

## QCD corrections to vector boson pair production in gluon fusion at the LHC

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We report on the calculation of the NLO QCD corrections to vector boson pair production in gluon fusion at hadron colliders. We focus in particular on the on-shell production of vector bosons, which is of fundamental importance, among the others, to check the consistency of the electroweak sector of the Standard Model. For similar studies, including the interference of the off-shell prompt  $gg \rightarrow V_1 V_2$  amplitudes with the Higgs production amplitudes  $gg \rightarrow H \rightarrow V_1 V_2$ , we refer to the proceedings of this conference.

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## 1. Introduction and motivation

The production of pairs of electroweak vector bosons at hadron colliders provides a large number of observables for the study of the electroweak symmetry breaking mechanism in the Standard Model and for searches of physics beyond the Standard Model. Experimentally, an increasing effort has been devoted to the study of these processes and a large number of results are already available from the CMS and ATLAS collaborations. The precision of the experimental studies is rapidly increasing and has already reached the percent level for different observables. Keeping up with this impressive effort is a primary goal for the theory community. Providing equally precise theoretical predictions to be compared with the experimental results requires full control on higher order corrections, in particular in QCD.

At the LHC, pairs of electroweak vector bosons  $V_1 V_2$  are produced through two main partonic channels. The most important one is the  $q\bar{q} \rightarrow V_1 V_2$  annihilation channel, which is since recently known in NNLO QCD, both inclusively and differentially [1–8]. The second channel is the gluon fusion channel  $gg \rightarrow V_1 V_2$ . NLO QCD corrections to this channel have been computed for  $\gamma\gamma$  production [9] and, more recently, for ZZ and WW production, both for on-shell and off-shell final states [10–12]. The  $gg$  channel, being loop induced, is suppressed by a power of  $\alpha_s$  and its two-loop corrections contribute to the full partonic process only starting at N<sup>3</sup>LO. Nevertheless, as we will argue in the following, their contribution cannot be neglected if we aim to produce theoretical prediction at the percent level accuracy.

	$\sigma_{NLO}^{q\bar{q}}$ [pb]	$\sigma_{NNLO}^{q\bar{q}}$ [pb]	$\sigma_{LO}^{gg}/(\sigma_{NNLO}^{q\bar{q}} - \sigma_{NLO}^{q\bar{q}})$
ZZ 13 TeV	$14.5 \pm 3\%$	$16.9 \pm 3\%$	$\approx 60\%$
WW 13 TeV	$106.0 \pm 3.5\%$	$118.7 \pm 3\%$	$\approx 35\%$

**Table 1:** Fully inclusive cross-sections for the production of ZZ and WW at the 13 TeV LHC. We use NNLO PDFs,  $\mu_{R,F} = m_Z, m_W$  respectively, variation  $0.5m_V < \mu_{R,F} < 2m_V$

A way to summarize why this is needed is provided in Table 1. Here we see that, at the energy of 13 TeV, the contribution of the LO gluon-fusion channel amounts to about 60% and 35% of the NNLO corrections to the inclusive cross-section for ZZ and WW production, respectively. For ZZ and WW, moreover, the full NNLO corrections ( $q\bar{q}$  and  $gg$  channel) amount to around  $\approx 12\%$  of the total cross section [3, 4]. This implies that, as far as the full NNLO cross-section goes, the LO gluon-fusion channel is expected to contribute around 7% and 4% for ZZ and WW production respectively [10, 11]. Now we should keep in mind that the LO prediction for the gluon-channel are expected to suffer from very large radiative corrections up to  $\mathcal{O}(100\%)$  [13]. This shows that, in order to claim full control on the theoretical uncertainty of the calculation at a few percent level, a precise account of the contribution of the gluon-fusion channel is mandatory.

The calculation of the NLO corrections to ZZ and WW production in gluon fusion is technically challenging and requires the interplay of cutting edge techniques. The calculation requires three distinct ingredients. The two-loop virtual amplitudes for  $gg \rightarrow V_1 V_2$ , the one-loop real-virtual amplitudes for  $gg \rightarrow V_1 V_2 g$ , and finally a subtraction scheme to regulate and cancel IR soft and collinear divergences. On the one hand, the computation of the one-loop amplitudes and the IR

subtractions are conceptually solved problems, at least as long as NLO computations go. We compute the one-loop amplitudes using unitarity methods [14]. In particular, we use numerical unitarity [15] for the cut-constructible part and analytical methods for the rational part [16, 17]. The subtraction of IR poles is performed, instead, using FKS [18] and  $q_T$ -subtraction [19], in order to allow for a consistency check of the results. On the other hand, the main bottleneck is undoubtedly the computation of the two-loop amplitudes, which have been computed analytically in [20, 21]. For the calculation described here we used the amplitudes computed in [21], which are publicly available as a C++ code<sup>1</sup>. For details on the computation we refer to the original papers [20, 21].

## 2. Results

With all amplitudes at hand, we can produce precise phenomenology studied for the LHC. We start considering the production of two on-shell Z bosons and their decay to leptonic pairs, in particular we focus on  $gg \rightarrow ZZ \rightarrow e^+e^- \mu^+ \mu^-$ . As the general set up, we generate the invariant masses of Z bosons around  $m_Z$  using Breit-Wigner distributions and we require the  $e^+e^-$  and  $\mu^+ \mu^-$  pairs to have invariant masses  $m_{l\bar{l}} \in (60, 120)$  GeV. We use as central scale  $\mu = \mu_R = \mu_F = 2m_Z$ , and vary it between  $\mu = m_Z$  and  $\mu = 4m_Z$ . We use leading (next-to-leading) order parton distribution functions for one- and two-loop calculations, respectively. We employ the NNPDF3.0 set of parton distribution functions and obtain the relevant values of the strong coupling constant from NNPDF routines [22]. For the 8 TeV LHC we find

$$\sigma_{\text{LO}}^{gg \rightarrow ZZ} = 0.97^{+0.3}_{-0.2} \text{ fb}, \quad \sigma_{\text{NLO}}^{gg \rightarrow ZZ} = 1.8^{+0.2}_{-0.2} \text{ fb}, \quad (2.1)$$

where the central values refer to the renormalization and factorization scales set to  $\mu = 2m_Z$  and the upper (lower) values to  $\mu = m_Z$  ( $\mu = 4m_Z$ ). It follows from Eq.(2.1) that the NLO cross section increases the LO cross section by  $\mathcal{O}(60\% - 110\%)$ , depending on the renormalization scale. A similar situation occurs at the 13 TeV LHC. We find

$$\sigma_{\text{LO}}^{gg \rightarrow ZZ} = 2.8^{+0.7}_{-0.6} \text{ fb}, \quad \sigma_{\text{NLO}}^{gg \rightarrow ZZ} = 4.7^{+0.4}_{-0.4} \text{ fb}. \quad (2.2)$$

The NLO QCD corrections to  $gg \rightarrow ZZ$  at 13 TeV LHC are again significant but somewhat smaller than those at 8 TeV. Indeed, the cross section increases by  $\mathcal{O}(40\% - 90\%)$ . In view of the discussion above, it is clear that such large radiative corrections on the gluon-fusion channel, have an important impact on the theory predictions for the NNLO cross section, which turns out to be increased by a non-negligible amount. For example, we find that in order to match our best prediction, the 8 TeV  $gg \rightarrow ZZ$  cross section of Ref. [3] should be increased by about 80%. This would lead to an increase in the total NNLO QCD correction to  $pp \rightarrow ZZ$  at 8 TeV from the current 12%, to 18%, which is beyond the  $\mathcal{O}(3\%)$  scale variation of the NNLO QCD result for  $pp \rightarrow ZZ$  [3]. See [10] for more details.

A similar analysis can be performed for the  $WW$  cross section. The most important difference with respect to our previous work is the treatment of the massive quark loops. In  $ZZ$  production, it is possible to separate the contribution of bottom and top loops, if one neglects the contribution of vector-axial triangle diagrams which are suppressed by the top mass. On the other hand, if

<sup>1</sup>See <https://vvamp.hepforge.org/>

$W$  bosons are radiated from the quark loop, such a separation is obviously not possible and we therefore neglect the contribution of the third generation entirely. We present results for the  $gg \rightarrow W^+W^- \rightarrow \nu_e e^+ \mu^- \bar{\nu}_\mu$  cross sections. To perform the computation, we take the masses of the  $W$  and  $Z$  bosons to be  $m_W = 80.398$  GeV and  $m_Z = 91.1876$  GeV, their widths to be  $\Gamma_W = 2.1054$  GeV and  $\Gamma_Z = 2.4952$  GeV and the Fermi constant  $G_F = 1.16639$  GeV<sup>-2</sup>. We use  $\mu = \mu_R = \mu_F = m_W$  as the central value for the renormalization and factorization scale, and estimate the effect of the scale variation by calculating the cross section at  $\mu = 2m_W$  and  $\mu = m_W/2$ . Again, we use LO and NLO NNPDF3.0 parton distribution functions and one- and two-loop running of the strong coupling, for our LO and NLO results, respectively. We do not include the contribution from Higgs-mediated amplitudes. As in the previous case,  $W$  bosons are produced on the mass shell using a Breit-Wigner distribution. As exemplification, we report results for the calculation for proton-proton collisions at 13 TeV. We find the LO and the NLO cross sections,

$$\sigma_{gg,LO}^{W^+W^-} = 56.5^{+15.4}_{-11.5} \text{ fb}, \quad \sigma_{gg,NLO}^{W^+W^-} = 79.5^{+4.2}_{-5.9} \text{ fb}. \quad (2.3)$$

The NLO corrections increase the cross section by a factor of 1.2 – 1.6, with an increase of 1.4 at the central scale. Differently with respect to the case of  $ZZ$  production, the NLO QCD corrections to  $gg \rightarrow W^+W^-$  increase the full NNLO cross section by only about 2% which, roughly, corresponds to the scale uncertainty of the NNLO QCD computation [4].

Until now we have considered only fully inclusive quantities. We can of course derive predictions for arbitrary differential observables and, in particular, we can study directly the cross-section in the fiducial region measured by ATLAS and CMS. This is particularly important in the case of  $WW$  production, as argued in [23]. We use the cuts describe there to reproduce ATLAS fiducial region and we find the results summarized in Table 2.

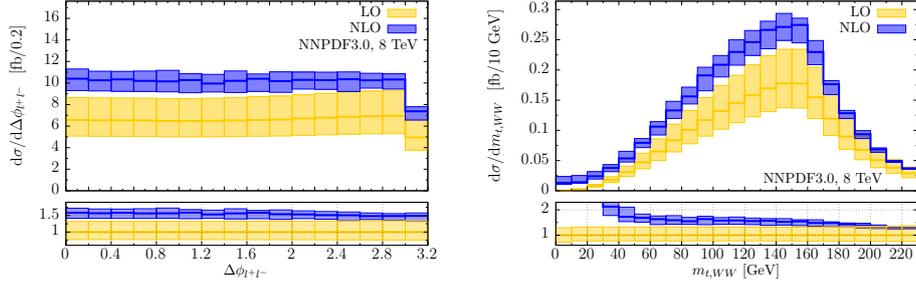
	$\sigma_{\mu\mu,8 \text{ TeV}}$	$\sigma_{ee,8 \text{ TeV}}$	$\sigma_{e\mu,8 \text{ TeV}}$
$\sigma_{gg,LO} [\text{fb}]$	$5.94^{+1.89}_{-1.35}$	$5.40^{+1.71}_{-1.23}$	$9.79^{+3.13}_{-2.24}$
$\sigma_{gg,NLO} [\text{fb}]$	$7.01^{+0.36}_{-0.17}$	$6.40^{+0.32}_{-0.16}$	$11.78^{+0.46}_{-0.34}$

**Table 2:** LO and NLO gluon-initiated fiducial cross sections for in the  $ee$ ,  $\mu\mu$ , and  $e\mu$  decay channels. For central scale NLO increases results of  $\approx 18 - 20\%$

As it is easy to see, the NLO corrections in the fiducial region are substantially smaller than for the inclusive cross-section; the reason is to be found mainly in the  $p_T$  veto on jets with  $p_T > 25$  GeV, which strongly suppresses hard gluons emissions<sup>2</sup>. This is very important, as it points to the necessity, from the theory side, to always compute cross-sections in the fiducial volume in order to allow for a proper comparison with the experiments, avoiding ambiguities in the extrapolation from the fiducial volume to the fully inclusive cross-section.

Indeed, with our set up we can apply any cuts and study differential distributions both for  $ZZ$  and  $WW$  production. As one example, in Figure 1 we show distributions for the azimuthal angle between the charged leptons  $\Delta\phi_{\ell^+\ell^-}$ , and the transverse mass of the  $W^+W^-$  system  $m_{T,WW}$  for the  $gg \rightarrow W^+W^- \rightarrow \nu_e e^+ \mu^- \bar{\nu}_\mu$  process at the 8 TeV LHC. We refer to [10, 11] for a more complete phenomenology discussion both for  $ZZ$  and  $WW$  production.

<sup>2</sup>See [23] for a thorough description of the cuts applied.



**Figure 1:** The azimuthal angle between the charged leptons  $\Delta\phi_{\ell^+\ell^-}$  (left), and the transverse mass of the  $W^+W^-$  system  $m_{T,WW}$  (right), in  $gg \rightarrow W^+W^- \rightarrow \nu_e e^+ \mu^- \bar{\nu}_\mu$  process at the  $\sqrt{s} = 8$  TeV LHC. LO results are shown in yellow, NLO results are shown in blue. The central scale is  $\mu = m_W$ ; the scale variation bands correspond to scale variations by a factor of two in either direction. The lower panes show the ratios of the LO and NLO distributions at each scale to the LO distribution at the central scale.

### 3. Conclusions

We described the calculation of the NLO QCD corrections to ZZ and WW production in gluon fusion at the LHC. We showed that, despite being suppressed by one power of  $\alpha_s$  compared to the  $q\bar{q}$  annihilation channel, QCD corrections to the gluon-fusion channel must be taken into account in order to provide reliable theory predictions at the percent level accuracy. We discussed precise theoretical prediction for LHC observables both at 8 and 13 TeV and showed that, in particular in the ZZ case, the inclusion of NLO QCD corrections to the  $gg$  channel shifts the cross-section of an amount which exceeds the pure NNLO theory uncertainty. The inclusion of the NLO QCD corrections to the gluon-fusion channel helps therefore to stabilize the scale dependence and the theory uncertainty. We performed also differential studies and, in particular, we computed the NLO QCD corrections to  $gg \rightarrow WW$  in the fiducial volume. We showed that, due to the  $p_T$  jet veto imposed by the experimental collaborations, the NLO QCD corrections are substantially reduced with respect to the fully inclusive cross-section. For a more detailed analysis we refer to [10, 11].

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