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Supersymmetric explanation of CP violation in $K \to \pi\pi$ decays

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Recent progress in the determination of hadronic matrix elements has revealed a tension between the measured value of ϵ'_K/ϵ_K, which quantifies direct CP-violation in $K \to \pi\pi$ decays, and the Standard-Model prediction. The well-understood indirect CP violation encoded in the quantity ϵ_K typically precludes large new-physics contributions to ϵ'_K/ϵ_K and challenges such an explanation of the discrepancy. We show that it is possible to cure the ϵ'_K/ϵ_K anomaly in the Minimal Supersymmetric Standard Model with squark masses above 3 TeV without overshooting ϵ_K. This solution exploits two features of supersymmetry, the possibility of large isospin-breaking contributions (enhancing ϵ'_K) and the Majorana nature of gluinos (permitting a suppression of ϵ_K). Our solution involves no fine-tuning of CP phases or other parameters.

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INTRODUCTION

Measurements of charge-parity (CP) violation are sensitive probes of physics beyond the Standard Model (SM). CP violation in $K \to \pi\pi$ decays is characterized by the two quantities ϵ_K and ϵ'_K, which describe indirect and direct CP violation, respectively. $|\epsilon_K| = (2.228\pm0.011)\cdot 10^{-3}$ measures CP violation in the K^0-$\overline{K}^0$ mixing amplitude, in which the strangeness quantum number S changes by 2 units [1]. ϵ'_K quantifies CP violation in the $|\Delta S| = 1$ amplitude triggering the decay $K \to \pi\pi$. To predict ϵ'_K in the SM one must calculate hadronic matrix elements of four-quark operators with non-perturbative methods. A determination of all operators by lattice QCD has only been obtained recently [2] and the predicted ϵ'_K lies substantially below the experimental value [3]:

$$\frac{\epsilon'_K}{\epsilon_K} = \begin{cases} (16.6 \pm 2.3) \times 10^{-4} & \text{(PDG [1])} \\ (1.4 \pm 4.3 \pm 1.4 \pm 0.6) \times 10^{-4} & \text{(SM-NLO)} \end{cases} \quad (1)$$

Our SM prediction [4] is based on the next-to-leading order (NLO) calculation of Wilson coefficients and anomalous dimensions [6, 7] and the hadronic matrix elements of Refs. [2, 8]. As in Ref. [9] we exploit CP-conserving data to reduce hadronic uncertainties. The two numbers in Eq. (1) disagree by $2.9\,\sigma$ [9]. This tension is underpinned by results found with the $1/N_c$ expansion [10, 11], which is a completely different calculational method [9]. In the near future the increasing precision of lattice calculations will sharpen the SM prediction in Eq. (1) further and answer the question about new physics (NP) in ϵ'_K.

An explanation of the puzzle in Eq. (1) by physics beyond the SM calls for a NP contribution which is seemingly even larger than the SM value. On general grounds, however, one expects that NP effects in a $|\Delta F| = 1$ four-quark process are highly suppressed once constraints from the corresponding $|\Delta F| = 2$ transition are taken into account. Here F denotes the flavour quantum number and $F = S$ in our case of $K \to \pi\pi$ decays. To explain the NP hierarchy in $|\Delta F| = 1$ vs. $|\Delta F| = 2$ transitions we specify to ϵ'_K and ϵ_K: the SM contributions to both quantities are governed by the combination

$$\tau = -\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*} \sim (1.5 - i0.6)\cdot 10^{-3} \quad (2)$$

of elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix with $\epsilon'^{\rm SM}_K \propto {\rm Im}\,\tau/M_W^2$ and $\epsilon^{\rm SM}_K \propto {\rm Im}\,\tau^2/M_W^2$. If the NP contribution comes with the $\Delta S = 1$ parameter δ and is mediated by heavy particles of mass M, one finds $\epsilon'^{\rm NP}_K \propto {\rm Im}\,\delta/M^2$, $\epsilon^{\rm NP}_K \propto {\rm Im}\,\delta^2/M^2$, and therefore

$$\frac{\epsilon'^{\rm NP}_K/\epsilon'^{\rm SM}_K}{\epsilon^{\rm NP}_K/\epsilon^{\rm SM}_K} = \mathcal{O}\left(\frac{{\rm Re}\,\tau}{{\rm Re}\,\delta}\right). \quad (3)$$

With $M \gtrsim 1\,$TeV NP effects can only be relevant for $|\delta| \gg |\tau|$ and Eq. (3) seemingly forbids detectable NP contributions to ϵ'_K. In this Letter we show that Eq. (3) can be overcome in the Minimal Supersymmetric Standard Model (MSSM) and one can reproduce the central value of the measured ϵ'_K in Eq. (1) with squark and gluino masses in the multi-TeV range. Our solution involves no fine-tuning of CP phases or other parameters.

ϵ'_K IN THE MSSM

The MSSM is a good candidate for physics beyond the SM, because it alleviates the hierarchy problem, improves gauge coupling unification and provides dark-matter candidates. Present collider bounds [12] (and the largish Higgs mass of 125 GeV [13, 14]) push the masses of colored superpartners into the TeV range, which makes supersymmetry an imperfect solution to the hierarchy problem but actually improves gauge coupling unification.

The master equation for ϵ'_K reads [9]:

$$\frac{\epsilon'_K}{\epsilon_K} = \frac{\omega_+}{\sqrt{2}|\epsilon_K^{\rm exp}|{\rm Re}A_0^{\rm exp}} \left\{ \frac{{\rm Im}A_2}{\omega_+} - \left(1 - \hat{\Omega}_{\rm eff}\right) {\rm Im}A_0 \right\}, \tag{4}$$

with $\omega_+ = (4.53 \pm 0.02) \cdot 10^{-2}$, the measured $|\epsilon_K^{\rm exp}|$, $\hat{\Omega}_{\rm eff} = (14.8 \pm 8.0) \cdot 10^{-2}$, and the amplitudes $A_I = \langle (\pi\pi)_I | \mathcal{H}^{|\Delta S|=1} | K^0 \rangle$ involving the effective $|\Delta S| = 1$ Hamiltonian $\mathcal{H}^{|\Delta S|}$. $I = 0, 2$ labels the strong isospin of the final two-pion state. ${\rm Im}A_2$ is under good control for some time [8]; the recent theory progress of Refs. [2, 11] concerns the QCD penguin contribution to ${\rm Im}A_0$. The MSSM contribution to ϵ'_K simply adds to the SM piece. Supersymmetric contributions to ϵ'_K/ϵ_K have been widely studied [15–20] in the past, but for a supersymmetry-breaking scale M_S in the ballpark of the electroweak scale, so that the suppression mechanism inferred from Eq. (3) is avoided.

In the absence of sizable left-right squark mixing the low-energy Hamiltonian reads

$$\mathcal{H}^{|\Delta S|=1}_{\text{eff, SUSY}} = \frac{G_F}{\sqrt{2}} \sum_q \Bigg[\sum_{i=1}^{2} c_i^q(\mu) Q_i^q(\mu) + \sum_{i=1}^{4} \left(c_i'^q(\mu) Q_i'^q(\mu) + \tilde{c}_i'^q(\mu) \tilde{Q}_i'^q(\mu) \right) \Bigg] + \text{H.c.}, \tag{5}$$

where G_F is the Fermi constant and

$$\begin{aligned} Q_1^q &= (\bar{s}_\alpha q_\beta)_{V-A} (\bar{q}_\beta d_\alpha)_{V-A}, \; Q_2^q = (\bar{s} q)_{V-A} (\bar{q} d)_{V-A}, \\ Q_1'^q &= (\bar{s} d)_{V-A} (\bar{q} q)_{V+A}, \; Q_2'^q = (\bar{s}_\alpha d_\beta)_{V-A} (\bar{q}_\beta q_\alpha)_{V+A}, \\ Q_3'^q &= (\bar{s} d)_{V-A} (\bar{q} q)_{V-A}, \; Q_4'^q = (\bar{s}_\alpha d_\beta)_{V-A} (\bar{q}_\beta q_\alpha)_{V-A}. \end{aligned} \tag{6}$$

Here $(\bar{s}d)_{V-A}(\bar{q}q)_{V\pm A} = (\bar{s}\gamma_\mu(1-\gamma_5)d)(\bar{q}\gamma^\mu(1\pm\gamma_5)q)$, α and β are color indices, and and opposite-chirality operators $\tilde{Q}_i'^q$ are found by interchanging $V-A \leftrightarrow V+A$. Our solution exploits two special features of supersymmetric theories: first, there are loops governed by the strong interaction which contribute to ${\rm Im}A_2$ entering Eq. (4) with the enhancement factor $1/\omega_+ = 22.1$ [19, 20]. These are gluino-box diagrams which feed the $(\pi\pi)_{I=2}$ final state if the right-handed up and down squarks ($\tilde{\bar{U}}$ and $\tilde{\bar{D}}$) have different masses (see Fig. 1). The FCNC parameter is the $(1,2)$ element of the left-handed down squark mass matrix M_Q^2 inducing $\tilde{s}_L$–$\tilde{d}_L$ mixing. Second, the Majorana nature of the gluino leads to a suppression of the gluino-squark contribution to ϵ_K, because there are *two* such diagrams (crossed and uncrossed boxes) with opposite signs. If the gluino mass $m_{\tilde{g}}$ equals roughly 1.5 times the average down squark mass M_S and if either left-handed or right-handed squark mixing is suppressed, both contributions to $\epsilon_K^{\rm SUSY}$ cancel [21]. For $m_{\tilde{g}} > 1.5 M_S$ the gluino-box contribution approximately behaves as $[m_{\tilde{g}}^2 - (1.5M_S)^2]/m_{\tilde{g}}^4$, with a shallow maximum at $m_{\tilde{g}} \simeq 2.5 M_S$ after which the $1/m_{\tilde{g}}^2$ decoupling sets in. In this parameter region also chargino, neutralino, and gluino-neutralino box diagrams are important [21] and are included in our numerics. The up-type squark mass matrix is $(V M_Q^2 V^\dagger)_{ij}$ (up to negligible $\mathcal{O}(v^2)$ terms, where v is the electroweak vev), so that also chargino diagrams are affected by squark flavour mixing. The measured ϵ_K agrees well with the SM expectation, if the global CKM fit uses the $|V_{cb}|$ measured in inclusive semileptonic B decays [22], but exceeds $\epsilon_K^{\rm SM}$ for the smaller $|V_{cb}|$ inferred from exclusive decays [23, 24]. Figure 2 shows that for both cases $\epsilon_K^{\rm SM} + \epsilon_K^{\rm SUSY}$ complies with $\epsilon_K^{\rm exp}$ over a wide parameter range without fine-tuning.

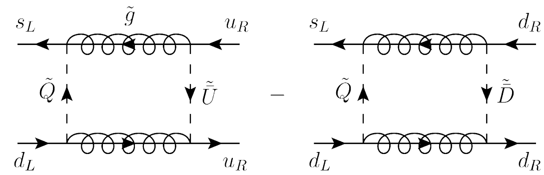

FIG. 1. Supersymmetric gluino box contribution to ϵ'_K/ϵ_K (called *Trojan penguin* in [20]). It contributes to ${\rm Im}\,A_2$ for $m_{\bar{U}} \neq m_{\bar{D}}$ and is the largest contribution in our scenario. There are also crossed box diagrams.

To get the desired large effect in ϵ'_K we need a contribution to the operators $Q'_{1,2}$ with $(V-A)\times(V+A)$ Dirac structure, whose matrix elements are chirally enhanced by a factor $(m_K/m_s)^2$. Therefore the flavour mixing has to be in the left-handed squark mass matrix. The opposite situation with right-handed flavour mixing and $\tilde{u}_L$–$\tilde{d}_L$ mass splitting is not possible, because $\text{SU}(2)_L$ invariance enforces $M^2_{\tilde{u}_L} - M^2_{\tilde{d}_L} = \mathcal{O}(v^2)$. Therefore, our scenario involves flavour mixing between left-handed squarks only. We use the following notation for the squark mass matrices: $M^2_{X,ij} = m_X^2 \left(\delta_{ij} + \Delta_{X,ij}\right)$, with $X = Q, \bar{U}$, or $\bar{D}$. Throughout this paper we use $m_Q^2 = m_{\bar{D}}^2 = M_S^2$ and vary $m_{\bar{U}}$. We have calculated all one-loop contributions to the coefficients in Eq. (5) in the squark mass eigenbasis and will present the full results elsewhere [25]. For the dominant "trojan penguin" contribution we confirm the result of Ref. [19] and find a typo in the expression for c'_4 in Ref. [20]. The second largest contribution to ϵ'_K stems from the chromomagnetic penguin operator and our coefficient is in agreement with Refs. [26, 27]. To our knowledge, the other coefficients have only been obtained in mass insertion approximation [15] and our results agree upon expansion in $\Delta_{X,ij}$. Our results also comply with the loop diagram results collected in Ref. [28]. The individual contributions to ϵ'_K/ϵ_K are shown in Fig. 3.

For the calculation of ϵ'_K/ϵ_K we must use the renormalization group (RG) equations to evolve the Wilson coefficients calculated at the high scale $\mu = M_S$ down to the hadronic scale $\mu_h = \mathcal{O}(1\text{ GeV})$ at which the operator matrix elements are calculated. In order to use the well-known NLO 10×10 anomalous dimensions for the SM

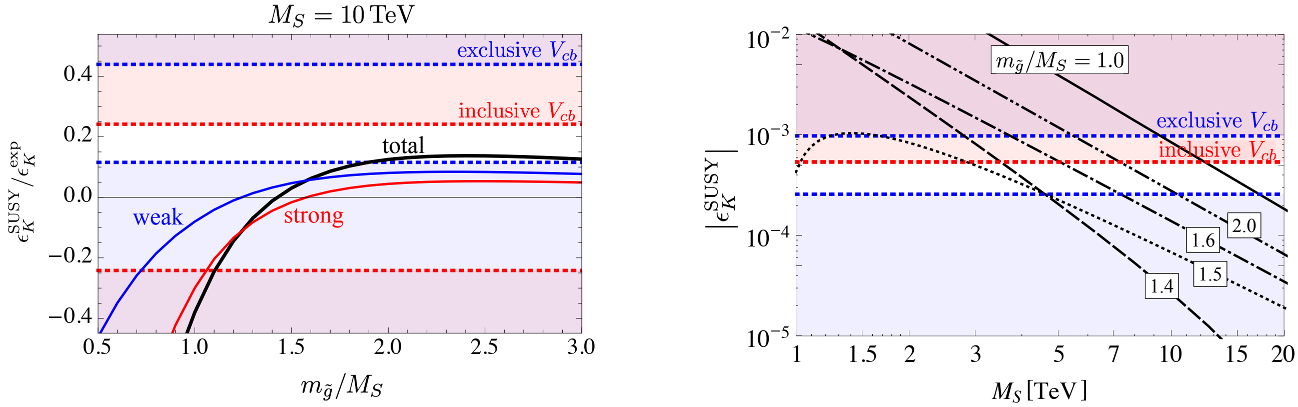

FIG. 2. The *left* plot shows $\epsilon_K^{\rm SUSY}/\epsilon_K^{\rm exp}$ as a function of the gluino-squark mass ratio $m_{\tilde g}/M_S$, where we take $M_S = m_Q = m_{\bar U} = m_{\bar D} = 10\,$TeV. The red line shows the gluino-gluino box contribution (with the zero crossing near $m_{\tilde g}/M_S = 1.5$ [21]), while the blue line denotes the sum of the box contributions with one or two winos. The total contribution is shown in black. The red (blue) regions are excluded by the measurement of ϵ_K at the 95 % confidence level (C.L.), if the SM prediction uses the inclusive (exclusive) measurement of $|V_{cb}|$ [23]. On the *right*, the black lines show $|\epsilon_K^{\rm SUSY}|$ for several gluino-squark mass ratios as a function of the squark mass.

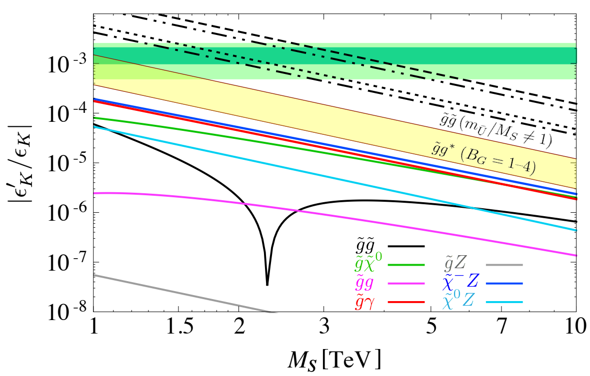

FIG. 3. Individual supersymmetric contributions to $|\epsilon'_K/\epsilon_K|$ as a function of $M_S = m_Q = m_{\bar D}$. $\tilde g\tilde g$, $\tilde g\tilde\chi^0$, $\tilde g g$, $\tilde g\gamma$, $\tilde g Z$, $\tilde\chi^- Z$, $\tilde\chi^0 Z$ and $\tilde g g^*$ represent the gluino-gluino and gluino-neutralino boxes, gluino gluon-, photon-, and Z-penguins, chargino and neutralino Z-penguins, and chromomagnetic contributions, respectively. The thick lines show the case of universal squark masses, $m_{\bar U} = M_S$. The broken black lines are the gluino-gluino box contributions for $m_{\bar U}/M_S = 0.5, 2.0, 0.8, 1.2$ from top to bottom. The ϵ'_K/ϵ_K discrepancy is resolved at $1\,(2)\,\sigma$ in the dark (light) green band.

four-fermion operator basis [6] we switch from Eq. (5) to

$$\mathcal{H}^{|\Delta S|=1}_{\text{eff, SUSY}} = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} \Big(C_i(\mu) Q_i(\mu) + \tilde C_i(\mu) \tilde Q_i(\mu) \Big) + \text{H.c.}, \tag{7}$$

where $Q_{1,\ldots,10}$ are given in Ref. [6, 7], and

$$\begin{aligned} C_{1,2}(\mu) &= c^u_{1,2}(\mu), \qquad \tilde C_{1,2}(\mu) = 0, \\ C_{3,4,5,6}(\mu) &= \frac{1}{3}\left(c'^{u}_{3,4,1,2}(\mu) + 2 c'^{d}_{3,4,1,2}(\mu) \right), \\ C_{7,8,9,10}(\mu) &= \frac{2}{3}\left(c'^{u}_{1,2,3,4}(\mu) - c'^{d}_{1,2,3,4}(\mu) \right), \end{aligned} \tag{8}$$

and the coefficients $\tilde C_{3,\ldots 10}$ of the opposite-chirality operators are found from $C_{3,\ldots 10}$ by replacing $c'^{q}_i \to \tilde c'^{q}_i$. Note that $C_{7,8}$ receive the contribution of Fig. 1.

For the evolution of the coefficients from $\mu = M_S$ to $\mu = \mu_h$ we use a new analytical solution of the RG equations which avoids the problem of a singularity in the NLO terms discussed in Ref. [29]. For $\mathcal{H}^{|\Delta S|=1}_{\text{eff, SUSY}}$ we employ proper threshold matching at the scales $\mu_{t,b,c}$ set by the top bottom, and charm quark masses with the usual threshold matching matrices [7]. In our analysis we take $\mu_h = 1.3$ GeV. For the SM prediction in Eq. (1) and the calculation of the MSSM prediction we have evolved the matrix elements of Refs. [2, 8] (which are given at $\mu = 1.531$ GeV for A_0 and at $\mu = 3.0$ GeV for A_2) to μ_h with three-flavour full NLO operator mixing. The use of NLO RG formulae for $\mathcal{H}^{|\Delta S|=1}_{\text{eff, SUSY}}$ involves a relative error of order $\alpha_s(M_S)$, because the two-loop corrections to the initial conditions of the Wilson coefficients are not included. However, the NLO corrections proportional to the much larger $\alpha_s(\mu_h)$ are all correctly included and independent of the renormalization scheme.

PHENOMENOLOGY OF ϵ_K AND ϵ'_K

In this section we study ϵ_K and ϵ'_K/ϵ_K in the MSSM parameter region in which the discrepancy in Eq. (1) is removed. As input we take $\alpha_s(M_Z) = 0.1185$, the GUT relation for gaugino masses, $m_{\tilde g}/M_S = 1.5$, and $m_Q = m_{\bar D} = \mu_{\rm SUSY} = M_S$, where $\mu_{\rm SUSY}$ is the higgsino mass parameter. Furthermore, the trilinear supersymmetry-breaking matrices A_q are set to zero, $\tan\beta = 10$, and the only non-zero off-diagonal elements of the squark mass matrices are $\Delta_{Q,12,13,23} = 0.1 \cdot \exp(-i\pi/4)$ and $(V\Delta_Q V^\dagger)_{ij}$ for the left-handed down and up sectors, respectively. For the CKM elements we use CKMfitter results [24].

Starting with ϵ_K, we first note that the phase of the SUSY contribution to the K^0-$\overline{K}^0$ mixing amplitude is essentially twice the phase of $\Delta_{Q,12}$. That is, our choice of $\pi/4$ for this phase maximizes the CP phase and is far away from a fine-tuned solution to suppress ϵ_K. We evaluate the MSSM Wilson coefficients for ϵ_K with the $\mathcal{O}(g_s^4, g_s^2 g^2, g^4)$ strong and weak contributions [21, 30]. For the RG evolution of the MSSM contribution the LO formula is sufficient [31]; lattice results for $|\Delta S| = 2$ hadronic matrix elements are available from several groups [32]. For an accurate SM prediction of ϵ_K one must include all NLO corrections [33] and the NNLO contributions involving the low charm scale [34]. At this level $\epsilon_K^{\rm SM}$ agrees with $\epsilon_K^{\rm exp}$, if the value of $|V_{cb}|$ measured in inclusive $b \to c\ell\nu$ decays is used for the calculation of the CKM elements. Figure 2 shows that the MSSM can accommodate this situation as well as the scenario with $|V_{cb}|$ taken from exclusive $B \to D^{(*)}\ell\nu$ decays [35], which calls for a new-physics contribution to ϵ_K. The left plot of Fig. 2 clearly reveals that the MSSM solution is not fine-tuned, but merely requires $m_{\tilde{g}}/M_S \gtrsim 1.5$. For our chosen parameters we roughly find $M_S \gtrsim 3$ TeV, with the possibility of slightly lighter squarks if the exclusive $|V_{cb}|$ is true.

We note that our results are stable if we switch on right-handed squark mixing as long as $\Delta_{\bar{D},12} \lesssim 10^{-5}$. Although in our scenario $\Delta_{\bar{D},12}$ is generated by radiative corrections, the value is smaller than 10^{-5} thanks to the small down Yukawa coupling.

We next turn to the discussion of ϵ'_K: the thick lines in Fig. 3 show the individual contributions to $|\epsilon'_K/\epsilon_K|$ for the case of universal squark masses. The broken lines show that already a moderate $\bar{U}$-$\bar{D}$ mass splitting suffices to explain the measured value (indicated by the green bands). The second-largest contribution from the chromomagnetic penguin diagram comes with a poorly known hadronic matrix element [36]. The B parameter parametrising this matrix element is estimated as $B_G = 1 \pm 3$ [18]. The yellow band in in Fig. 3 is for $1 \le B_G \le 4$. Next we remark that in our parameter region the gluino-photon (red line) and chargino-Z (blue line) penguins have opposite sign and almost cancel each other. (This picture changes with non-zero trilinear terms, e.g. $|A_{d,21}| = 0.1 M_S$ ($|A_{u,31} A_{u,32}| = 0.1 M_S^2$) can lift the chromomagnetic (chargino-Z) contribution by about 40 % (140 %).) We have neglected the gluino-W penguin and the gluino-chargino box contributions, which matches onto $c_{1,2}^u$ at $\mu = M_S$ and gives at most an $\mathcal{O}(10^{-5})$ contribution to ϵ'_K/ϵ_K.

Figure 4 shows our main result, the portion of the squark mass plane which simultaneous explains ϵ'_K/ϵ_K and ϵ_K. The figure uses the complete supersymmetric results except for the chromomagnetic contribution to ϵ'_K because of the uncertainty in B_G. The red region is excluded by the measurement of ϵ_K at 95 % C.L. in combination with the inclusive V_{cb}, while the region between the blue-dashed lines can explain the ϵ_K discrepancy at

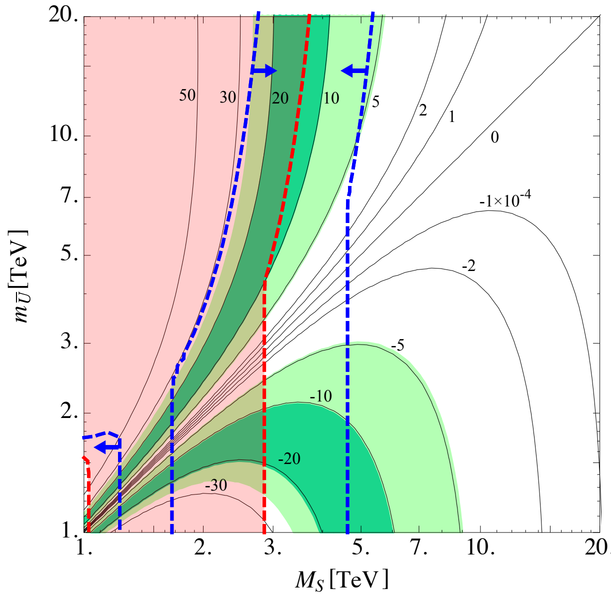

FIG. 4. Contours of the supersymmetric contributions to ϵ'_K/ϵ_K in units of 10^{-4}. The ϵ'_K/ϵ_K discrepancy is resolved at 1 (2) σ in the dark (light) green region. The red shaded region is excluded by ϵ_K with inclusive $|V_{cb}|$ at 95 % C.L., while the region between the blue-dashed lines can explain the ϵ_K discrepancy which is there for the exclusive $|V_{cb}|$. The green regions labeled with negative ϵ'_K/ϵ_K correspond to the change $\Delta_{Q,12,13,23} = 0.1\exp(-i\pi/4) \to \Delta_{Q,12,13,23} = 0.1\exp(i3\pi/4)$, which flips the sign of $\epsilon_K^{\prime\rm SUSY}$ (making it positive) while leaving ϵ_K essentially unchanged.

95 % C.L. for the exclusive value of $|V_{cb}|$. Note that we also found that there are no constraints from the mass difference of neutral Kaon, D^0-$\overline{D}^0$ mixing [37] and the neutron EDM [38].

CONCLUSIONS

In this Letter, we have calculated ϵ'_K in the MSSM and have shown that the large contributions needed to solve the discrepancy in Eq. (1) can be obtained for squark and gluino masses in the multi-TeV range. The constraint from ϵ_K, which in generic models of new physics precludes large effects in ϵ'_K, can be fulfilled without fine-tuning.

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