Sum Rules of Charm CP Asymmetries beyond the $\mathrm{SU}(3)_F$ Limit*

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We find new sum rules between direct CP asymmetries in D meson decays with coefficients that can be determined from a global fit to branching ratio data. Our sum rules eliminate the penguin topologies P and PA, which cannot be determined from branching ratios. In this way we can make predictions about direct CP asymmetries in the Standard Model without ad-hoc assumptions on the sizes of penguin diagrams. We consistently include first-order $\mathrm{SU}(3)_F$ breaking in the topological amplitudes extracted from the branching ratios. By confronting our sum rules with future precise data from LHCb and Belle II one will identify or constrain new-physics contributions to P or PA. The first sum rule correlates the CP asymmetries a_{CP}^{dir} in $D^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$, and $D^0 \to \pi^0\pi^0$. We study the region of the $a_{CP}^{\mathrm{dir}}(D^0 \to \pi^+\pi^-)$–$a_{CP}^{\mathrm{dir}}(D^0 \to \pi^0\pi^0)$ plane allowed by current data and find that our sum rule excludes more than half of the allowed region at 95% CL. Our second sum rule correlates the direct CP asymmetries in $D^+ \to \bar{K}^0K^+$, $D_s^+ \to K^0\pi^+$, and $D_s^+ \to K^+\pi^0$.

INTRODUCTION

Decays of charmed mesons are currently the only way to probe flavour violation in the up-quark sector. A major goal of experimental charm physics is the discovery of CP violation in nonleptonic charm decays (see e.g. [1–4]). To this end it is promising to study singly Cabibbo-suppressed (SCS) decays d whose direct CP asymmetries may be large enough to be detected in the near future:

$$a_{CP}^{\mathrm{dir}}(d) \equiv \frac{|\mathcal{A}^{\mathrm{SCS}}(d)|^2 - |\overline{\mathcal{A}}^{\mathrm{SCS}}(d)|^2}{|\mathcal{A}^{\mathrm{SCS}}(d)|^2 + |\overline{\mathcal{A}}^{\mathrm{SCS}}(d)|^2}\,. \tag{1}$$

We decompose the decay amplitude $\mathcal{A}^{\mathrm{SCS}}(d)$ as $\mathcal{A}^{\mathrm{SCS}}(d) = \mathcal{A}_{sd}^{\mathrm{SCS}}(d) + \mathcal{A}_b^{\mathrm{SCS}}(d)$ with

$$\mathcal{A}_{sd}^{\mathrm{SCS}}(d) = \lambda_{sd}\mathcal{A}_{sd}(d)\,, \qquad \mathcal{A}_b^{\mathrm{SCS}}(d) = -\frac{\lambda_b}{2}\,\mathcal{A}_b(d),$$

and the shorthand notations $\lambda_q \equiv V_{cq}^* V_{uq}$ and $\lambda_{sd} \equiv (\lambda_s - \lambda_d)/2$ for the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix involved. Adopting the PDG convention (with $\lambda_s > 0$) one may safely neglect subleading powers of $\lambda_b/\lambda_{sd} \sim 10^{-3}$. Eq. (1) reads

$$a_{CP}^{\mathrm{dir}}(d) = \frac{\mathrm{Im}\,\lambda_b\ \mathrm{Im}\left[e^{-i\delta(d)}\mathcal{A}_b(d)\right]}{|\mathcal{A}_{sd}^{\mathrm{SCS}}(d)|} \tag{2}$$

in terms of the strong phase $\delta(d) \equiv \arg\left[\mathcal{A}_{sd}(d)\right]$. The smallness of $\mathrm{Im}\,\lambda_b$ renders $a_{CP}^{\mathrm{dir}}(d)$ highly sensitive to physics beyond the Standard Model (SM). To establish a "smoking gun" signal of new physics one needs reliable SM predictions for $a_{CP}^{\mathrm{dir}}(d)$ or at least robust theoretical upper bounds on $|a_{CP}^{\mathrm{dir}}(d)|$ which cannot be exceeded within the SM. The difficulty of such theory predictions can be witnessed from $\Delta a_{CP}^{\mathrm{dir}} \equiv a_{CP}^{\mathrm{dir}}(D^0 \to K^+K^-) - a_{CP}^{\mathrm{dir}}(D^0 \to \pi^+\pi^-)$: estimates vary between $\mathcal{O}(0.01\%)$ [5], $\mathcal{O}(0.1\%)$ [6–10], $\sim -0.25\%$ [11] and $\sim -0.4\%$ [12], not excluding an enhanced SM value between $\sim -0.6\%$ and $\sim -0.8\%$ [12–17]. There are claims that CP-violating effects in charm physics can be $\mathcal{O}(1\%)$ [18]. The situation is not any better in CP violation induced by D–$\overline{D}$ mixing [19]. The key problem is our lack of knowledge of the penguin amplitude entering $\mathcal{A}_b(d)$ in Eq. (2) [12, 14, 15].

All theoretical analyses of $D \to PP'$ decays, where P,P' denote pseudoscalar mesons, rely on the approximate $\mathrm{SU}(3)_F$ symmetry of the strong interaction [7, 10–18, 20–41]. In analyses of branching ratios one can include first-order $\mathrm{SU}(3)_F$ breaking [7, 10–17, 27, 28, 30, 32, 33, 35, 37, 38, 40, 41]. An intuitive way to exploit $\mathrm{SU}(3)_F$ relations involves topological amplitudes (pioneered in Refs. [25, 26, 42]) which characterize the flavor-flow in terms of tree (T), color-suppressed tree (C), exchange (E), annihilation (A), penguin ($P_{d,s,b}$), and penguin annihilation ($PA_{d,s,b}$) diagrams. This method has been extended to include linear $\mathrm{SU}(3)_F$ breaking in applications to B [43] and D [41] decays.

With our inability to predict individual CP asymmetries it is natural to study correlations among several asymmetries. There are two sum rules which hold in the limit of exact $\mathrm{SU}(3)_F$ symmetry [7, 16, 44]:

$$a_{CP}^{\mathrm{dir}}(D^0 \to K^+K^-) + a_{CP}^{\mathrm{dir}}(D^0 \to \pi^+\pi^-) = 0\,, \tag{3}$$

$$a_{CP}^{\mathrm{dir}}(D^+ \to \bar{K}^0K^+) + a_{CP}^{\mathrm{dir}}(D_s^+ \to K^0\pi^+) = 0\,, \tag{4}$$

No relations among direct CP asymmetries hold to first order in $\mathrm{SU}(3)_F$ breaking [44]. Analyses of branching ratios permit the determination of $|\mathcal{A}_{sd}^{\mathrm{SCS}}(d)|$ in Eq. (2) and,

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through global fits, also to constrain the phase $\delta(d)$. The branching ratios of the decays entering Eqs. (3) and (4) exhibit sizable $\mathrm{SU}(3)_F$ breaking which limits the power of these $\mathrm{SU}(3)_F$-limit sum rules to test the SM. In this letter we derive new sum rules which incorporate $\mathrm{SU}(3)_F$ breaking in $\mathcal{A}_{sd}^{\mathrm{SCS}}(d)$ to linear order. To this end we use the result of our global fit to $D \to PP'$ branching ratios in Ref. [41] in two ways: On one hand we extract $|\mathcal{A}_{sd}^{\mathrm{SCS}}(d)|$ and $\delta(d)$ for the decays of interest from the fit to find the numerical relation between $a_{CP}^{\mathrm{dir}}(d)$ and $\mathcal{A}_b(d)$ in Eq. (2). On the other hand the same fit also returns the values of the topological amplitudes entering the fitted branching ratios. However, the desired $\mathcal{A}_b(d)$ of individual decay modes involve additional topological amplitudes (new combinations of $P_{d,s,b}$ and $PA_{d,s,b}$) which do not appear in the branching ratios. Our sum rules are constructed in a way to eliminate these unknowns. Unlike Eqs. (3) and (4) these new sum rules use the $\mathrm{SU}(3)_F$ limit for the eliminated $P_{d,s,b}$, $PA_{d,s,b}$ in $\mathcal{A}_b(d)$ only, while consistently including $\mathrm{SU}(3)_F$ breaking in T, A, C, E.

CP ASYMMETRY SUM RULES

Our conventions and the methodology of the global fit can be found in Ref. [41]. The CKM-subleading part of the SCS amplitude is expanded in terms of topological amplitudes $\mathcal{T}_i = C, P_{\mathrm{break}}, \ldots$ as

$$\mathcal{A}_b(d) = c_{sd}^d \mathcal{A}_{sd}(d) + \sum_i c_i^d \mathcal{T}_i\,, \qquad (5)$$

with the coefficients c_i^d specified in Table I. With the first term (with $c_{sd}^d = \pm 1$) we eliminate the numerically large parameters T and E from the sum in Eq. (5). Since $e^{-i\delta(d)}\mathcal{A}_{sd}(d) = |\mathcal{A}_{sd}(d)|$ is real, $c_{sd}^d \mathcal{A}_{sd}(d)$ does not contribute to $a_{CP}^{\mathrm{dir}}(d)$ in Eq. (2). The branching fractions involve the $\mathrm{SU}(3)_F$-breaking penguin topology $P_{\mathrm{break}} \equiv P_s - P_d$, with s, d denoting the quark flavour in the penguin loop. The new penguin topologies in $a_{CP}^{\mathrm{dir}}(d)$ are $P \equiv P_d + P_s - 2P_b$ and $PA \equiv PA_d + PA_s - 2P_b$. Since we consistently neglect subleading terms (such as $PA_{\mathrm{break}} \equiv PA_s - PA_d$ which vanishes in the SU(3) limit) in $\mathcal{A}_b(d)$, we can use the fitted values in Eq. (5).

Our strategy involves three steps: in step 1, we determine $A_i^{(1)}$, C, $C_3^{(1)}$, P_{break}, $|\mathcal{A}_{sd}(d)|$, and the phases $\delta(d)$ in a global fit to branching ratio data as described in Ref. [41]. In step 2 we eliminate all hadronic parameters but P and PA from $a_{CP}^{\mathrm{dir}}(d)$. To this end we define

$$\begin{aligned}\mathcal{S}(d) &\equiv 2ie^{i\delta(d)}\left[\frac{|\mathcal{A}_{sd}^{\mathrm{SCS}}(d)|a_{CP}^{\mathrm{dir}}(d)}{\mathrm{Im}\,\lambda_b} - \mathrm{Im}\,X(d)\right] \\ &= 2ie^{i\delta(d)}\mathrm{Im}\left[e^{-i\delta(d)}\mathcal{A}_b(d) - X(d)\right]\,, \qquad (6)\end{aligned}$$

with $\mathrm{Im}\,X(d)$ given in Table II. The right column of Table II shows the expansion of $\mathcal{S}(d)$ in terms of P, PA, their complex conjugates P^*, PA^*, and $\delta(d)$ which is determined from the fit. In step 3 we construct sum rules to eliminate P, PA, P^*, and PA^*. In this last step $\mathrm{SU}(3)_F$ breaking in P, PA is neglected. From the expressions in Table II one can read off the two possible independent linear combinations, connecting three $a_{CP}^{\mathrm{dir}}(d)$ each:

CP Asymmetry Sum Rule 1:

$$\begin{aligned}&\frac{\mathcal{S}(D^0 \to K^+K^-) - \mathcal{S}(D^0 \to \pi^+\pi^-)}{e^{2i\delta(D^0 \to K^+K^-)} - e^{2i\delta(D^0 \to \pi^+\pi^-)}} - \\ &\frac{\mathcal{S}(D^0 \to K^+K^-) + \sqrt{2}\mathcal{S}(D^0 \to \pi^0\pi^0)}{e^{2i\delta(D^0 \to K^+K^-)} - e^{2i\delta(D^0 \to \pi^0\pi^0)}} = 0\,. \qquad (7)\end{aligned}$$

CP Asymmetry Sum Rule 2:

$$\begin{aligned}&\frac{\mathcal{S}(D^+ \to \bar{K}^0K^+) - \mathcal{S}(D_s^+ \to K^0\pi^+)}{e^{2i\delta(D^+ \to \bar{K}^0K^+)} - e^{2i\delta(D_s^+ \to K^0\pi^+)}} - \\ &\frac{\mathcal{S}(D^+ \to \bar{K}^0K^+) + \sqrt{2}\mathcal{S}(D_s^+ \to K^+\pi^0)}{e^{2i\delta(D^+ \to \bar{K}^0K^+)} - e^{2i\delta(D_s^+ \to K^+\pi^0)}} = 0\,. \qquad (8)\end{aligned}$$

If some of the phases in the denominators of the sum rules are equal (covering the $\mathrm{SU}(3)_F$ limit as a special case) one finds: for $e^{2i\delta(D^0 \to K^+K^-)} = e^{2i\delta(D^0 \to \pi^+\pi^-)}$ Eq. (7) collapses to

$$\mathcal{S}(D^0 \to K^+K^-) - \mathcal{S}(D^0 \to \pi^+\pi^-) = 0\,, \qquad (9)$$

while for $e^{2i\delta(D^0 \to K^+K^-)} = e^{2i\delta(D^0 \to \pi^0\pi^0)}$ the sum rule becomes

$$\mathcal{S}(D^0 \to K^+K^-) + \sqrt{2}\mathcal{S}(D^0 \to \pi^0\pi^0) = 0\,. \qquad (10)$$

If all three phase factors are equal, Eqs. (9) and (10) hold simultaneously. The special cases of Sum Rule 2 are obtained from those in Eqs. (9)–(10) by obvious replacements.

When linking Sum Rule 2 to experimental quantities, we use $a_{CP}^{\mathrm{dir}}(D^+ \to K_SK^+) = a_{CP}^{\mathrm{dir}}(D^+ \to \bar{K}^0K^+)$ and $a_{CP}^{\mathrm{dir}}(D_s^+ \to K_S\pi^+) = a_{CP}^{\mathrm{dir}}(D_s^+ \to K^0\pi^+)$, meaning that in our definition of a_{CP}^{dir} kaon CP violation is properly subtracted [45]. The two sum rules probe the $SU(3)_F$ limit in P and $P + PA$. If future experiments find deviations of order 30%, one will ascribe those to $SU(3)_F$-breaking hadronic effects. The smallness of $\mathrm{Im}\,\lambda_b$ makes the sum rules highly sensitive to new physics, which may well violate the sum rules at a far higher level.

SM PREDICTION OF CP ASYMMETRIES

We combine the sum rules Eqs. (7)–(8) with the branching ratio fit presented in Ref. [41]: for each point in the parameter space complying with all measured branching fractions (and the strong phase $\delta_{K\pi}$) we determine $S(d)$ for the decays entering the sum rules. In the same step Eqs. (7) and (8) are used to predict one CP asymmetry in terms of the other two. In our fit we

Decay ampl. $\mathcal{A}(d)$	$A_{sd}(d)$	$A_1^{(1)}$	$A_2^{(1)}$	$A_3^{(1)}$	$C+A$	$C_3^{(1)}$	$P_{\rm break}$	$P{+}2A$	$P{+}PA$
$\mathcal{A}(D^0\to K^+K^-)$	1	0	0	0	0	0	-1	0	1
$\mathcal{A}(D^0\to \pi^+\pi^-)$	-1	0	0	0	0	0	1	0	1
$\mathcal{A}(D^0\to \pi^0\pi^0)$	-1	0	0	0	0	0	$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$
$\mathcal{A}(D^+\to \bar{K}^0K^+)$	1	0	0	2	0	0	-1	1	0
$\mathcal{A}(D_s^+\to K^0\pi^+)$	-1	2	2	0	0	0	1	1	0
$\mathcal{A}(D_s^+\to K^+\pi^0)$	1	0	0	0	$\sqrt{2}$	$\sqrt{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0

TABLE I. The coefficients c_i^d of the topological decomposition of $\mathcal{A}_b(d)$ in Eq. (5).

Decay d	$\mathrm{Im}\,[X(d)]$	$\mathcal{S}(d)$
$D^0\to K^+K^-$	$\mathrm{Im}\left[e^{-i\delta\left(D^0\to K^+K^-\right)}\left(-P_{\rm break}\right)\right]$	$(P+PA)-e^{2i\delta\left(D^0\to K^+K^-\right)}\left(P+PA\right)^*$
$D^0\to \pi^+\pi^-$	$\mathrm{Im}\left[e^{-i\delta\left(D^0\to \pi^+\pi^-\right)}\left(P_{\rm break}\right)\right]$	$(P+PA)-e^{2i\delta\left(D^0\to \pi^+\pi^-\right)}\left(P+PA\right)^*$
$D^0\to \pi^0\pi^0$	$\mathrm{Im}\left[e^{-i\delta\left(D^0\to \pi^0\pi^0\right)}\left(-\frac{1}{\sqrt{2}}P_{\rm break}\right)\right]$	$\frac{1}{\sqrt{2}}\left(-P-PA\right)-e^{2i\delta\left(D^0\to \pi^0\pi^0\right)}\frac{1}{\sqrt{2}}\left(-P-PA\right)^*$
$D^+\to \bar{K}^0K^+$	$\mathrm{Im}\left[e^{-i\delta\left(D^+\to \bar{K}^0K^+\right)}\left(-2A^{\rm fac}_{D^+\to\bar{K}^0K^+}+2\delta_A-P_{\rm break}\right)\right]$	$P-e^{2i\delta\left(D^+\to \bar{K}^0K^+\right)}P^*$
$D_s^+\to K^0\pi^+$	$\mathrm{Im}\left[e^{-i\delta\left(D_s^+\to K^0\pi^+\right)}\left(2A^{\rm fac}_{D_s^+\to K^0\pi^+}+2\delta_A+P_{\rm break}\right)\right]$	$P-e^{2i\delta\left(D_s^+\to K^0\pi^+\right)}P^*$
$D_s^+\to K^+\pi^0$	$\mathrm{Im}\left[e^{-i\delta\left(D_s^+\to K^+\pi^0\right)}\left(\sqrt{2}C+\sqrt{2}C_3^{(1)}+\frac{1}{\sqrt{2}}P_{\rm break}\right)\right]$	$\frac{1}{\sqrt{2}}\left(-P\right)-e^{2i\delta\left(D_s^+\to K^+\pi^0\right)}\frac{1}{\sqrt{2}}\left(-P\right)^*$

TABLE II. Definitions of $X(d)$ and results for $\mathcal{S}(d)$ as used and defined in Eq. (6) in case of the $\mathrm{SU}(3)_F$ fit including $1/N_c$ counting. For the sign conventions of $A^{\rm fac}(d)$ see Ref. [41]. Note that $A^{\rm fac}_{D^+\to\bar{K}^0K^+}=0$ by isospin symmetry.

Observable	Measurement	References
$\Delta a_{CP}^{\rm dir}$	-0.00253 ± 0.00104	[1–3, 46–51]
$\Sigma a_{CP}^{\rm dir}$	-0.0011 ± 0.0026	†[1, 2, 47, 49, 52]
$a_{CP}^{\rm dir}(D^0\to K_SK_S)$	-0.23 ± 0.19	[53]
$a_{CP}^{\rm dir}(D^0\to \pi^0\pi^0)$	-0.0004 ± 0.0064	†[4, 53]
$a_{CP}^{\rm dir}(D^+\to \pi^0\pi^+)$	$+0.029\pm0.029$	[54]
$a_{CP}^{\rm dir}(D^+\to K_SK^+)$	$+0.0011\pm0.0017$	†[54–58]
$a_{CP}^{\rm dir}(D_s^+\to K_S\pi^+)$	$+0.006\pm0.005$	†[54, 56, 58–60]
$a_{CP}^{\rm dir}(D_s^+\to K^+\pi^0)$	$+0.266\pm0.228$	[54]

TABLE III. Current data on SCS charm CP asymmetries with subtracted indirect CP violation from kaon and charm mixing [45, 61], see Appendix A of Ref. [16]. We use the notation $\Sigma a_{CP}^{\rm dir}\equiv a_{CP}^{\rm dir}(D^0\to K^+K^-)+a_{CP}^{\rm dir}(D^0\to\pi^+\pi^-)$. No correlations between CP asymmetries are taken into account in the fits. †Our average. Table adapted from Ref. [62].

demand $|(C{+}\delta_A)/T^{\rm fac}|, |(E{+}\delta_A)/T^{\rm fac}|\leq 1.3$ to enforce proper $1/N_c$ counting. ($T^{\rm fac}$ and $A^{\rm fac}$ are the factorized tree and annihilation amplitudes.) Apart from the fit with current data (see Table X of Ref. [41]) we also consider a hypothetical future scenario with improved branching ratios by scaling their errors with a factor $1/\sqrt{50}$. To illustrate the impact of the $1/N_c$ counting for the SM predictions, we perform an additional fit without $1/N_c$ input. This plain $\mathrm{SU}(3)_F$ fit relies on the topological parameterization of Table III in Ref. [41] with the $\mathrm{SU}(3)_F$ counting described in Sec. IIIB of Ref. [41]. The redundancy of the four $\mathrm{SU}(3)_F$-limit topologies [41] is removed by absorbing A into T, C and E. We further demand $|A_i^{(1)}/T|\leq 50\%$ to respect the $\mathrm{SU}(3)_F$ counting.

The experimental values of the CP asymmetries included in the fit are summarized in Table III. Our global fit results are shown in Fig. 1. The χ^2 of the global minima range from 0.0 to 2.0 in the considered scenarios, i.e. with or without $1/N_c$ counting and with current or future data, indicating an excellent fit. The sum rules have nontrivial implications for direct CP asymmetries, especially when combined with the input from $1/N_c$ counting. With current data we see the largest impact of the sum rules in Fig. 1(a): roughly 47% of the 95% CL region allowed by the measurements of the CP asymmetries is excluded by our global fit result.

Our results show that future improved measurements of branching ratios will play a key role to sharpen our predictions: drastic examples are the prediction of $a_{CP}^{\rm dir}(D_s^+\to K^+\pi^0)$ in Figs. 1(c) and 1(d) and the correlation of $a_{CP}^{\rm dir}(D^0\to\pi^0\pi^0)$ and $a_{CP}^{\rm dir}(D^0\to\pi^+\pi^-)$ in Fig. 1(a), where one of the two overlapping ellipses vanishes in our future-data scenario. In Fig. 1(b) a smaller region is excluded than in Fig. 1(a), because $a_{CP}^{\rm dir}(D^0\to\pi^0\pi^0)$ is measured less precisely than $\Delta a_{CP}^{\rm dir}$ (see Table III). In general, the correlation of two CP asymmetries can be better predicted once improved data for the third one appearing in Eqs. (7) and (8) becomes available. In case of the fit without $1/N_c$ counting the smaller errors of the branching ratios in our future scenario do not improve the predictions for CP asymmetries. Note that with current data the predicted ranges barely depend on the additional input from $1/N_c$ counting.

In principle one can obtain the quantities P and $P + PA$, which we eliminate through our sum rules, from the individual CP asymmetries. If future data challenge our sum rules at a level which cannot be explained with $\mathrm{SU}(3)_F$ breaking in P and $P + PA$, this will point to new physics which couples differently to s and d quarks. (For an $\mathrm{SU}(3)_F$ analysis of such models see e.g. Ref. [16].) However, $\mathrm{SU}(3)_F$-symmetric new physics in P and $P + PA$ vanishes from the sum rules. (A similar situation can be found in the isospin sum rules of Ref. [63] which are insensitive to new physics in $\Delta I = 1/2$ amplitudes.)

CONCLUSIONS

To find reliable SM predictions for charm CP asymmetries we derive two sum rules which treat T, A, C, E, P_{break} correctly to linear order in $\mathrm{SU}(3)_F$ breaking and eliminate the penguin topologies P and $P + PA$ to leading order in $\mathrm{SU}(3)_F$. Thus we treat large tree-level parameters at sub-leading order to increase the precision in the extraction of loop-induced quantities sensitive to new physics. Unlike previously known $\mathrm{SU}(3)_F$-limit sum rules our new sum rules correlate three CP asymmetries each. The interplay of the two sum rules probes both the quality of $\mathrm{SU}(3)_F$ for penguin topologies and new physics. Future branching ratio measurements play a key role in order to reduce the uncertainties of the consequent SM predictions for charm CP asymmetries.

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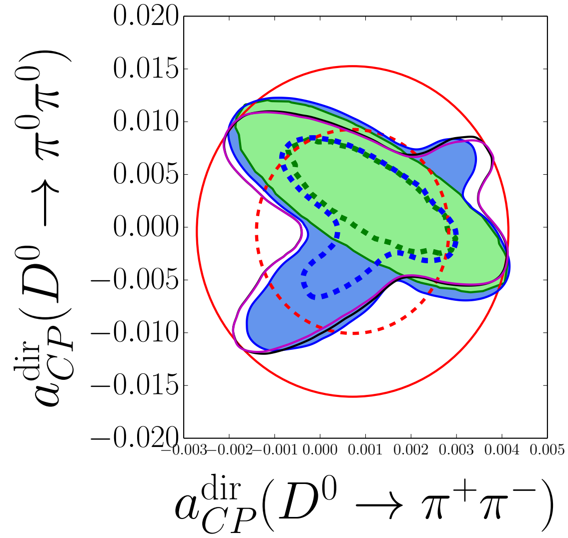
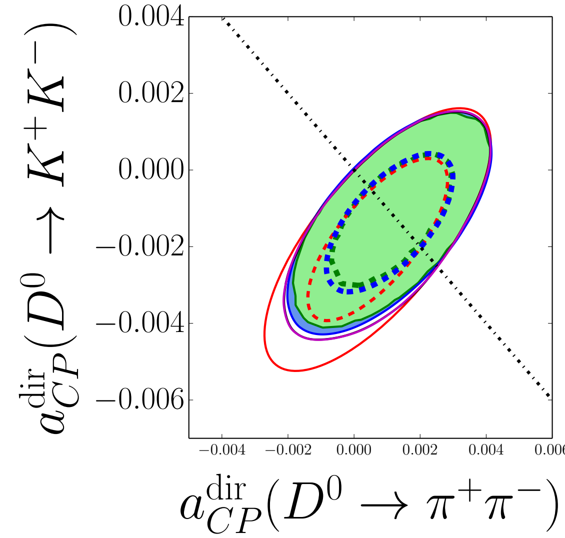
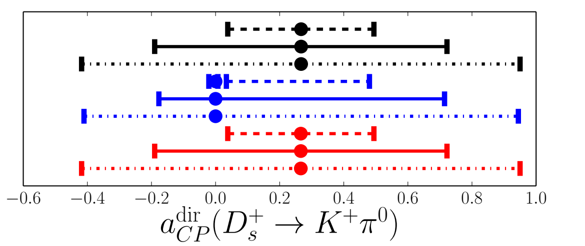


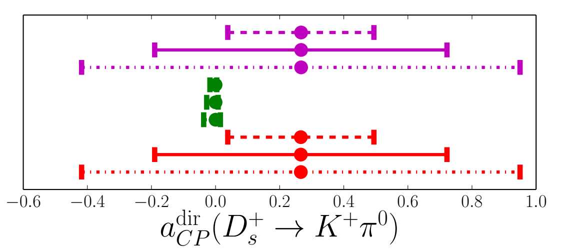
(a) Fit including $1/N_c$ counting in the topological amplitudes with current (future) branching ratio data in blue (green). The black (magenta) line delimits the 95% CL region found in a fit without using $1/N_c$ counting in the topological amplitudes for current (future) branching ratio data. Note that the black and magenta curves lie on top of each other.



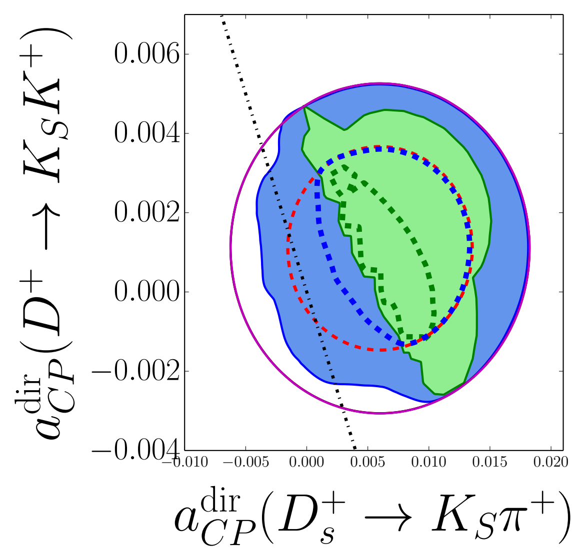
(b) Color coding as in Fig. 1(a). The dashed-dotted line denotes the $\mathrm{SU}(3)_F$-limit sum rule of Eq. (3).



(c) Prediction from current branching ratio data. The dashed, solid, and dashed-dotted lines correspond to the 1σ, 2σ, and 3σ intervals, respectively. The blue bars show our fit results for current branching ratio data. The corresponding result without $1/N_c$ counting (i.e. only using $\mathrm{SU}(3)_F$) is shown in black.



(d) Same as in Fig. (c), but for our future scenario with smaller errors of the branching ratios. The green (magenta) bars correspond to the analysis with (without) $1/N_c$ counting.



(e) Future scenario assuming $a_{CP}^{\mathrm{dir}}(D_s^+ \to K^+\pi^0) = -1\%$ and branching ratios with current (future) errors including $1/N_c$ counting in blue (green). The corresponding allowed area found in a fit without $1/N_c$ counting is delimited by the black (magenta) line. Note that the black, magenta, and red curves lie on top of each other. The dashed-dotted line denotes the $\mathrm{SU}(3)_F$-limit sum rule of Eq. (4).

FIG. 1. SM predictions for CP asymmetries obtained from our global fit. In Figures (a),(b),(e) the dashed (solid) red lines delimit the experimental 68% (95%) CL regions. The other dashed (solid) lines are the 68% (95%) CL regions of the respective fit scenarios explained in the captions of the subfigures. The generic error of order $\sim 30\%$ from $\mathrm{SU}(3)_F$ breaking in P and $P + PA$ is not shown. The experimental error ellipses are obtained by scaling the errors quoted in Table III by factors of $\sqrt{2.28}$ and $\sqrt{5.99}$ in order to obtain the two-dimensional 68% and 95% CL regions from the corresponding one-dimensional ranges. The experimental error ellipse shown in Fig. 1(b) is calculated from the corresponding one for $\Delta a_{CP}^{\mathrm{dir}}$ and $\Sigma a_{CP}^{\mathrm{dir}}$.

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