$ should not be discarded. In this case larger enhancements than in Eq. (12) are possible, with the generic strong correlation between $\mathcal{B}(K^+ \to \pi^+ \nu\overline{\nu})$ and $\mathcal{B}(K_L \to \pi^0 \nu\overline{\nu})$ found in [22], which holds for any model with flavour mixing only among left-handed quarks. (For detailed plots see Ref. [6].) Thus also the scenario with $\theta \approx \pm 90°$ is falsifiable by combining NA62 and KOTO-step2 data. If, however, the two experiments both find substantial enhancement following the pattern of Ref. [22], the CP phase θ will be accurately pinned down to a value near $\pm 90°$.

An interesting prediction of our MSSM scenario is a strict correlation between $\mathcal{B}(K_L \to \pi^0 \nu\overline{\nu})$ and the hierarchy between the masses $m_{\bar{U}}$, $m_{\bar{D}}$ of the right-handed up-squark and down-squark:

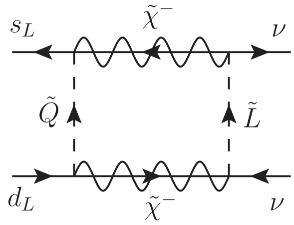

Figure 4 – Dominant contribution to $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ in the MSSM scenario of Refs.[5,6]. $\widetilde{L}$ denotes a charged slepton. Neutralino diagrams sum to a smaller contribution.

The (positive) sign of the MSSM contribution to ϵ'_K implies

$$\text{sgn}\left[\mathcal{B}(K_L \to \pi^0 \nu\overline{\nu}) - \mathcal{B}^{\text{SM}}(K_L \to \pi^0 \nu\overline{\nu})\right] = \text{sgn}\,(m_{\bar{U}} - m_{\bar{D}}).$$

Thus a precise measurement of $\mathcal{B}(K_L \to \pi^0 \nu\overline{\nu})$ will tell whether the right-handed up squark is heavier or lighter than the right-handed down squark.

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