

Recent progress on CP violation in $K \rightarrow \pi\pi$ decays in the SM and a supersymmetric solution

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Abstract. Using the recent first lattice results of the RBC-UKQCD collaboration for $K \rightarrow \pi\pi$ decays, we perform a phenomenological analysis of ϵ'_K/ϵ_K and find a discrepancy between SM prediction and experiments by $\sim 3\sigma$. We discuss an explanation by new physics. The well-understood value of ϵ_K , which quantifies indirect CP violation, however, typically prevents large new physics contributions to ϵ'_K/ϵ_K . In this talk, we show a solution of the ϵ'_K/ϵ_K discrepancy in the Minimal Supersymmetric Standard Model with squark masses above 3 TeV without fine-tuning of CP phases. In this solution, the *Trojan penguin* diagram gives large isospin-breaking contributions which enhance ϵ'_K , while the contribution to ϵ_K is suppressed thanks to the Majorana nature of gluinos.

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1. Introduction to ϵ'_K/ϵ_K discrepancy

In $K \rightarrow \pi\pi$ decays, one distinguishes between two types of charge-parity (CP) violation: direct and indirect CP violations which are parametrized by ϵ'_K and ϵ_K , respectively. Both types of CP violation have been quantified by many kaon experiments precisely. While ϵ_K is a per mille effect in the Standard Model (SM), ϵ'_K is smaller by another three orders of magnitude: $\epsilon'_K \sim \mathcal{O}(10^{-6})$. This strong suppression comes from the suppression of the isospin-3/2 amplitude w.r.t. the isospin-1/2 amplitude ($\Delta I = 1/2$ rule) and an accidental cancellation of leading contributions in the SM. In Fig. 1, the contributions of individual operators to ϵ'_K/ϵ_K are shown. Q_3 – Q_6 are called QCD penguin operators, while Q_7 – Q_{10} are called EW penguin operators. The leading contributions come from Q_6 and Q_8 , having opposite sign, and thus a cancellation emerges. Remarkably, this figure also shows that even if one includes sub-leading contributions, the cancellation still exists with high precision.

The compilation of representative SM predictions and the experimental values for $\text{Re } \epsilon'_K/\epsilon_K$ is given in Fig. 2. The SM predictions (colored bars) are taken from: Bertolini *et al.* (BEFL '97) [1], Pallante *et al.* (PPS '01) [2], Hambye *et al.* (HPR '03) [3], Buras and Gérard (BG '15) [4] with lattice result for $I = 2$ (BG '15+Lat.), RBC-UKQCD lattice result [5], Buras *et al.* (BGJJ '15) [6], and Kitahara *et al.* (KNT '16) [7]. The experimental values (black bars) are

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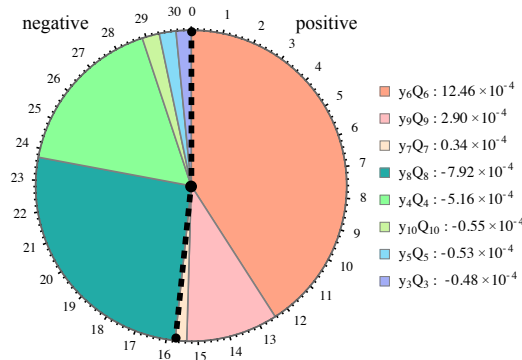


Figure 1. The composition of ϵ'_K/ϵ_K with respect to the operator basis. The right and left side of the dashed lines represent positive and negative contributions to ϵ'_K/ϵ_K , respectively. This figure is based on the result of Ref. [7].

taken from: E371 [8], NA31 [9], NA48 [10] and KTeV [11] collaborations, and the black thick one is the world average of the experimental values [12],

$$\text{Re}\left(\frac{\epsilon'_K}{\epsilon_K}\right) = (16.6 \pm 2.3) \times 10^{-4} \text{ (PDG average)}. \quad (1)$$

In order to predict ϵ'_K in the SM, one has to calculate the hadronic matrix elements of four-quark operators with nonperturbative methods. The magenta bars in Fig. 2 have utilized analytic approaches to calculating them: chiral quark model (BEFL '97), chiral perturbation theory (PPS '01) with minimal hadronic approximation (HPR '03), and the dual QCD approach (BG '15). Note that the dual QCD approach predicts an upper bound on ϵ'_K/ϵ_K . On the other hand, a determination of all hadronic matrix elements from lattice QCD has been obtained only recently by the RBC-UKQCD collaboration [5], and the blue bars are based on the lattice result:

$$\frac{\epsilon'_K}{\epsilon_K} = \begin{cases} (1.9 \pm 4.5) \times 10^{-4} & \text{(BGJJ '15),} \\ (1.06 \pm 5.07) \times 10^{-4} & \text{(KNT '16).} \end{cases} \quad (2)$$

These results are obtained by next-to-leading order (NLO) calculations exploiting CP -conserving data to reduce hadronic uncertainties and include isospin-violating contributions [13] which are not included in the lattice result. Furthermore, the latter result includes an additional $\mathcal{O}(\alpha_{EM}^2/\alpha_s^2)$ correction, which appears only in this order, and also utilizes a new analytic solution of the renormalization group (RG) equation which avoids the problem of singularities in the NLO terms. The two numbers in Eq. (2) disagree with the experimental value in Eq. (1) by 2.9σ [6] and 2.8σ [7], respectively. The uncertainties are dominated by the lattice statistical and systematic uncertainties for the $I = 0$ amplitude. Therefore, in the near future, the increasing precision of lattice calculations will further sharpen the SM predictions in Eq. (2) and answer the question about new physics (NP) in ϵ'_K/ϵ_K .

The main difference between each result of analytic approaches and the lattice result is in the hadronic parameter $B_6^{(1/2)}$, which controls the largest positive contribution to ϵ'_K/ϵ_K , the $y_6 Q_6$ component in the Fig. 1. In chiral perturbation theory, typically large values are obtained: $B_6^{(1/2)} \sim 1.6$ (BEFL '97), ~ 1.6 (PPS '01), and ~ 3 (HPR '03, [4]). On the other hand, the dual QCD approach predicts a smaller number, $B_6^{(1/2)} \leq B_8^{(3/2)} \sim 0.8$ (BG '15). The lattice result is

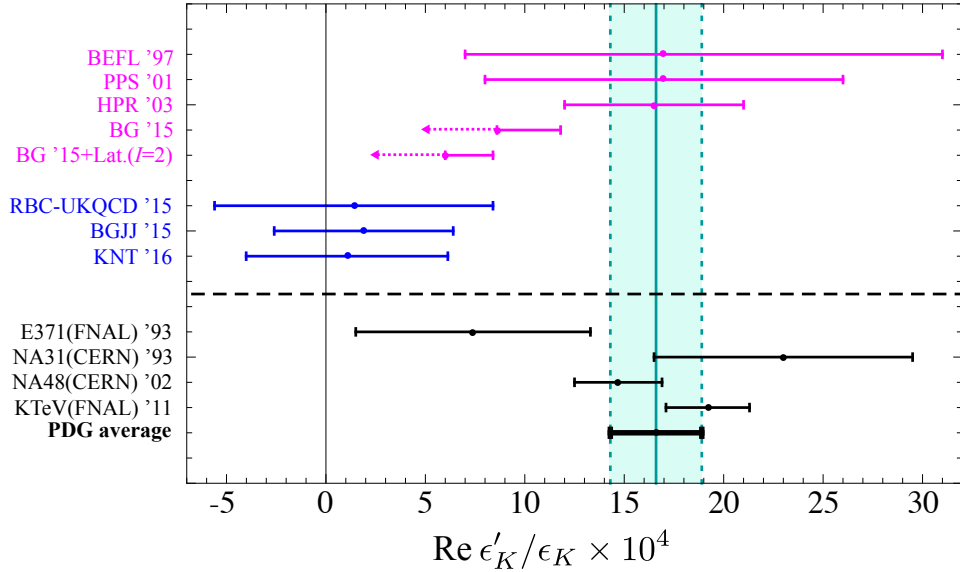


Figure 2. Compilation of representative SM predictions and the experimental values for $\text{Re } \epsilon'_K/\epsilon_K$. All error bars represent 1σ range. The SM predictions are taken from Bertolini *et al.* (BEFL '97) [1], Pallante *et al.* (PPS '01) [2], Hambye *et al.* (HPR '03) [3], Buras and Gérard (BG '15) [4], RBC-UKQCD lattice result [5], Buras *et al.* (BGJJ '15) [6], and Kitahara *et al.* (KNT '16) [7], where magenta bars are based on analytic approaches to hadronic matrix elements, while blue bars are based on lattice results. The black thick one is the world average of the experimental values [12].

consistent with the latter result: $B_6^{(1/2)} = 0.56 \pm 0.20$ [5, 7]. Note that the lattice calculation *includes* final-state interaction of the two pions in accordance with Ref. [14].

We also should comment on the $\Delta I = 1/2$ rule, which denotes the largeness of the ratio of the CP -conserving amplitudes, $(\text{Re}A_0/\text{Re}A_2)_{\text{exp.}} = 22.45 \pm 0.05$. Although none of the analytic approaches can explain such a large value, the first lattice calculation has found a consistent value within 1σ , $(\text{Re}A_0/\text{Re}A_2)_{\text{Lat.}} = 31.0 \pm 11.1$ [5, 15].

2. ϵ_K in the MSSM

An explanation of the puzzle between Eq. (1) and Eq. (2) by physics beyond the SM requires a NP contribution which is seemingly even larger than the SM contribution. However, it is known that once constraints from the corresponding $|\Delta S| = 2$ transition are taken into account, one expects that NP effects in a $|\Delta S| = 1$ four-quark process are highly suppressed. To explain the NP hierarchy in $|\Delta S| = 1$ vs $|\Delta S| = 2$ transitions, we specify to ϵ'_K and ϵ_K : The SM contributions are governed by the combination $\tau = -V_{td}V_{ts}^*/(V_{ud}V_{us}^*) \sim (1.5 - i0.6) \times 10^{-3}$ with $\epsilon_K^{\prime\text{SM}} \propto \text{Im } \tau/M_W^2$ and $\epsilon_K^{\text{SM}} \propto \text{Im } \tau^2/M_W^2$. If the NP contribution enters through the $\Delta S = 1$ parameter δ and is mediated by heavy particles of mass M , one obtains $\epsilon_K^{\prime\text{NP}} \propto \text{Im } \delta/M^2$, $\epsilon_K^{\text{NP}} \propto \text{Im } \delta^2/M^2$, and therefore the experimental constraint $|\epsilon_K^{\text{NP}}| \leq |\epsilon_K^{\text{SM}}|$ leads to

$$\left| \frac{\epsilon_K^{\prime\text{NP}}}{\epsilon_K^{\prime\text{SM}}} \right| \leq \frac{|\epsilon_K^{\prime\text{NP}}/\epsilon_K^{\prime\text{SM}}|}{|\epsilon_K^{\text{NP}}/\epsilon_K^{\text{SM}}|} = \mathcal{O} \left(\frac{\text{Re } \tau}{\text{Re } \delta} \right). \quad (3)$$

If NP enters through a loop with particles of mass $M \gtrsim 1$ TeV, the NP effects can be relevant only for $|\delta| \gg |\tau|$, and thus Eq. (3) seemingly forbids detectable NP contributions to ϵ'_K .

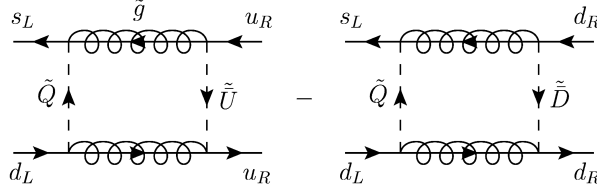


Figure 3. Trojan penguin contributions to $\text{Im } A_2$ for $m_{\tilde{U}} \neq m_{\tilde{D}}$.

In the Minimal Supersymmetric Standard Model (MSSM), there is a bypass to Eq. (3). The Majorana nature of the gluino leads to a suppression of gluino-squark box contributions to ϵ_K . This is so, because there are two such diagrams (crossed and uncrossed boxes) with opposite signs. If the gluino mass $m_{\tilde{g}}$ equals roughly 1.5 times the average down squark mass M_S , both contributions to ϵ_K^{SUSY} cancel [16]. For $m_{\tilde{g}} > 1.5M_S$, the gluino-box contribution approximately behaves as $[m_{\tilde{g}}^2 - (1.5M_S)^2]/m_{\tilde{g}}^4$, and the $1/m_{\tilde{g}}^2$ decoupling sets in. Note that this suppression appears only when a hierarchy $\Delta_{Q,12} \gg \Delta_{\tilde{D},12}$ of $\Delta_{Q,12} \ll \Delta_{\tilde{D},12}$ is satisfied, where the following notation is used for the squark mass matrices: $M_{X,ij}^2 = m_X^2 (\delta_{ij} + \Delta_{X,ij})$, with $X = Q, \tilde{U},$ or \tilde{D} .

3. ϵ'_K/ϵ_K in the MSSM

The master equation for ϵ'_K is given by [6]

$$\frac{\epsilon'_K}{\epsilon_K} = \frac{\omega_+}{\sqrt{2}|\epsilon_K^{\text{exp}}|\text{Re}A_0^{\text{exp}}} \left\{ \frac{\text{Im}A_2}{\omega_+} - (1 - \hat{\Omega}_{\text{eff}}) \text{Im}A_0 \right\}, \quad (4)$$

with $\hat{\Omega}_{\text{eff}} = (14.8 \pm 8.0) \times 10^{-2}$, the measured $|\epsilon_K^{\text{exp}}|$, $\omega_+ = (4.53 \pm 0.02) \times 10^{-2}$, and the amplitudes $A_I = \langle (\pi\pi)_I | \mathcal{H}^{|\Delta S|=1} | K^0 \rangle$ involving the effective $|\Delta S| = 1$ Hamiltonian $\mathcal{H}^{|\Delta S|}$. $I = 0, 2$ represents the strong isospin of the final two-pion state.

The MSSM contributions to ϵ'_K/ϵ_K have been widely studied in the past. However, the supersymmetry-breaking scale M_S was considered in the ballpark of the electroweak scale, so that the suppression mechanism inferred from Eq. (3) is avoided. The low-energy Hamiltonian in the case of small left-right squark mixing reads

$$\mathcal{H}_{\text{eff, SUSY}}^{|\Delta S|=1} = \frac{G_F}{\sqrt{2}} \sum_q \left[\sum_{i=1}^2 c_i^q(\mu) Q_i^q(\mu) + \sum_{i=1}^4 [c_i^{\prime q}(\mu) Q_i^{\prime q}(\mu) + \tilde{c}_i^{\prime q}(\mu) \tilde{Q}_i^{\prime q}(\mu)] \right] + \text{H.c.}, \quad (5)$$

where G_F is the Fermi constant and

$$\begin{aligned} Q_1^q &= (\bar{s}_\alpha q_\beta)_{V-A} (\bar{q}_\beta d_\alpha)_{V-A}, & Q_2^q &= (\bar{s}q)_{V-A} (\bar{q}d)_{V-A}, \\ Q_1^{\prime q} &= (\bar{s}d)_{V-A} (\bar{q}q)_{V+A}, & Q_2^{\prime q} &= (\bar{s}_\alpha d_\beta)_{V-A} (\bar{q}_\beta q_\alpha)_{V+A}, \\ Q_3^{\prime q} &= (\bar{s}d)_{V-A} (\bar{q}q)_{V-A}, & Q_4^{\prime q} &= (\bar{s}_\alpha d_\beta)_{V-A} (\bar{q}_\beta q_\alpha)_{V-A}. \end{aligned} \quad (6)$$

Here $(\bar{s}d)_{V-A}(\bar{q}q)_{V\pm A} = [\bar{s}\gamma_\mu(1 - \gamma_5)d][\bar{q}\gamma^\mu(1 \pm \gamma_5)q]$, α and β represent color indices, and opposite-chirality operators $\tilde{Q}_i^{\prime q}$ are given by interchanging $V - A \leftrightarrow V + A$.

In our analysis [17] we found that the dominant supersymmetric contribution comes from a certain gluino box diagrams^{#1}, called a *Trojan penguin* in Ref. [19], which are shown in Fig. 3. It contributes to $\text{Im } A_2$ when $m_{\tilde{U}} \neq m_{\tilde{D}}$. Because these contributions are governed by the strong

^{#1} The other supersymmetric solution focusing the chargino Z -penguin contribution has been studied in Ref. [18].

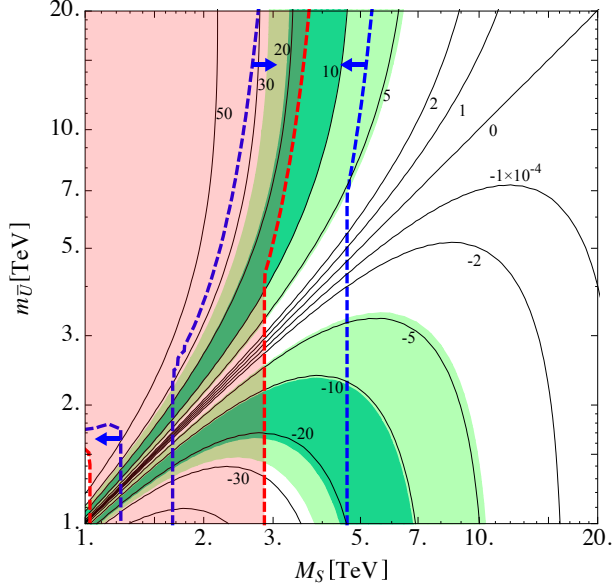


Figure 4. The black contour represents the supersymmetric contributions to ϵ'_K/ϵ_K in units of 10^{-4} . The ϵ'_K/ϵ_K discrepancy is resolved at 1σ (2σ) in the dark (light) green region. The red shaded region (region between the blue dashed lines) is excluded (preferred) by ϵ_K with inclusive (exclusive) $|V_{cb}|$ at 95% C.L.

interaction and there is an enhancement factor $1/\omega_+ = 22.1$ for the $\text{Im}A_2$ term in (4), they easily become the largest contribution to ϵ'_K/ϵ_K [19]. In order to obtain the desired large effect in ϵ'_K , one needs a contribution to the operators $Q'_{1,2}$ with $(V-A) \times (V+A)$ Dirac structure, whose matrix elements are chirally enhanced by a factor $(m_K/m_s)^2$. Hence, the flavor mixing has to be in the left-handed squark mass matrix. The opposite situation with right-handed flavor mixing and \tilde{u}_L - \tilde{d}_L mass splitting is not possible because of the $\text{SU}(2)_L$ invariance.

For the calculation of supersymmetric contributions to ϵ'_K/ϵ_K , one has to use the RG equations to evolve the Wilson coefficients calculated at the high scale M_S down to the hadronic scale $\mu_h = \mathcal{O}(1 \text{ GeV})$ at which the hadronic matrix elements are calculated [5, 7]. To use the well-known NLO 10×10 anomalous dimensions for the SM four-fermion operator basis [20], we switch from Eq. (5) to

$$\mathcal{H}_{\text{eff, SUSY}}^{|\Delta S|=1} = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} [C_i(\mu) Q_i(\mu) + \tilde{C}_i(\mu) \tilde{Q}_i(\mu)] + \text{H.c.}, \quad (7)$$

where $Q_{1,\dots,10}$ are given in Ref. [20] and

$$C_{1,2}(\mu) = c_{1,2}^u(\mu), \quad \tilde{C}_{1,2}(\mu) = 0, \\ C_{3,4,5,6}(\mu) = \frac{1}{3} [c_{3,4,1,2}'^u(\mu) + 2c_{3,4,1,2}'^d(\mu)], \quad C_{7,8,9,10}(\mu) = \frac{2}{3} [c_{1,2,3,4}'^u(\mu) - c_{1,2,3,4}'^d(\mu)], \quad (8)$$

and the coefficients $\tilde{C}_{3,\dots,10}$ for the opposite-chirality operators can be obtained from $C_{3,\dots,10}$ by replacing $c_i'^q \rightarrow \tilde{c}_i'^q$. Note that the contribution of Fig. 3 is collected into the coefficients $C_{7,8}$. For the RG evolution of the coefficients, we use the new analytic solution of the RG equations discussed in Ref. [7].

Our main result is given in Fig. 4, where the portion of the squark mass plane which simultaneously explains ϵ'_K/ϵ_K and ϵ_K is shown. As input, we take the grand-unified theory

relation for gaugino masses, $\alpha_s(M_Z) = 0.1185$, $m_{\tilde{g}}/M_S = 1.5$ for the suppressed ϵ_K , and $m_Q = m_{\bar{D}} = \mu_{\text{SUSY}} = M_S$ with varying $m_{\tilde{U}}$. Furthermore, the trilinear supersymmetry-breaking matrices A_q are set to zero, $\tan\beta = 10$, and the only nonzero off-diagonal elements of the squark mass matrices are $\Delta_{Q,12,13,23} = 0.1 \exp(-i\pi/4)$ for the left-handed squark sectors. We have calculated all relevant one-loop contributions to the coefficients in Eq. (5) in the squark mass eigenbasis. The ϵ'_K/ϵ_K discrepancy can be resolved at 1σ (2σ) in the dark (light) green region. The red region is already excluded by the measurement of ϵ_K at 95% C.L. in combination with the inclusive V_{cb} . On the other hand, the region between the blue dashed lines can explain the ϵ_K discrepancy at 95% C.L. for the exclusive value of $|V_{cb}|$. The area in Fig. 4 labeled with negative values of ϵ'_K/ϵ_K becomes feasible by adding π to the phase of $\Delta_{Q,ij}$, which flips the sign of ϵ'_K while keeping ϵ_K unchanged.

4. Conclusions

In this talk, we have discussed the supersymmetric contributions to ϵ'_K , and it is found that the large contributions required to solve the discrepancy between Eq. (1) and Eq. (2) can be obtained in the multi-TeV squark mass range thanks to the Trojan penguin diagrams. Using a relatively heavy gluino, the severe constraint from ϵ_K can be fulfilled without fine-tuning.

Acknowledgments

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