Supersymmetric explanation of $CP$ violation in $K \rightarrow \pi \pi$ decays

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Recent progress in the determination of hadronic matrix elements has revealed a tension between the measured value of $|\epsilon_K|/|\epsilon_K|$, which quantifies direct $CP$-violation in $K \rightarrow \pi \pi$ decays, and the Standard-Model prediction. The well-understood indirect $CP$ violation encoded in the quantity $\epsilon_K$ typically precludes large new-physics contributions to $\epsilon_K/\epsilon_K$ and challenges such an explanation of the discrepancy. We show that it is possible to cure the $\epsilon_K/\epsilon_K$ anomaly in the Minimal Supersymmetric Standard Model with squark masses above 3 TeV without overshooting $\epsilon_K$. This solution exploits two features of supersymmetry, the possibility of large isospin-breaking contributions (enhancing $\epsilon_K$) and the Majorana nature of gluinos (permitting a suppression of $\epsilon_K$). Our solution involves no fine-tuning of $CP$ phases or other parameters.

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INTRODUCTION

Measurements of charge-parity ($CP$) violation are sensitive probes of physics beyond the Standard Model (SM). $CP$ violation in $K \rightarrow \pi \pi$ decays is characterized by the two quantities $\epsilon_K$ and $\epsilon'_K$, which describe indirect and direct $CP$ violation, respectively. $|\epsilon_K| = (2.228 \pm 0.011) \times 10^{-3}$ measures $CP$ violation in the $K^0, \bar{K}^0$ mixing amplitude, in which the strangeness quantum number $S$ changes by 2 units [1]. $\epsilon'_K$ quantifies $CP$ violation in the $|\Delta S| = 1$ amplitude triggering the decay $K \rightarrow \pi \pi$. To predict $\epsilon'_K$ in the SM one must calculate hadronic matrix elements of four-quark operators with non-perturbative methods. A determination of all operators by lattice QCD has only been obtained recently [2] and the predicted $\epsilon'_K$ lies substantially below the experimental value [3]:

$$\frac{\epsilon'_K}{\epsilon_K} = \left\{ \begin{array}{l} (16.6 \pm 2.3) \times 10^{-4} \quad \text{(PDG [1])} \\ (1.4 \pm 4.3 \pm 1.4 \pm 0.6) \times 10^{-4} \quad \text{(SM-NLO)} \end{array} \right. \quad (1)$$

Our SM prediction [4] is based on the next-to-leading order (NLO) calculation of Wilson coefficients and anomalous dimensions [6, 7] and the hadronic matrix elements of Refs. [2, 8]. As in Ref. [9] we exploit $CP$-conserving data to reduce hadronic uncertainties. The two numbers in Eq. (1) disagree by 2.9σ [9]. This tension is underpinned by results found with the $1/N_c$ expansion [10, 11], which is a completely different calculational method [9]. In the near future the increasing precision of lattice calculations will sharpen the SM prediction in Eq. (1) further and answer the question about new physics (NP) in $\epsilon'_K$.

An explanation of the puzzle in Eq. (1) by physics beyond the SM calls for a NP contribution which is seemingly even larger than the SM value. On general grounds, however, one expects that NP effects in a $|\Delta F| = 1$ four-quark process are highly suppressed once constraints from the corresponding $|\Delta F| = 2$ transition are taken into account. Here $F$ denotes the flavour quantum number and $F = S$ in our case of $K \rightarrow \pi \pi$ decays. To explain the NP hierarchy in $|\Delta F| = 1$ vs. $|\Delta F| = 2$ transitions we specify to $\epsilon'_K$ and $\epsilon_K$: the SM contributions to both quantities are governed by the combination

$$\tau = \frac{V_{ud}V_{us}^*}{V_{ud}V_{us}} \sim (1.5 - i0.6) \times 10^{-3} \quad (2)$$

of elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix with $\epsilon_K^{SM} \propto \Im \tau/M_W^2$ and $\epsilon'_K^{SM} \propto \Im \tau^2/M_W^2$. If the NP contribution comes with the $\Delta S = 1$ parameter $\delta$ and is mediated by heavy particles of mass $M$, one finds $\epsilon'_K^{NP} \propto \Im \delta/M^2, \epsilon'_K^{NP} \propto \Im \delta^2/M^2$, and therefore

$$\frac{\epsilon'_K^{NP}/\epsilon_K^{NP}}{\epsilon'_K^{SM}/\epsilon_K^{SM}} \propto \mathcal{O} \left( \frac{\Re \tau}{\Re \delta} \right). \quad (3)$$

With $M \gtrsim 1$ TeV NP effects can only be relevant for $|\delta| \gg |\tau|$ and Eq. (3) seemingly forbids detectable NP contributions to $\epsilon'_K$. In this Letter we show that Eq. (3) can be overcome in the Minimal Supersymmetric Standard Model (MSSM) and one can reproduce the central value of the measured $\epsilon'_K$ in Eq. (1) with squark and gluino masses in the multi-TeV range. Our solution involves no fine-tuning of $CP$ phases or other parameters.

$\epsilon'_K$ IN THE MSSM

The MSSM is a good candidate for physics beyond the SM, because it alleviates the hierarchy problem, improves gauge coupling unification and provides dark-matter candidates. Present collider bounds [12] (and the large Higgs mass of 125 GeV [13, 14]) push the masses of colored superpartners into the TeV range, which makes supersymmetry an imperfect solution to the hierarchy problem but actually improves gauge coupling unification.
with $\omega_+ = (4.53 \pm 0.02) \cdot 10^{-2}$, the measured $|\epsilon^\text{CPS}|$, $\Omega_{\text{eff}} = (14.8 \pm 8.0) \cdot 10^{-2}$, and the amplitudes $A_1 = (\pi \pi) |H| [\Delta S = 1]|K^0$ involving the effective $|\Delta S| = 1$ Hamiltonian $H[\Delta S]$, $I = 0, 2$ labels the strong isospin of the final two-pion state. Im$A_2$ is under good control for some time [8]; the recent theory progress of Refs. [2, 11] concerns the QCD penguin contribution to Im$A_0$. The MSSM contribution to $\epsilon'_K$ simply adds to the SM piece. Supersymmetric contributions to $\epsilon'_K/\epsilon_K$ have been widely studied [15–20] in the past, but for a supersymmetry-breaking scale $M_S$ in the ballpark of the electroweak scale, so that the suppression mechanism inferred from Eq. (3) is avoided.

In the absence of sizable left-right squark mixing the low-energy Hamiltonian reads

$$\begin{align*}
\mathcal{H}_{\text{eff, SUSY}}^{[\Delta S=1]} &= \frac{G_F}{\sqrt{2}} \sum_q \sum_{\mu=1}^2 \left[ \frac{2}{c_1^q(\mu)} Q^q(\mu) \right. \\
&+ \frac{4}{c_1^q(\mu)} Q^q(\mu) + c_1^q(\mu) \tilde{Q}^q(\mu) \left. \right] + \text{H.c.,}
\end{align*}$$

where $G_F$ is the Fermi constant and

$$Q^q = (\bar{s}_q \gamma_\mu D_L \gamma_\mu \bar{d}_q)_{\mu\bar{\mu}} = \frac{1}{\sqrt{2}} (\bar{s}_q \gamma_\mu \bar{d}_q)_{\mu\bar{\mu}}, Q^q_{s} = (\bar{s}_q \gamma_\mu \bar{d}_q)_{\mu\bar{\mu}}, Q^q_{d} = (\bar{d}_q \gamma_\mu \bar{q}_d)_{\mu\bar{\mu}}, Q^q_{\epsilon} = (\bar{s}_q \gamma_\mu \bar{q}_d)_{\mu\bar{\mu}}.$$

Here $(\bar{s}_q \gamma_\mu \bar{d}_q)_{\mu\bar{\mu}}$ is the $V-A$ component of the gluino-box diagrams which feed the $(\pi \pi)_{\pi\pi}$ final state if the right-handed up and down squarks ($\tilde{U}$ and $\tilde{D}$) have different masses (see Fig. 1). The FCNC parameter is the $(1, 2)$ element of the left-handed down squark mass matrix $M_D$ induced by $\tilde{D}_L - \tilde{D}_L$ mixing. Second, the Majorana nature of the gluino leads to a suppression of the gluino-squark contribution to $\epsilon_K$, because there are two such diagrams (crossed and uncrossed boxes) with opposite signs. If the gluino mass $m_3$ equals roughly 1.5 times the average down squark mass $M_D$ and if either left-handed or right-handed squark mixing is suppressed, both contributions to $\epsilon'_K$ cancel [21]. For $m_3 > 1.5 M_D$ the gluino-box contribution approximately

\[\text{FIG. 1. Supersymmetric gluino box contribution to } \epsilon'_K/\epsilon_K \text{ (called Trojan penguin in [20]). It contributes to Im}\ A_2 \text{ for } m_U \neq m_D \text{ and is the largest contribution in our scenario. There are also crossed box diagrams.}\]

\[\text{behaves as } [m_3^2 - (1.5 M_D)^2]/m_3^4, \text{ with a shallow maximum at } m_3 \approx 2.5 M_D \text{ after which the } 1/m_3^2 \text{ decoupling sets in. In this parameter region also chargino, neutralino, and gluino-neutralino box diagrams are important [21] and are included in our numerics. The up-type squark mass matrix is } (V M_D^- V^t)_{ij} \text{ (up to negligible } \mathcal{O}(v^2) \text{ terms, where } v \text{ is the electroweak vev), so that also chargino diagrams are affected by squark flavour mixing. The measured } \epsilon_K \text{ agrees well with the SM expectation, if the global CKM fit uses the } |V_{ud}| \text{ measured in inclusive semileptonic } B \text{ decays [22], but exceeds } \epsilon^K_{\text{SM}} \text{ for the smaller } |V_{cb}| \text{ inferred from exclusive decays [23, 24]. Figure 2 shows that for both cases } \epsilon^{SM}_K + \epsilon^{SUSY}_K \text{ complies with } \epsilon^{exp}_K \text{ over a wide parameter range without fine-tuning.}\]

\[\text{To get the desired large effect in } \epsilon_K \text{ we need a contribution to the operators } Q^q_{s} \text{ with } (V - A) \times (V + A) \text{ Dirac structure, whose matrix elements are chirally enhanced by a factor } (m_3/m_s)^2. \text{ Therefore the flavour mixing has to be in the left-handed squark mass matrix. The opposite situation with right-handed flavour mixing and } \tilde{u}_L - \tilde{d}_L \text{ mass splitting is not possible, because SU(2)_L \text{ invariance enforces } M^2_{d_L} - M^2_{d_L} = \mathcal{O}(v^2). \text{ Therefore, our scenario involves flavour mixing between left-handed squarks only. We use the following notation for the squark mass matrices: } M^X_{X,ij} = m_X (\delta_{ij} + \Delta_{X,ij}), \text{ with } X = Q, U, \text{ or } D. \text{ Throughout this paper we use } m^2_3 = M^2_D = M^2_3 \text{ and vary } m_0. \text{ We have calculated all one-loop contributions to the coefficients in Eq. (5) in the squark mass eigenbasis and will present the full results elsewhere [25]. For the dominant “trojan penguin” contribution we confirm the result of Ref. [19] and find a typo in the expression for } c'_4 \text{ in Ref. [20]. The second largest contribution to } \epsilon'_K \text{ stems from the chromomagnetic penguin operator and our coefficient is in agreement with Refs. [26, 27]. To our knowledge, the other coefficients have only been obtained in mass insertion approximation [15] and our results agree upon expansion in } \Delta_{X,ij}. \text{ Our results also comply with the loop diagram results collected in Ref. [28]. The individual contributions to } \epsilon'_K/\epsilon_K \text{ are shown in Fig. 3.}\]

\[\text{For the calculation of } \epsilon'_K/\epsilon_K \text{ we must use the renormalization group (RG) equations to evolve the Wilson coefficients calculated at the high scale } \mu = M_S \text{ down to the hadronic scale } \mu_h = \mathcal{O}(1 \text{ GeV}) \text{ at which the operator matrix elements are calculated. In order to use the well-known NLO } 10 \times 10 \text{ anomalous dimensions for the SM}\]
FIG. 2. The left plot shows $\epsilon_K^{\text{SUSY}} / \epsilon_K^{\exp}$ as a function of the gluino-squark mass ratio $m_{\tilde{g}} / M_S$, where we take $M_S = m_Q = m_{\tilde{g}} = 10$ TeV. The red line shows the gluino-gluino box contribution (with the zero crossing near $m_{\tilde{g}} / M_S = 1.5$ [21]), while the blue line denotes the sum of the box contributions with one or two winos. The total contribution is shown in black. The red (blue) regions are excluded by the measurement of $\epsilon_K$ at the 95% confidence level (C.L.), if the SM prediction uses the inclusive (exclusive) measurement of $|V_{cb}|$ [23]. On the right, the black lines show $|\epsilon_K^{\text{SUSY}}|$ for several gluino-squark mass ratios as a function of the squark mass.

FIG. 3. Individual supersymmetric contributions to $|\epsilon_K^{\text{SUSY}} / \epsilon_K^{\exp}|$ as a function of $M_S = m_Q = m_{\tilde{g}}$. $\tilde{g}\tilde{g}, \tilde{g}\tilde{g}_s, \tilde{g}g, \tilde{g}g, \tilde{g}Z, 3\tilde{g}Z$ and $\tilde{g}g$ represent the gluino-gluino and gluino-neutralino boxes, gluino-photino-, and Z-penguins, chargino and neutralino Z-penguins, and chromomagnetic contributions, respectively. The thick lines show the case of universal squark masses, $m_{\tilde{Q}} = M_S$. The broken black lines are the gluino-gluino box contributions for $m_{\tilde{g}} / M_S = 0.5, 2.0, 8.0, 12.0$ from top to bottom. The $\epsilon_K/\epsilon_K$ discrepancy is resolved at 1(2)σ in the dark (light) green band.

four-fermion operator basis [6] we switch from Eq. (5) to

$$
H_{\text{eff, SUSY}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} \left( C_i(\mu)Q_i(\mu) + \tilde{C}_i(\mu)\tilde{Q}_i(\mu) \right) + \text{H.c.},
$$

where $Q_{1,...,10}$ are given in Ref. [6, 7], and

$$
C_{1,2}(\mu) = \epsilon_{1,2}\left(\mu\right), \quad \tilde{C}_{1,2}(\mu) = 0,
$$

$$
C_{3,4,5,6}(\mu) = \frac{1}{3} \left( \epsilon_{3,4,5,6}\left(\mu\right) + 2\epsilon_{3,4,5,6}\left(\mu\right) \right),
$$

$$
C_{7,8,9,10}(\mu) = \frac{2}{3} \left( \epsilon_{7,8,9,10}\left(\mu\right) - \epsilon_{7,8,9,10}\left(\mu\right) \right),
$$

and the coefficients $\tilde{C}_{3,...,10}$ of the opposite-chirality operators are found from $C_{3,...,10}$ by replacing $\epsilon_{i} \rightarrow \epsilon_{i}$. Note that $C_{7,8}$ receive the contribution of Fig. 1.

For the evolution of the coefficients from $\mu = M_S$ to $\mu = \mu_h$ we use a new analytical solution of the RG equations which avoids the problem of a singularity in the NLO terms discussed in Ref. [29]. For $H_{\text{eff, SUSY}}^{\Delta S=1}$ we employ proper threshold matching at the scales $\mu_{t,b,c}$ set by the top bottom, and charm quark masses with the usual threshold matching matrices [7]. In our analysis we take $\mu_h = 1.3$ GeV. For the SM prediction in Eq. (1) and the calculation of the MSSM prediction we have evolved the matrix elements of Refs. [2, 8] (which are given at $\mu = 1.531$ GeV for $A_0$ and at $\mu = 3.0$ GeV for $A_2$) to $\mu_h$ with three-flavour full NLO operator mixing. The use of NLO RG formulae for $H_{\text{eff, SUSY}}^{\Delta S=1}$ involves a relative error of order $\alpha_s(M_S)$, because the two-loop corrections to the initial conditions of the Wilson coefficients are not included. However, the NLO corrections proportional to the much larger $\alpha_s(\mu_h)$ are all correctly included and independent of the renormalization scheme.

PHENOMENOLOGY OF $\epsilon_K$ AND $\epsilon_K^{\text{SUSY}}$

In this section we study $\epsilon_K$ and $\epsilon_K^{\text{SUSY}}$ in the MSSM parameter region in which the discrepancy in Eq. (1) is removed. As input we take $\alpha_s(M_Z) = 0.1185$, the GUT relation for gaugino masses, $m_{\tilde{g}}/M_S = 1.5$, and $m_Q = m_{\tilde{Q}} = m_{\tilde{g}} = M_S$, where $m_{\tilde{g}}$ is the higgsino mass parameter. Furthermore, the trilinear supersymmetry-breaking matrices $A_q$ are set to zero, $\tan \beta = 10$, and the only non-zero off-diagonal elements of the squark mass matrices are $\Delta_{Q_{12,13,23}} = 0.1 \cdot \exp(-i\pi/4)$ and $(V_{Q_{1}}V')_{ij}$ for the left-handed down and up sectors, respectively. For the CKM elements we use CKMfitter results [24].

Starting with $\epsilon_K$, we first note that the phase of
the SUSY contribution to the $K^0$-$\bar{K}^0$ mixing amplitude is essentially twice the phase of $\Delta_{Q,12}$. That is, our choice of $\pi/4$ for this phase maximizes the CP phase and is far away from a fine-tuned suppression of $\epsilon_K$. We evaluate the MSSM Wilson coefficients for $\epsilon_K$ with the $O(g_s^4, g_s^2g^2, g^4)$ strong and weak contributions [21, 30]. For the RG evolution of the MSSM contribution the LO formula is sufficient [31]; lattice results for $|\Delta S| = 2$ hadronic matrix elements are available from several groups [32]. For an accurate SM prediction of $\epsilon_K$ one must include all NLO corrections [33] and the NNLO contributions involving the low charm scale [34]. At this level $\epsilon_K^{\text{SM}}$ agrees with $\epsilon_K^{\exp}$, if the value of $|V_{cb}|$ measured in inclusive $b \rightarrow c\ell\nu$ decays is used for the calculation of the CKM elements. Figure 2 shows that the MSSM can accommodate this situation as well as the scenario with $|V_{cb}|$ taken from exclusive $B \rightarrow D^{(*)}\ell\nu$ decays [35], which calls for a new-physics contribution to $\epsilon_K$. The left plot of Fig. 2 clearly reveals that the MSSM solution is not fine-tuned, but merely requires $m_3/M_S \gtrsim 1.5$. For our chosen parameters we roughly find $M_S \gtrsim 3$ TeV, with the possibility of slightly lighter squarks if the exclusive $|V_{cb}|$ is true.

We note that our results are stable if we switch on right-handed squark mixing as long as $\Delta_{Q,12} \lesssim 10^{-5}$. Although in our scenario $\Delta_{Q,12}$ is generated by radiative corrections, the value is smaller than $10^{-5}$ thanks to the small down Yukawa coupling.

We next turn to the discussion of $\epsilon_K'$: the thick lines in Fig. 3 show the individual contributions to $|\epsilon_K'/\epsilon_K|$ for the case of universal squark masses. The broken lines show that already a moderate $U$-$D$ mass splitting suffices to explain the measured value (indicated by the green bands). The second-largest contribution from the chromomagnetic penguin diagram comes with a poorly known hadronic matrix element [36]. The $B$ parameter parametrising this matrix element is estimated as $B_G = 1 \pm 3$ [18]. The yellow band in in Fig. 3 is for $1 \leq B_G \leq 4$. Next we remark that in our parameter region the gluino-photon (red line) and chargino-$Z$ (blue line) penguins have opposite sign and almost cancel each other. (This picture changes with non-zero trilinear terms, e.g. $|A_{d,21}| = 0.1 M_S$ ($|A_{u,31}A_{u,32}| = 0.1 M_S^2$) can lift the chromomagnetic (chargino-$Z$) contribution by about 40% (140%).) We have neglected the gluino-$W$ penguin and the gluino-chargino box contributions, which matches onto $\epsilon_K'$ at $\mu = M_S$ and gives at most an $O(10^{-5})$ contribution to $\epsilon_K'/\epsilon_K$.

Figure 4 shows our main result, the portion of the squark mass plane which simultaneously explains $\epsilon_K'/\epsilon_K$ and $\epsilon_K$. The figure uses the complete supersymmetric results except for the chromomagnetic contribution to $\epsilon_K'$ because of the uncertainty in $B_G$. The red region is excluded by the measurement of $\epsilon_K$ at 95% C.L. in combination with the inclusive $V_{cb}$, while the region between the blue-dashed lines can explain the $\epsilon_K$ discrepancy at 95% C.L. for the exclusive value of $|V_{cb}|$. Note that we also found that there are no constraints from the mass difference of neutral Kaon, $D^0$-$\bar{D}^0$ mixing [37] and the neutron EDM [38].

**CONCLUSIONS**

In this Letter, we have calculated $\epsilon_K'$ in the MSSM and have shown that the large contributions needed to solve the $\epsilon_K'/\epsilon_K$ discrepancy in Eq. (1) can be obtained for squark and gluino masses in the multi-TeV range. The constraint from $\epsilon_K'$, which in generic models of new physics precludes large effects in $\epsilon_K'$, can be fulfilled without fine-tuning.

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The first uncertainty stems from the lattice results. The second and third are from perturbative higher-order corrections and the isospin-violating terms [5], respectively. The uncertainty from the CKM factor $\text{Im} V_{ud} V_{ud}$ is $0.1 \times 10^{-4}$ [9].