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Chapter 1

Sum Rules for Leptons

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There is a wide class of models which give a dynamical description of the origin of flavour in terms of spontaneous symmetry breaking of an underlying symmetry. Many of these models exhibit sum rules which relate on the one hand mixing angles and the Dirac CP phase with each other and/or on the other hand neutrino masses and Majorana phases with each other. We will briefly sketch how this happens and discuss briefly the impact of renormalisation group corrections to the mass sum rules.

1. Introduction

The origin of flavour in the Standard Model of Particle Physics (SM) is still a big puzzle. It is not clear why there are three generations of fermions exhibiting this very peculiar patterns of masses and mixing parameters. A very popular approach in recent years has been the use of non-Abelian (discrete) family symmetries driven by the rather large mixing angles in the lepton sector, for recent reviews, see, e.g., [1].

Nevertheless, in this proceedings we do not want to dive into cumbersome model building details. Instead we want to focus on two classes of predictions which appear in a very wide class of flavour models. To be precise we want to discuss two kinds of sum rules. The first type, mixing sum rules, relates the leptonic mixing angles to the Dirac CP violating phase while while the second type, mass rum rules, relates the neutrino masses to the Majorana phases.

After discussing the two cases separately we will give an example where both kinds of sum rules appear which makes the model extremely predictive. And then we will summarise and conclude.

2. A Short Note on Mixing Sum Rules

Probably better known and studied then the mass sum rules are so-called mixing sum rules. Typically, they emerge when the effective light Majorana neutrino mass

matrix exhibits a symmetry pattern, like, for instance, bimaximal mixing [2] or tri-bimaximal mixing [3]. If the charged lepton mass matrix would be diagonal in that basis, the leptonic mixing angles would be exactly predicted at the symmetry breaking scale. But since many of the popular mixing schemes exhibited a vanishing reactor mixing angle $\theta_{13} = 0$, this setup is disfavoured by the measurement of $\theta_{13} \approx 9°$.

There are plenty of possible modifications on the market which we cannot all discuss here exhaustively. Instead, we chose a case which we consider to be well motivated. Namely, that the charged lepton mass matrix is not simply diagonal, but has a sizeable mixing of the order of the Cabibbo angle for the first two generations. This is exactly what one would expect in a grand unified setup where the Yukawa matrices of the leptons are related to the quark Yukawa couplings.

Then one would find, for instance, for bimaximal mixing

$$\sin^2 \theta_{12} \approx \frac{1}{2} + \sin \theta_{13} \cos \delta \tag{1}$$

and for tri-bimaximal mixing

$$\sin^2 \theta_{12} \approx \frac{1}{3} + \frac{2\sqrt{2}}{3} \sin \theta_{13} \cos \delta \tag{2}$$

to leading order in the θ_{13} expansion, see, for instance, [4] and references therein.

Nowadays all mixing angles have been measured such that these sum rules can be translated into constraints on the Dirac CP violating phase δ. For the bimaximal case CP should be almost conserved($\cos \delta \approx -1$) while for the tri-bimaximal case CP should be strongly violated ($\cos \delta \lesssim 0$).

3. Neutrino Mass Sum Rules

Neutrino mass sum rules emerge somewhat accidental in flavour models. They are not related to a special family symmetry or a subgroup thereof. They are also not specific to any seesaw mechanism. The reason for them is simply that due to a very economic breaking of the family symmetry it can happen that the three light complex Majorana neutrino masses depend effectively on two complex parameters only. From simply counting the degrees of freedom it is clear that there should be two relations which can be expressed as a complex sum rule for the neutrino masses including the Majorana phases.

For instance in the $SU(5) \times A_5$ flavour model in [5] we had a type I seesaw mechanism where the neutrino Yukawa matrix Y and the right neutrino mass matrix M_{RR} had the structures

$$Y \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \text{ and } M_{RR} \sim \begin{pmatrix} 2\sqrt{\frac{2}{3}}(v_2 + v_3) & -\sqrt{3}v_2 & -\sqrt{3}v_2 \\ -\sqrt{3}v_2 & \sqrt{6}v_3 & -\sqrt{\frac{2}{3}}(v_2 + v_3) \\ -\sqrt{3}v_2 & -\sqrt{\frac{2}{3}}(v_2 + v_3) & \sqrt{6}v_3 \end{pmatrix} , \tag{3}$$

Table 1.   Summary table of the sum rules existing in the literature. The table is taken from [6].

Sum rule	References	c_1	c_2	d	$\Delta\chi_{13}$	$\Delta\chi_{23}$
1	[7–15]	1	1	1	π	π
2	[16]	1	2	1	π	π
3	[7, 10–14, 17–20]	1	2	1	π	0
4	[21]	1/2	1/2	1	π	π
5	[22]	$\frac{2}{\sqrt{3}+1}$	$\frac{\sqrt{3}-1}{\sqrt{3}+1}$	1	0	π
6	[5, 7–9, 23, 24]	1	1	-1	π	π
7	[7, 18–20, 25–27]	1	2	-1	π	0
8	[28]	1	2	-1	0	π
9	[29]	1	2	-1	π	$\pi/2, 3\pi/2$
10	[30, 31]	1	2	1/2	$\pi, 0, \pi/2$	$0, \pi, \pi/2$
11	[32]	1/3	1	1/2	π	0
12	[33]	1/2	1/2	$-1/2$	π	π

where v_2 and v_3 are complex flavon vacuum expectation values which break the family symmetry A_5. Then it is obvious that the light neutrino mass matrix will depend only on two effective parameters and in fact in this case we find the mass sum rule

$$\frac{\mathrm{e}^{\mathrm{i}\,\phi_1}}{m_1} + \frac{\mathrm{e}^{\mathrm{i}\,\phi_2}}{m_2} = \frac{1}{m_3}\,, \tag{4}$$

where m_i, $i = 1, 2, 3$, are the three light neutrino masses and ϕ_1 and ϕ_2 the Majorana phases.

But this is not the only known sum rule in the literature. We found in total twelve different sum rules which can all be parametrised in the following form

$$s \equiv c_1 \left(m_1 \mathrm{e}^{\mathrm{i}\,\phi_1}\right)^d \mathrm{e}^{\mathrm{i}\,\Delta\chi_{13}} + c_2 \left(m_2 \mathrm{e}^{\mathrm{i}\,\phi_2}\right)^d \mathrm{e}^{\mathrm{i}\,\Delta\chi_{23}} + m_3^d \overset{!}{=} 0\,, \tag{5}$$

where c_i, $\Delta\chi_{ij}$ and d are given by the underlying model but can only take discrete values. In the previous example, for instance, $c_1 = c_2 = 1$, $d = -1$ and $\Delta\chi_{13} = \Delta\chi_{23} = \pi$. A complete list of the sum rules we found in the literature is given in Table 1.

The sum rules had been known before, for recent overviews, see, e.g. [32, 34, 35]. But so far it was not studied how the predictions of the mass sum rules are affected by renormalisation group equation (RGE) corrections which we did in [6]. We do not want to discuss here the numerical results, for which the reader is kindly referred to the original publication [6]. Instead we will discuss some analytical estimates wich show that one of the most important qualitative features of the sum rules is robust under RGE corrections.

The typical size of RGE corrections in the minimal supersymmetric extension

of the Standard Model (MSSM) can be estimated to be [36]

$$\delta\theta_{ij} \sim 10^{-6}(1 + \tan^2\beta)\frac{m^2}{\Delta m^2} \ , \tag{6}$$

$$\delta\phi_i \sim 10^{-6}(1 + \tan^2\beta)\frac{m^2}{\Delta m^2} \ , \tag{7}$$

$$\delta m_i \sim (\mathcal{O}(1) + 10^{-6}(1 + \tan^2\beta))m_i \ , \tag{8}$$

where m^2 and Δm^2 stands for the corresponding neutrino masses and mass squared differences. In the Standard Model without supersymmetry there is no factor of $\tan\beta$ such that the RGE corrections there are expected to be rather small. On the other hand they can become quite sizeable in the MSSM for large $\tan\beta$ and a large neutrino mass scale. So in this case one might wonder if the corrections are large enough to allow a neutrino mass ordering which would be forbidden on tree level.

As an example we study sum rule 2, cf. Table 1 which reads

$$m_1 \mathrm{e}^{-\mathrm{i}\,\phi_1} + 2m_2 \mathrm{e}^{-\mathrm{i}\,\phi_2} - m_3 = 0 \ . \tag{9}$$

The sum rules can as well be interpreted geometrically in the complex plane, cf. [6, 35], as a closed triangle. Then we find for one of its angles on tree level for sum rule 2 and inverted ordering

$$\cos\alpha^{\mathrm{tree}} = \frac{m_1^2 - 4m_2^2 - m_3^2}{4m_2 m_3} < -\frac{1}{4}\left(3\frac{m_2^2}{m_3^2} + 1\right) < -1 \ . \tag{10}$$

Since the modulus of the cosine of an angle in a triangle is restricted to be smaller than one we see that inverted ordering cannot be realised on tree level. Two sides of the triangle are too short compared to the third side to close the triangle.

An approximation of the RGE effects on $\cos\alpha$ gives in the MSSM where the effect is expected to be sizeable

$$\delta(\cos\alpha)^{\mathrm{RGE}} \approx -\underbrace{\frac{Cy_\tau^2}{192\pi^2}}_{>0}\underbrace{\frac{2.8m_1^2 - 0.4m_2^2 + 0.1m_3^2}{m_2 m_3}}_{>0}\underbrace{\log\frac{M_S}{M_Z}}_{>0} < 0 \ . \tag{11}$$

For details on the derivation and the notation, please see the original paper [6]. The important thing is, that the correction is negative and hence points in the wrong direction. This statement is true for most of the sum rules in the overwhelming part of the parameter space. For the very few cases where the sign is correct one would still need very extreme parameter choices, for instance $\tan\beta > 500$ or $m_1 > 1$ eV, to reconstitute the forbidden ordering by RGE effects.

Note that this is particular to RGE corrections. Other kind of corrections, like higher-dimensional operators, flavon misalignment and so on, have in principle an arbitrary sign and might reconstitute forbidden orderings in a plausible parameter range. But this is subject of another study in progress [37]

4. A Powerful Example

As we have seen there are two kinds of sum rules. On the one hand there are sum rules predicting the Dirac CP violating phase from the mixing angles and on the other hand there are sum rules predicting the Majorana phases from the neutrino masses.

Now we want to briefly show how powerful it can be if a model incorporates both sum rules making the model extremely predictive. As an example we want to take again the $SU(5) \times A_5$ model [5]. In this model the neutrino sector exhibited golden ratio type A mixing [38]

$$\tan \theta_{12}^\nu = \frac{2}{1 + \sqrt{5}} \,, \quad \theta_{23}^\nu = \frac{\pi}{4} \,, \quad \theta_{13}^\nu = 0 \,. \tag{12}$$

Due to its GUT nature and the use of alternative new GUT relations [39] there is only one sizeable mixing angle in the charged lepton sector for the first two generations

$$\theta_{12}^e \approx \theta_C \Rightarrow \theta_{13} \approx \frac{\theta_C}{\sqrt{2}} \approx 9^\circ \,, \tag{13}$$

where $\theta_C \approx 12^\circ$ is the Cabibbo angle. Related to this is the mixing sum rule

$$\theta_{12} \approx \theta_{12}^\nu + \theta_C \cos \delta \,. \tag{14}$$

As mentioned above the model also has the mass sum rule

$$\frac{e^{i\,\phi_1}}{m_1} + \frac{e^{i\,\phi_2}}{m_2} = \frac{1}{m_3} \,. \tag{15}$$

An additional important information is that to get the correct GUT Yukawa relations a rather large $\tan \beta \gtrsim 30$ which implies that RGE corrections can be sizeable depending on the neutrino mass scale.

From the mass sum rule we can estimate that

$$0.011 \text{ eV} \lesssim m_1 \qquad\qquad \text{for normal odering,} \tag{16}$$
$$0.028 \text{ eV} \lesssim m_3 \lesssim 0.454 \text{ eV} \qquad \text{for inverted ordering.} \tag{17}$$

Especially the lower bound for the neutrino masses is interesting since it will give a lower bound on the running effects.

From the mixing sum rule we find an allowed range for θ_{12} at the high scale M_S

$$24^\circ \lesssim \theta_{12}(M_S) \lesssim 39^\circ \,. \tag{18}$$

From the lower bound on $\tan \beta$ and the neutrino mass scale we can now estimate the high scale range of θ_{12} by evolving the low energy 3σ range up to the high scale and we find

$$\theta_{12}(M_S) \lesssim 33.5^\circ \qquad\qquad \text{for normal odering,} \tag{19}$$
$$\theta_{12}(M_S) \lesssim 5.7^\circ \qquad\qquad \text{for inverted ordering.} \tag{20}$$

By comparing the ranges derived from the model at the high scale and the RGE evolved ranges from the low scale we see that the inverted ordering is by far excluded. This would not have been the case, if there had been no mixing angle sum rule on top of the mass sum rule. Hence, the combination of two kinds of sum rules can be extremely powerful.

5. Summary and Conclusions

Sum rules are a common feature in flavour models and they can appear in two different incarnations, either as mixing sum rules which relate the Dirac CP phase to the mixing angles and/or as neutrino mass sum rules relating the neutrino masses to the Majorana phases.

Both of these sum rules are easily testable in the near future. The Dirac CP phase might be measured already very soon while mass sum rules are more difficult to test. The most promising observable to distinguish between different sum rules is the effective neutrino mass which can be determined from neutrinoless double beta decay where no precision measurement is foreseeable in the near future. But even there by simply measuring the ordering of the neutrino masses and the absolute neutrino mass scale several of the sum rules and hence a lot of models would be immediately excluded.

Nevertheless, the testability of a high scale model should always be questioned taking renormalisation group effects into account which can be very large in the neutrino sector and alter the predictions at low energies. In fact, as we have demonstrated RGE corrections have been crucial to understand why the inverted ordering of the neutrino masses was excluded in our example $SU(5) \times A_5$ model.

Finally, to really understand flavour at the high energy scale we will have to test the TeV scale extensively at colliders to discover the mechanism which makes the Higgs boson mass natural or to abandon the notion of naturalness altogether.

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