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Three-loop quark form factor at high energy: the leading mass corrections

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Abstract

We compute the leading mass corrections to the high-energy behavior of the massive quark vector form factor to three loops in QCD in the double-logarithmic approximation.

Keywords: QCD perturbation theory, asymptotic expansion, form factor

The vector form factor of a quark is a crucial building block in the perturbative analysis of many processes in quantum chromodynamics. It is also the simplest scattering amplitude which can be used to study the infrared structure of perturbative QCD. The form factors of a massless quark have been evaluated through the three-loop approximation [1, 2] and even to four loops in the leading-color approximation [3]. For a massive quark however only the two-loop result is available so far [4, 5]. The complete calculation of the three-loop corrections is quite a challenging problem for the existing computational techniques. Only recently the leading-color contribution of the planar three-loop Feynman diagrams has been found analytically in terms of Goncharov polylogarithms retaining the full dependence on the quark mass m_q [6]. At the same time the full mass dependence is often excessive for practical applications and proper expansion of the result in a given kinematical region could be sufficient (see *e.g.* [7, 8, 9, 10, 11]). In particular, in the high-energy limit the corrections to the form factor can be expanded in a small ratio $\rho = m_q^2/Q^2$, where Q_μ is the large momentum transfer. The resulting series is asymptotic with the coefficients dominated by the double-logarithmic contribution enhanced by the second power of the large logarithm $\ln\rho$ per each power of the strong coupling constant α_s. In the leading order of the small-mass expansion the origin and structure of the "Sudakov" double logarithms have been established long time ago [12, 13]. The analysis has been subsequently generalized to subleading logarithms [14, 15, 16] and the leading-power result for the massive quark form factor is currently known through three loops up to the $\mathcal{O}(\alpha_s^3)$ nonlogarithmic contribution, which is only available in the leading-color approximation (see [17] and references therein). By contrast, the logarithmic structure of the power suppressed terms is not well understood and currently is under study in various contexts [18, 19, 20]. In particular, the leading power corrections to the form factor in QED have been recently evaluated in the double-logarithmic approximation to all orders in the coupling constant [18]. The result determines the abelian part of the corrections to the quark form factor. In the present paper we complete the analysis of the three-loop contribution by evaluating its nonabelian part and derive the $\mathcal{O}(\rho\ln^6\!\rho\,\alpha_s^3)$ correction to the form factor in QCD.

The amplitude $\mathcal{F}$ of the quark scattering in an external singlet vector field can be parametrized in the standard way by the Dirac and Pauli form factors

$$\mathcal{F} = \bar{q}(p_2)\left(\gamma_\mu F_1 + \frac{i\sigma_{\mu\nu}Q^\nu}{2m_q}F_2\right)q(p_1)\,. \tag{1}$$

The Pauli form factor F_2 does not contribute in the approximation discussed in this paper and we focus on the high-energy behavior of the Dirac form factor F_1. We consider the on-shell quark $p_1^2 = p_2^2 = m_q^2$ and the large Euclidean momentum transfer $Q^2 = -(p_2-p_1)^2$ corresponding to positive values of the parameter ρ. The asymptotic expansion of the Dirac form factor can be written as follows

$$F_1 = S_\varepsilon \sum_{n=0}^{\infty} \rho^n F_1^{(n)}\,, \tag{2}$$

where $F_1^{(n)}$ are given by the power series in α_s with the coefficients depending on ρ

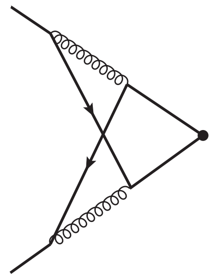

Figure 1: The two-loop diagram generating the $\mathcal{O}(\rho)$ double-logarithmic contribution. The blob stands for the color singlet vector current.

only logarithmically. The factor

$$S_\varepsilon = \exp\left[-\frac{\alpha_s}{2\pi}\frac{\Gamma^{(1)}}{\varepsilon}\right] \tag{3}$$

accounts for the singular dependence on the parameter of the dimensional regularization $d = 4 - 2\varepsilon$ used to treat the infrared divergences of the amplitude. Here $\Gamma^{(1)}$ is the one-loop cusp anomalous dimension. In the high-energy limit $\rho \to 0$ it reads [21]

$$\Gamma^{(1)} = C_F \ln\rho\left(1 + \mathcal{O}(\rho^2)\right), \tag{4}$$

where $C_F = \frac{N_c^2-1}{2N_c}$, $N_c = 3$. In the double-logarithmic approximation the leading term is given by the Sudakov exponent [12, 13]

$$F_1^{(0)} = e^{-C_F x}, \tag{5}$$

where

$$x = \frac{\alpha_s}{4\pi}\ln^2\rho \tag{6}$$

is the double-logarithmic variable. The goal of this paper is to compute the leading power correction coefficient $F_1^{(1)}$ to $\mathcal{O}(x^3)$. The origin of the $\mathcal{O}(\rho)$ double-logarithmic corrections is quite peculiar. They are induced by the emission of soft virtual fermions rather than gauge bosons responsible for the Sudakov logarithms [18, 20]. The mass suppression factor in this case comes from the helicity flip term in the soft fermion propagator, which effectively becomes scalar and is sufficiently singular at small momentum to develop the double-logarithmic contribution. In the case of the form factor the $\mathcal{O}(\rho)$ double-logarithmic contribution is associated with the soft scalar quark pair exchange and appears first in the two-loop nonplanar vertex diagram, Fig. 1 [18]. The higher-order double-logarithmic corrections are obtained by dressing this diagram with extra soft gluons. The relevant three-loop diagrams are given in Fig. 2.

Let us briefly describe how the diagrams are evaluated in the double-logarithmic approximation [18, 19, 20]. Since two soft quark propagators provide the explicit mass suppression factor, the double logarithmic asymptotic of the integral over the virtual momenta can be obtained by the technique originally applied to the analysis of the leading-power term [12]. To introduce the main idea of the method we consider the evaluation of the two-loop diagram, Fig. 1. The double-logarithmic contribution originates from the momentum configuration when the large external momenta flow through the edges of the diagram. In the infrared region all the propagators with the external momenta are eikonal and the edges of the diagram effectively turn into the light-cone Wilson lines. At the same time the momenta l_i of the exchanged quark pair are soft and the corresponding propagators in the infrared region become scalar. The effective Feynman rules for this momentum region, which retain the leading infrared behavior of the full theory, are given in [20]. To separate the double-logarithmic contribution the Sudakov parametrization $l_i = u_i p_1 + v_i p_2 + l_{i\perp}$ is used for each virtual soft quark momentum. The integration over the transverse components $l_{i\perp}$ is performed by taking the residues of the soft propagators. In general the resulting expression has double-logarithmic scaling when $u_i,\ v_i \ll 1$ and the Sudakov parameters are ordered along the Wilson lines. For the nonplanar diagram under consideration this condition reads $v_2 \ll v_1 \ll 1$, $u_1 \ll u_2 \ll 1$. An additional constraint $\rho \ll u_i v_i$ ensures that the soft quark propagators can go on-shell. This condition also suggests that $\rho \ll u_i,\ v_i$, which sets the infrared cutoff on the integral over the Sudakov parameters. Thus the quark mass regulates both collinear and soft divergences and the result for the diagram is infrared finite. In this way the two-loop contribution can be reduced to the following expression [18][1]

$$F_1^{(1,2l)} \;=\; 2\,(C_A - 2C_F)\,x^2 \int K(\eta_1,\eta_2,\xi_1,\xi_2)\mathrm{d}\eta_1\mathrm{d}\eta_2\mathrm{d}\xi_1\mathrm{d}\xi_2\,, \tag{7}$$

where $C_A = N_c$, $\eta_i = \ln v_i/\ln\rho$ and $\xi_i = \ln u_i/\ln\rho$ are the normalized logarithmic integration variables, the integration goes over the four-dimensional cube $0 < \eta_i,\ \xi_i < 1$, and the kernel

$$K(\eta_1,\eta_2,\xi_1,\xi_2) \;=\; \theta(1-\eta_1-\xi_1)\theta(1-\eta_2-\xi_2)\theta(\eta_2-\eta_1)\theta(\xi_1-\xi_2) \tag{8}$$

selects the kinematically allowed region of double-logarithmic integration discussed above. After integrating Eq. (7) one gets

$$F_1^{(1,2l)} = \frac{C_F\,(C_A - 2C_F)}{6}x^2\,, \tag{9}$$

in agreement with [4]. The three-loop correction can be represented as a sum over the contribution of the diagrams in Fig. 2

$$F_1^{(1,3l)} = \frac{C_F\,(C_A - 2C_F)}{2}\sum_\lambda c_\lambda d_\lambda\, x^3\,, \tag{10}$$

where the diagrams (d)-(i) with a symmetric counterpart should be counted twice. Here c_λ stands for a reduced color factor and the three-loop integrals are converted

[1] The detailed derivation can be found in Ref. [20] in the context of two-loop analysis of Bhabha scattering. The relevant contribution is proportional to the integral I_1 in the Appendix A.

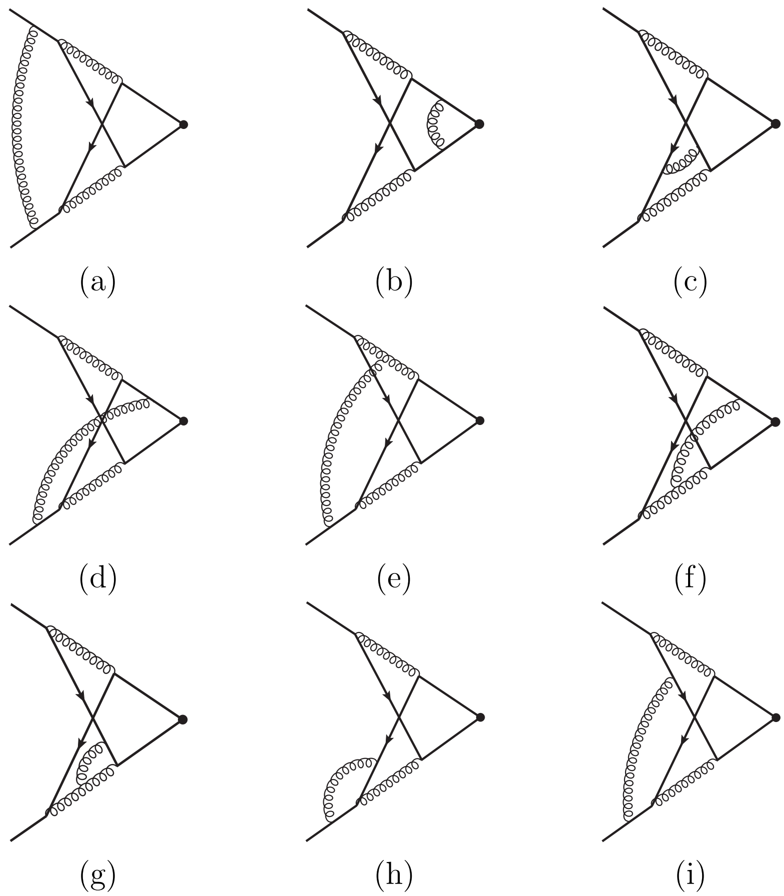

Figure 2: The three-loop diagrams contributing to the $\mathcal{O}(\rho)$ double-logarithmic corrections. Symmetric diagrams are not shown. The remaining diagrams either do not have the double-logarithmic integration region or have vanishing color factor.

into the following form

$$d_\lambda = 4 \int w_\lambda(\eta, \xi) K(\eta_1, \eta_2, \xi_1, \xi_2) \mathrm{d}\eta_1 \mathrm{d}\eta_2 \mathrm{d}\xi_1 \mathrm{d}\xi_2 \,, \tag{11}$$

where w_λ is the weight function resulting from the double-logarithmic integration over the soft gluon momentum. The results for w_λ, d_λ, and c_λ are listed in Table 1. Examples of the calculation of the functions w_λ are given in the Appendix A. Note that the diagram Fig. 2 (a) has an infrared divergent contribution which reproduces the factorized singular structure of Eq. (2) and is not included into Eq. (10).

Collecting the contributions of the individual diagrams we get

$$F_1^{(1,3l)} = \frac{8C_F^3 - 2C_A C_F^2 - C_A^2 C_F}{30} x^3 \tag{12}$$

and

$$F_1^{(1)} = \frac{C_F\,(C_A - 2C_F)}{6} x^2 \left[1 - \frac{C_A + 4C_F}{5} x + \mathcal{O}(x^2)\right]. \tag{13}$$

λ	w_λ	d_λ	c_λ
a	$-((\eta_2+2)\eta_2+(\xi_1-2\eta_2+2)\xi_1-1)$	$-\frac{17}{45}$	$-C_F$
b	$2\xi_2\eta_1$	$\frac{1}{45}$	$-C_F$
c	$2(\xi_1-\xi_2)(\eta_2-\eta_1)$	$\frac{1}{15}$	C_A-C_F
d	$-\eta_1(\eta_1-2\xi_1+2)$	$-\frac{1}{10}$	C_A-C_F
e	$(\eta_2-\eta_1)(2-2\xi_1+\eta_1+\eta_2)$	$\frac{8}{45}$	$-\frac{C_A}{2}$
f	$2\eta_1(\xi_1-\xi_2)$	$\frac{1}{30}$	$-\frac{C_A}{2}$
g	$2\eta_2(\xi_1-\xi_2)$	$\frac{1}{10}$	$-\frac{C_A}{2}$
h	$\eta_1(\eta_1-2\xi_1+2)$	$\frac{1}{10}$	$\frac{C_A}{2}-C_F$
i	$\eta_2(\eta_2-2\xi_1+2)$	$\frac{5}{18}$	$\frac{C_A}{2}-C_F$

Table 1: The weights w_λ, integrals d_λ, and color factors c_λ of the diagrams in Fig. 2. To obtain w_a the singilar part of the infrared divergent diagram (a) is subtracted as discussed in the Appendix A.

Thus, we have evaluated the dominant power corrections to the tree-loop massive quark vector form factor at high energy. Only the nonplanar diagrams contribute to Eq. (13) and the result has the subleading color factor C_A-2C_F which scales as $1/N_c$ in the large N_c limit. This agrees with the leading-color analysis of Ref. [6], where such term is absent and the $\mathcal{O}(\rho\,\alpha_s^3)$ contribution has at most the fifth power of the large logarithm. Our result can be used as a cross check for the future exact calculation of the three-loop corrections. It can be used also to identify and extend the domain where the high energy approximation [17] is applicable. An interesting and important problem is to extend Eq. (13) to all orders in x. So far all-order resummation of the non-Sudakov double logarithms has been performed only in abelian gauge theories [18, 19, 22]. Generalization of the analysis to the nonabelian case can be crucial in particular for the analysis of the light quark effects in Higgs boson production [19], and our result can be considered as the first step towards this goal.

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Appendix A. Evaluation of the soft gluon momentum integrals

Besides the integration over two soft quark momenta, the three-loop diagrams include an extra integration over the soft gluon momentum. In general, this integration can be performed in the double-logarithmic approximation within the Sudakov method outlined above. However, the analysis of the diagrams with the soft gluon emission from the on-shell external or soft quark lines is more subtle due to soft divergences which are not regulated by the quark mass as in the two-loop case. We describe how this problem is treated for the two typical cases of the diagrams (a) and (h) in Fig. 2.

Fig. 2 (a) is the only diagram with the infrared divergence in the final result. The integration over the soft gluon momentum l_3 in this diagram is double-logarithmic when one can neglect it in the eikonal propagators with the the soft quark momenta $l_{1,2}$. This defines the conditions $l_3p_1 \ll l_2p_1$, $l_3p_2 \ll l_1p_2$ corresponding to the ordering of the Sudakov parameters $v_3 \ll v_2$, $u_3 \ll u_1$. Thus l_3 should be retained only in the propagators without the soft quark momenta and the integral over the soft gluon momentum is reduced to

$$\frac{2iQ^2}{\pi^2}\int \frac{d^4l_3}{l_3^2((p_1+l_3)^2-m_q^2)((p_2+l_3)^2-m_q^2)}\,, \tag{A.1}$$

with the above restriction on l_3 and the prefactor introduced for convenience. In the double-logarithmic approximation the propagators in this expression take the following form

$$\begin{aligned}
&\frac{1}{l_3^2} \approx -i\pi\delta(Q^2u_3v_3+l_{3\perp}^2)\,,\\
&\frac{1}{(p_1+l_3)^2-m_q^2} \approx \frac{1}{Q^2(v_3+2\rho u_3)}\,,\\
&\frac{1}{(p_2+l_3)^2-m_q^2} \approx \frac{1}{Q^2(u_3+2\rho v_3)}\,.
\end{aligned} \tag{A.2}$$

After integrating Eq. (A.1) over $l_{3\perp}$ with the double-logarithmic accuracy we get

$$2\int_{\rho u_3}^{v_2}\frac{\mathrm{d}v_3}{v_3}\int_{\rho v_3}^{u_1}\frac{\mathrm{d}u_3}{u_3}\,. \tag{A.3}$$

Eq. (A.3) has soft divergence when v_3 and u_3 simultaneously become small. This divergence can be removed by subtracting the factorized expression

$$2\int_{\rho u_3}^{1}\frac{\mathrm{d}v_3}{v_3}\int_{\rho v_3}^{1}\frac{\mathrm{d}u_3}{u_3}\,. \tag{A.4}$$

The subtraction term does not depend on the soft quark momenta. It is equivalent to the double-logarithmic approximation of the one-loop correction to the form factor and gives the following contribution to Eq. (2)

$$-\left(\frac{\alpha_s}{2\pi}\frac{\Gamma^{(1)}}{\varepsilon}+C_Fx\right)\rho F_1^{(1,2l)}\,. \tag{A.5}$$

The first term of Eq. (A.5) reproduces the singular $\mathcal{O}(\rho\alpha_s^3)$ part of Eq. (2) while the second term should be included in Eq. (10). The subtracted expression reads

$$\begin{aligned}
&-\;2\left(\int_{v_2}^{1}\frac{\mathrm{d}v_3}{v_3}\int_{\rho v_3}^{u_1}\frac{\mathrm{d}u_3}{u_3}+\int_{\rho u_3}^{v_2}\frac{\mathrm{d}v_3}{v_3}\int_{u_1}^{1}\frac{\mathrm{d}u_3}{u_3}+\int_{v_2}^{1}\frac{\mathrm{d}v_3}{v_3}\int_{u_1}^{1}\frac{\mathrm{d}u_3}{u_3}\right)\\
&=\;-\left(\ln v_2\left(\ln v_2+2\ln\rho\right)+\ln u_1\left(\ln u_1-2\ln v_2+2\ln\rho\right)\right)\,.
\end{aligned} \tag{A.6}$$

After converting to the logarithmic variables the above equation together with the nonsingular term of Eq. (A.5) gives the expression for w_a in Table. 1.

A characteristic feature of the diagram Fig. 2 (h) is that the soft gluon is emitted by a soft quark. In this case the Sudakov parametrization of its virtual momentum should be defined with respect to the corresponding soft quark momentum $l_3 = u_3 l_1 + v_3 p_2 + l_{3\perp}$. As in the two-loop contribution the integration over the transverse component of l_2 is performed by taking the residue of a soft quark propagator pole and there exist two contributions corresponding to the on-shell propagators on either side of the soft gluon emission vertex. When in Fig. 2 (h) the soft quark propagator above the vertex is on the mass shell, the soft gluon momentum has to flow through the quark propagator below the vertex and the integral over l_3 coincides with the one-loop correction to the on-shell form factor with the external momenta p_1 and l_2. Using the same normalization as in Eq. (A.1) it can be written as follows

$$-\frac{2i(p_2-l_1)^2}{\pi^2}\int\frac{d^4l_3}{l_3^2((p_2+l_3)^2-m_q^2)((l_1+l_3)^2-m_q^2)} \tag{A.7}$$

and in the standard way reduces to the integral over the Sudakov parameters

$$2\int_{\rho u_3/u_1}^{1}\frac{\mathrm{d}v_3}{v_3}\int_{\rho v_3/u_1}^{1}\frac{\mathrm{d}u_3}{u_3}\,, \tag{A.8}$$

where we used the relation $(p_2-l_1)^2 \approx -Q^2u_1$. When in Fig. 2 (h) the soft quark propagator below the vertex is on the mass shell, the soft gluon momentum has to flow through the quark propagator above the vertex and instead of Eq. (A.7) one gets

$$-\frac{2i(p_2-l_1)^2}{\pi^2}\int\frac{d^4l_3}{l_3^2((p_2+l_3)^2-m_q^2)((l_1-l_3)^2-m_q^2)}\,, \tag{A.9}$$

with an additional condition $p_1l_3 \ll p_1l_1$ or $v_3 \ll v_1$ on the double-logarithmic integration region. This gives

$$-\,2\int_{\rho u_3/u_1}^{v_1}\frac{\mathrm{d}v_3}{v_3}\int_{\rho v_3/u_1}^{1}\frac{\mathrm{d}u_3}{u_3}\,. \tag{A.10}$$

Both Eq. (A.8) and Eq. (A.10) are infrared divergent. However, their sum

$$2\int_{v_1}^{1}\frac{\mathrm{d}v_3}{v_3}\int_{\rho v_3/u_1}^{1}\frac{\mathrm{d}u_3}{u_3} = \ln v_1\,(\ln v_1 - 2\ln u_1 + 2\ln\rho) \tag{A.11}$$

is finite and after converting to the logarithmic variables coincides with the expression for w_h in Table. 1.

The evaluation of the rest of the diagrams poses no new technical problem and can be performed in the same way.

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