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Precision Measurements in Electron-Positron Annihilation: Theory and Experiment

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Theory results on precision measurements in electron-positron annihilation at low and high energies are collected. These cover pure QCD calculations as well as mixed electroweak and QCD results, involving light and heavy quarks. The impact of QCD corrections on the W-boson mass is discussed and, last not least, the status and the perspectives for the Higgs boson decay rate into $b\bar{b}$, $c\bar{c}$ and into two gluons.

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1. Introduction

The determination of the strong coupling α_s in clean experimental conditions is one of the important issues in ongoing theoretical and experimental investigations. During the past years significant progress has been made in perturbative calculations of a large variety of processes. In this talk a number of benchmark processes is identified and the corresponding predictions are presented to the highest presently available order.

During the past forty years calculations in the framework of perturbative QCD have developed from a quantitative description of a few benchmark processes to precise predictions of numerous hadronic processes, albeit typically at relatively high energies and/or for inclusive reactions. Many of these are closely related to electron-positron annihilation into hadrons, at lower energies through the electromagnetic, at higher energies through the neutral current. QCD corrections to the decay of the W-boson into hadrons through the vector and the axial vector current can be evaluated in a similar way and are, in turn, closely related to QCD corrections of the τ-lepton decay rate. The decay of the Higgs boson into hadrons, on the other hand, proceeds through the scalar current and can be treated with very similar methods. Finally the running of the strong coupling constant from low energies, say m_τ, up to the mass of the Higgs boson and beyond, is governed by the beta-function, can be calculated with similar techniques, is now available in five-loop order and will also be discussed in this context.

2. Electron-positron annihilation at low energies

The cross section for electron-positron annihilation into hadrons is well described by perturbative QCD, at least in the regions away from the various quark thresholds. The result of the BESSII collaboration [1], consisting of an average of measurements at 3.650 GeV and 3.6648 GeV,

$$\bar{R} = 2.224 \pm 0.019 \pm 0.089 \tag{2.1}$$

is in good agreement with the theoretical expectation

$$\bar{R} = 3(Q_u^2 + Q_d^2 + Q_s^2)(1 + a_s + 1.64010 a_s^2 - 10.28395 a_s^3 - 104.78910 a_s^3) \tag{2.2}$$

adopting as value of the strong coupling $\alpha_s = 0.31 \pm 0.14$. Although the precision of this experiment cannot compete with those at LEP (to be discussed below), the agreement between theory and experiment is, nevertheless, remarkable already now. Any further improvement of the experimental precision would be welcome and would allow the comparison of results for α_s at low and high energies. Let us mention in passing, that there is in principle the (very small) singlet contribution contribution proportional $(\sum_i Q_i)^2$, which starts contributing in order α_s^3 and is also available up to order α_s^4. For the three-flavour case $(\sum_i Q_i)^2$ happens to vanish, for the four- and five-flavour case the term is numerically small [2, 3].

3. Z-production and -decay in electron-positron annihilation

From the theory side there is only one slight complication when moving from low to high energies: the axial current starts contributing and, correspondingly, QCD corrections specific for

this case start contributing in order α_s^2. Of course, also a singlet piece, starting in order α_s^3, is present, just as for the electromagnetic current. The corrections for the three different pieces, each evaluated to order α_s^4, are shown separately in Figs. 1–4. Note that $\alpha_s(M_Z) = 0.1190$ and $n_l = 5$ are adopted in Figs. 2–4.

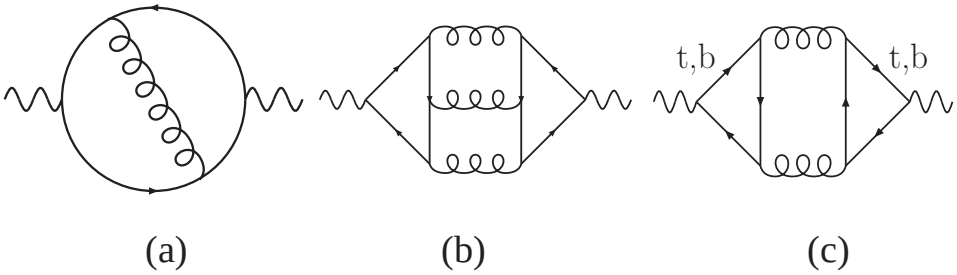

Figure 1: Different contributions to r-ratios: (a) non-singlet, (b) vector singlet and (c) axial vector singlet.

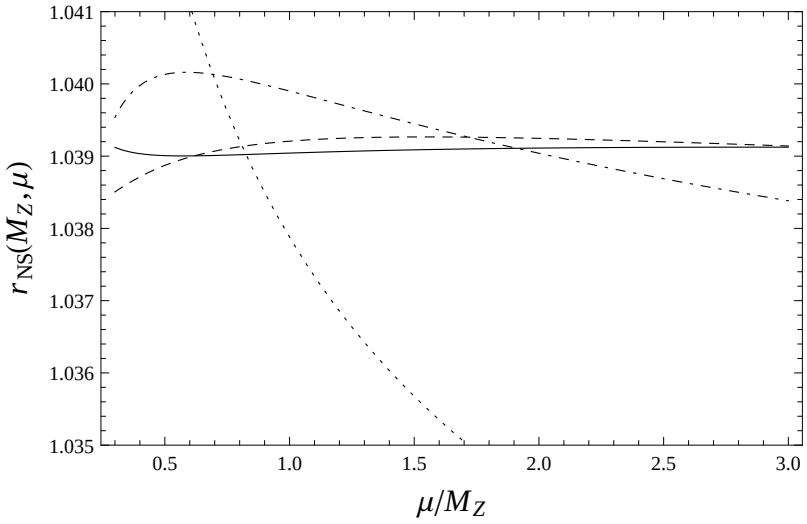

Figure 2: Scale dependence of non-singlet r_{NS}. Dotted, dash-dotted, dashed and solid curves refer to $\mathcal{O}(\alpha_s)$ up to $\mathcal{O}(\alpha_s^4)$ predictions.

The result

$$\alpha_s(M_Z) = 0.1190 \pm 0.0026 \tag{3.1}$$

still exhibits a sizeable error, significantly larger than the theory error which has been estimated to [2, 3] $\delta\Gamma_{NS} = 101$ keV, $\delta\Gamma_S^V = 2.7$ keV, and $\delta\Gamma_S^A = 42$ keV. Summing these errors linearly, one arrives at a theory uncertainty of 146 keV, which corresponds to a shift in α_s of about 3×10^{-4} and is thus about a factor ten smaller than the current experimental error, based on Z decays, $\alpha_s = 0.1190 \pm 0.0026$.

4. Mixed electroweak and QCD corrections for Z decays: light and heavy quarks

As a consequence of the virtual top quark one expects a significant difference between the electroweak corrections for Z decays into $d\bar{d}$ and $u\bar{u}$ on the one hand and into $b\bar{b}$ on the other hand.

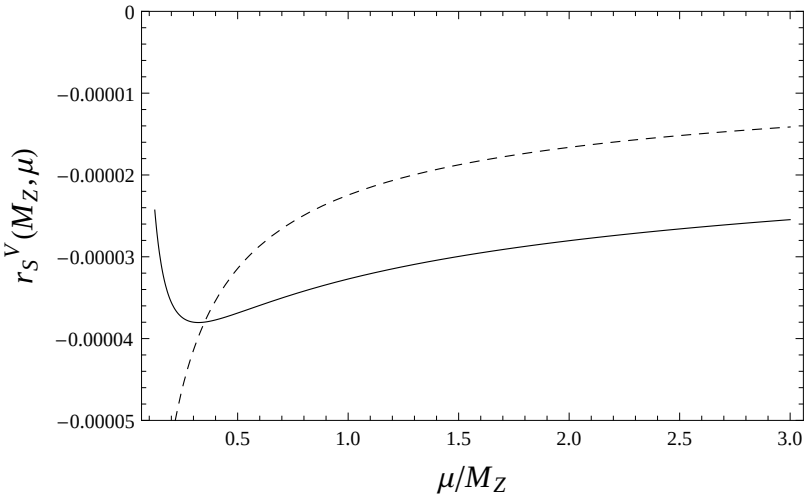

Figure 3: Scale dependence of the vector singlet r_S^V. Dashed and solid curves efer to $\mathcal{O}(\alpha_s^3)$ up to $\mathcal{O}(\alpha_s^4)$ predictions.

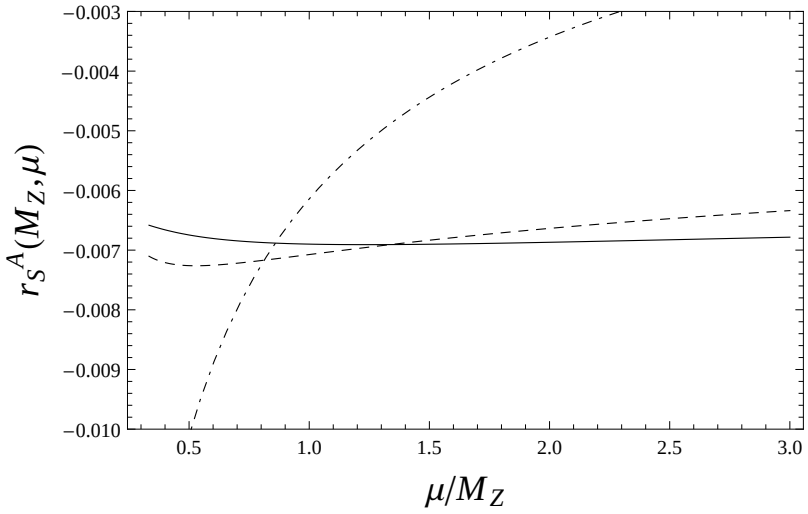

Figure 4: Scale dependence of the axial vector singlet $r_{S;t,b}^A$. Dotted, dash-dotted, dashed and solid curves refer to $\mathcal{O}(\alpha_s)$ up to $\mathcal{O}(\alpha_s^4)$ predictions.

This pattern repeats itself in the mixed electroweak and QCD corrections of order $\alpha_{weak}\alpha_s$. For light quarks the two-loop corrections of order $\alpha\alpha_s$ have been evaluated about twenty years ago. The final result which makes the non-factorizing terms explicit can be cast into the form [4]

$$\Delta\Gamma \equiv \Gamma(\text{two loop} : \text{EW} \times \text{QCD}) - \Gamma_{Born}\delta_{EW}^{NLS}\delta_{QCD}^{NLO} = -0.59(3)\ \text{MeV} \tag{4.1}$$

which is sufficient for the present experimental precision of 2 MeV for the hadronic decay rate. On the other hand, given an expected experimental precision of $\delta\Gamma \approx 0.1$ MeV, as advertised for a future electron-positron collider [5, 6], the next, not yet available three-loop term might eventually be required.

The situation is qualitatively similar for the $Z \to b\bar{b}$ decay mode which, however, receives also contributions from virtual top quarks. The precision of the measured branching ratio of $15.12 \pm 0.05\%$ is, at present, quite close to the size of the two-loop term, which is given by [7]

$$\Gamma_b - \Gamma_d = (-5.69 - 0.79 + 0.50 + 0.06)\ \text{MeV} \tag{4.2}$$

and has been split into one- and two-loop contributions and into the m_t^2-enhanced piece and the rest. Let us mention in passing that part of the three-loop corrections, the non-singlet piece, has been evaluated in [8]. It amounts to about 0.1 MeV, is irrelevant in the moment, but of potential importance at a future electron-positron collider.

Many top-induced corrections become significantly smaller, if the top quark mass is expressed in the $\overline{\rm MS}$ convention. The relation between pole and $\overline{\rm MS}$ mass has been evaluated in three- [9] and recently even four-loop [10] approximation and reads

$$\bar{m}_t(\bar{m}_t) = m_{\rm pole}(1 - 1.33\,a_s - 6.46\,a_s^2 - 60.27\,a_s^3 - 704.28\,a_s^4) = (163.45 \pm 0.72|_{m_t} \pm 0.19|_{\alpha_s} \pm ?|_{th})\ \text{GeV} \tag{4.3}$$

with a theory error of about 100 MeV.

5. The W boson mass from G_F, M_Z, α and the rest

The present precision [11] of $M_W = 80.385 \pm 0.015$ MeV is based on a combination of LEP, TEVATRON and LHC results. In contrast, at a future linear or circular electron-positron collider a precision better than 1 MeV is advertised [5, 6]. In Born approximation the W boson mass can be derived from the Fermi coupling G_F, the Z boson mass and the electromagnetic coupling α. The rest of the parameters, in particular the masses of fermions and the Higgs boson, enter through radiative corrections. Numerically one finds for the shift in the W-boson mass induced by virtual contributions of the top quark

$$\delta M_W \approx \frac{1}{2} M_W \frac{\cos^2\theta_W}{\cos^2\theta_W - \sin^2\theta_W} \approx 5.7 \times 10^4\, \delta\rho\,[(\text{MeV})], \tag{5.1}$$

with the ρ parameter calculated in three-[12, 13] and even four-loop [14, 15] approximation

$$\delta\rho_t = 3X_t(1 - 2.8599\,a_s - 14.594\,a_s^2 - 93.1\,a_s^3) \tag{5.2}$$

The three- and four-loop terms correspond to shifts of $\delta M_W = 9.5$ MeV and $\delta M_W = 2.1$ MeV respectively. The three-loop term is quite comparable to the current experimental sensitivity, the four-loop term would become relevant at a future electron-positron collider.

At this point it should be emphasized that in three-loop approximation a variety of mixed QCD and electroweak corrections are available [16], which amount to 2.5 MeV for the mixed terms proportional $\alpha_s X_t^2$ and to 0.2 MeV for the purely weak terms of order X_t^3. While these are certainly below the anticipated experimental precision for the near future, they might well become relevant at a future e^+e^- collider. At the same time a number of not yet calculated terms might eventually become relevant, for example four-loop tadpoles of order $\alpha_s^2 X_t^2$ or even five-loop terms of order $\alpha_s^4 X_t$. Although not yet relevant for the moment, these corrections might well enter the analysis of experiments at a future linear or circular e^+e^- collider.

Let us also mention that many corrections are significantly smaller if the top quark mass is expressed in terms of the $\overline{\rm MS}$-mass, or closely related quantities, like the potential subtracted (PS) [17], 1S [18, 19, 20] or renormalon subtracted (RS) [21] one. In other words, a large part of the corrections can be absorbed in the relation between the $\overline{\rm MS}$- and the pole mass, discussed above. Let us emphasize that e.g. the potential subtracted top quark mass (and as well as other "short-distance" masess) could be determined at electron-positron colliders with a significantly higher precision, reaching 20 to 30 MeV.

The present, relatively large experimental error in the top mass is necessarily connected to its determination at a hadron collider. The situation would be significantly better at an e^+e^- machine, where uncertainties around or even below 50 MeV might be possible[22], and even 10 to 20 MeV have been quoted [5, 6].

Let us mention in passing that the total cross section for electron-positron annihilation into hadrons at low energies, below the Z resonance, receives QED corrections connecting initial and final state in order α^2 and hence two loop only. This is a consequence of Yang's theorem which forbids contributions from triangular fermion graphs. This is different in the full electroweak theory, where mixed triangular contributions with vector and axial vector couplings start to contribute in one-loop approximation already. In addition there is a huge tail from ISR QED corrections which increases the cross section by about a factor three and must be carefully controlled to achieve a realistic result for the R ratio.

6. Perspectives for $e^+e^- \to Z + H(\to hadrons)$

One of the most important reactions at a future electron-positron collider will be the production of the Higgs boson in the process $e^+e^- \to Z + H$ with the subsequent decay of the Higgs boson into hadrons, i.e. quarks and gluons. Let us demonstrate the status of recent calculations in a few selected examples:

The Higgs boson decay into bottom-antibottom quarks is of course governed by the mass of the bottom quark, evaluated at the scale of m_H. In total the rate is given by [23]

$$\Gamma(H \to b\bar{b}) = \frac{G_F M_H}{4\sqrt{2}\pi} m_b^2(\mu^2(M_H^2)) R_S(s = M_H^2, \mu^2) \tag{6.1}$$

with

$$\begin{aligned} R_S(s = M_H^2, \mu^2 = M_H^2) &= 1 + 5.667\frac{\alpha_s}{\pi} + 29.147\frac{\alpha_s}{\pi}^2 + 41.758\frac{\alpha_s}{\pi}^3 - 825.7\frac{\alpha_s}{\pi}^4 & (6.2)\\ &= 1 + 0.1948 + 0.03444 + 0.0017 - 0.0012 = 1.2298 & (6.3) \end{aligned}$$

Here $\alpha_s = \alpha_s(M_H) = 0.108$, corresponding to $\alpha_s(M_z) = 0.118$ has been adopted. The decay rate depends on two phenomenological parameters, the strong coupling and the bottom quark mass. To avoid the appearance of large logarithms of the type $\ln(\mu^2/M_H^2)$, the parameter μ should be chosen around M_H. However, the starting value of m_b is typically determined at much smaller values, typically around 5 to 10 GeV [24]. The evolution from this low scale to $\mu = M_H$ is governed by the quark mass anomalous dimensionγ_m and the β function, both of which must be known in five-loop order [25, 26] in order to match the accuracy of the fixed order result. For

the quark mass value $m_b(10\text{GeV}) = 3610 - (\frac{\alpha_s(M_Z)-0.118}{0.002})^2 \times 12 \pm 11$ MeV one finds $m_b(M_H) = 2759 \pm 8|_{m_b} \pm 27|_{\alpha_s}$ MeV. The remaining theory uncertainty from our ignorance of higher order corrections amounts to about 1.5 permille and is completely negligible.

Let us list the potential improvements which might develop during the coming years: The strong coupling constant might be known to $\delta\alpha_s(M_Z) = 2 \times 10^{-4}$ and the bottom quark mass with a relative precision of $\delta m_b/m_b \approx 10^{-3}$. In total this would lead to a relative precision

$$\frac{\delta\Gamma(H \to b\bar{b})}{\Gamma(H \to b\bar{b})} = \pm 2 \times 10^{-3}|_{m_b} \pm 1.3 \times 10^{-3}|_{\alpha_s} \pm 1 \times 10^{-3}|_{theory} \tag{6.4}$$

which corresponds to a dramatic improvement compared to present theory estimates.

Similar statements do apply for the $H \to c\bar{c}$ mode with its rate being smaller by about a factor $((m_c(M_H)/m_b(M_H))^2$. In this case the reduction of $\delta m_c(3\text{GeV})$ from 13 MeV to 5 MeV seems conceivable, reducing the uncertainty from $\delta m_c(3\text{GeV})/m_c(3\text{GeV}) = 13\text{ MeV}/986\text{ MeV}$ to $5\text{ MeV}/986\text{ MeV}$. At the scale of M_H this would lead to a reduction of the error in $m_c(M_H)$ from $m_c(M_H) = (609 \pm 8|_{m_c} \pm 9|_{\alpha_s})$ MeV to ± 3 MeV. This, in turn, would lead to a reduction of the relative error of $\delta\Gamma(H \to c\bar{c})/\Gamma(H \to c\bar{c})$ from 5.5×10^{-2} to 1×10^{-2}. In absolute terms the errors of $H \to c\bar{c}$ and $H \to b\bar{b}$ are then compatible.

Finally, let us briefly mention another prominent decay mode of the Higgs boson, its decay into two gluons, which is available in order α_s^5 and given by [27]

$$\Gamma(H \to gg) = K\Gamma_{Born}(H \to gg) \tag{6.5}$$

with

$$K = 1 + 17.9167a_s + (156.81 - 5.71\ln\frac{M_t^2}{M_H^2})\,a_s^2 + (467.68 - 122.44\ln\frac{M_t^2}{M_H^2} + 10.94\ln^2\frac{M_t^2}{M_H^2})\,a_s^3. \tag{6.6}$$

For the specific choice $M_t = 175$ GeV, $M_H = 125$ GeV and $a_s = \alpha_s^{(5)}(M_t)/\pi = 0.0363$ one finds a correction factor

$$K = 1 + 17.9167\,a_s + 152.5\,a_s^2 + 381.5\,a_s^3 = 1 + 0.65038 + 0.20095 + 0.01825 = 1.86957 \tag{6.7}$$

Considering the claim that the experimental precision at a future electron-positron collider might reach 1.4%, experimental and theoretical uncertainties would match nicely.

Although the decay of the Higgs boson into photons constitutes only a small fraction of events, this is partly compensated by the fact that these events are particularly clean and thus can be dug out from a huge background. The one- and two-loop corrections can be written in the form [28]

$$\Gamma(H \to \gamma\gamma) = \frac{M_H^3}{64\pi}(A_{\rm LO}^2 + \frac{\alpha}{\pi}(2A_{\rm LO}A_{\rm NLO-EW}) + \frac{\alpha_s}{\pi}(2A_{\rm LO}A_{\rm NLO-QCD}) \tag{6.8}$$

$$+ \frac{\alpha_s}{\pi}^2(2A_{\rm LO}\,{\rm Re}(A_{\rm NNLO}) + A_{\rm NLO}^2)), \tag{6.9}$$

where the two-loop electroweak correction was taken from [29]. For the actual values $M_H = 126$ GeV, $m_t(M_H) = 166$ GeV and $\alpha_s(M_H)/\pi = 0.0358$ one finds

$$\Gamma(H \to \gamma\gamma) = (9.398 \times 10^{-6} - 1.48 \times 10^{-7} + 1.68 \times 10^{-7} + 7.93 \times 10^{-9})\text{ GeV} = 9.425 \times 10^{-6}\text{ GeV}, \tag{6.10}$$

where the four terms describe Born approximation, electroweak correction, QCD correction and order α_s and order α_s^2 respectively. Upon closer inspection one finds that this prediction is good to about one permille, which should be sufficient in the foreseeable future.

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