Penguin contributions to CP phases in $B_{d,s}$ decays to charmonium

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The precision of the CP phases $2\beta$ and $2\beta_s$ determined from the mixing-induced CP asymmetries in $B_d \to J/\psi K_S$ and $B_s \to J/\psi \phi$, respectively, is limited by the unknown long-distance contribution of a penguin diagram involving up quarks. We analyze the infrared QCD structure of this contribution and find that all soft and collinear divergences either cancel between different diagrams or factorize into matrix elements of local four-quark operators up to terms suppressed by $\Lambda_{QCD}/m_\psi$, where $m_\psi$ denotes the $J/\psi$ mass. Our results allow us to calculate the penguin-to-tree ratio $P/T$ in terms of the matrix elements of these operators and to constrain the penguin contribution to the phase $2\beta$ as $|\Delta \phi_0| \leq 0.68^\circ$. The penguin contribution to $2\beta_s$ is bounded as $|\Delta \phi_0^\circ| \leq 1.03^\circ$, $|\Delta \phi_0^\circ| \leq 1.28^\circ$, and $|\Delta \phi_0^\circ| \leq 0.95^\circ$ for the case of longitudinal, parallel, and perpendicular CP and $J/\psi$ polarizations, respectively. We further place bounds on $|\Delta \phi_0|$ for $B_d \to \psi(2S)K_S$ and the polarization amplitudes in $B_d \to J/\psi K^*$. In our approach it is further possible to constrain $P/T$ for decays in which $P/T$ is Cabibbo-suppressed and we derive upper limits on the penguin contribution to the mixing-induced CP asymmetries in $B_d \to J/\psi \pi^0$, $B_d \to J/\psi \phi$, $B_s \to J/\psi K_S$, and $B_s \to J/\psi K^*$. For all studied decay modes we also constrain the sizes of the direct CP asymmetries.

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INTRODUCTION

The mixing-induced CP asymmetry in the decay $B_d \to J/\psi K_S$ is the key quantity to measure the CP phase of the $B_d - \bar{B}_d$ mixing amplitude. Within the Standard Model (SM) this CP asymmetry determines the angle $\beta = \arg[-V_{ub}V_{ud}^*/(V_{ub}V_{ud})]$ of the unitarity triangle. $B_s \to J/\psi \phi$ plays the same role for the $B_s - \bar{B}_s$ system. Since the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix essentially fixes $\beta_s = \arg[-V_{ub}^*V_{us}/(V_{ub}V_{us})] = 1.0^\circ$, CP studies of $B_s \to J/\psi \phi$ directly probe physics beyond the SM. The decay amplitude $A_f$ for an $\overrightarrow{T} \to \pi \pi$ decay $B_q \to f$, where $f$ is a CP eigenstate consisting of a charmonium state and a light meson, can be written as

$$A_f = \lambda^*_f T_f + \lambda^*_u P_f$$  \hspace{1cm} (1)

with $\lambda^*_p = V_{pb}^* V_{ps}$, $p = u, c$, and

$$T_f = \frac{G_F}{\sqrt{2}} |f| C_1 Q_1^* + C_2 Q_2^* - \sum_j C_j Q_j |B_q|, \hspace{2cm} (2)$$

$$P_f = \frac{G_F}{\sqrt{2}} |f| C_1 Q_1^* + C_2 Q_2^* - \sum_j C_j Q_j |B_q|. \hspace{2cm} (3)$$

Here, $Q_1 = \pi^* \gamma^\mu (1 - \gamma_5) q^\mu q^\nu \gamma^\nu (1 - \gamma_5) b^\sigma$ and $Q_2 = \pi^* \gamma^\mu (1 - \gamma_5) q^\mu q^\nu \gamma^\nu (1 - \gamma_5) b^\sigma$ are the current-current operators. The index $j$ labels the penguin operators $Q_j$, which involve the CKM elements $\lambda^*_f = -\lambda^*_u - \lambda^*_s$. While the QCD penguin operators $Q_{3-6}$ and $Q_{SG}$ are important for this paper (see Ref. [1] for their definition), electroweak penguin operators have negligible effects. The time-dependent CP asymmetry $A_{CP}^{B_q \to f}(t) \equiv |\Gamma(B_q(t) \to f) - \Gamma(B_q(t) \to \bar{f})|/|\Gamma(B_q(t) \to f) + \Gamma(B_q(t) \to \bar{f})|$ reads

$$A_{CP}^{B_q \to f}(t) = \frac{S_f \sin(\Delta M_q t) - C_f \cos(\Delta M_q t)}{\cosh(\Delta \Gamma_q t/2) + \cos(\Delta \Gamma_q \sinh(\Delta \Gamma_q t/2))/2}. \hspace{2cm} (4)$$

Here $\Delta M_q$ and $\Delta \Gamma_q$ are the mass and width difference, respectively, between the mass eigenstates of the $B_q - \bar{B_q}$ system. We write $S_f = \eta_q \sin(\phi_q + \Delta \phi_0)$, where $CP$ | $f$ = $\eta_q |f$ and $\phi_q$ is the CP phase in the limit $P_f = 0$. The SM predictions are $\phi_i = 2\beta$ and $\phi_s = -2\beta_s$. To first order in $\epsilon = |V_{us}V_{ub}/(V_{ub}V_{us})| \approx 0.02$ one has

$$\tan(\Delta \phi_0) \approx 2 \epsilon \sin \gamma \Re P_f / T_f. \hspace{2cm} (5)$$

$T_f$ and $P_f$ are non-perturbative multi-scale matrix elements, which defy calculations from first principles of QCD.

For the prediction of the branching ratio $B(B_d \to J/\psi K_S)$ one only needs $T_f$, which was addressed with the method of QCD factorization [2] in Ref. [3]: in the limit of infinite charm and bottom masses $T_f$ can be expressed in terms of the $J/\psi$ decay constant and the $B_d \to K_S$ form factor. The result of Ref. [3] underestimates $B(B_d \to J/\psi K_S)$ by a factor of 8. This failure, however, is not surprising, because the corrections to the infinite-mass limit are of order $\Lambda_{QCD}/(m_c \alpha_s)$ and therefore numerically unsuppressed for the actual value of the charm mass [2, 4]. The standard approach to quantify $P_f/T_f$ in $B_d \to J/\psi K_S$ uses the approximate SU(3)$_C$ symmetry of QCD (or its U-spin subgroup) which relates the decay of interest to $b \to c \bar{d}d$ modes like $B_s \to J/\psi K_S$ and $B_d \to J/\psi \pi^0$ [5, 6]. A drawback of this method is
is useful to express the factorized matrix element and normalize the matrix elements (for $j \geq 3$) hadronic parameters is known numerical accident entailing that the weak decay $f \rightarrow B(J/\psi K_S)$ at the scale $\mu = m_{\psi}$ one finds $C_0 = 0.13$ and $C_8 = 2.2$. The smallness of $C_0$ is a well-known numerical accident entailing that the weak decay produces the $(c, \tau)$ pair almost in a color octet state. We normalize the matrix elements (for $j = 0, 8$) as

$$\langle Q_{jV} | V_{0}, v \rangle, \langle Q_{jA} | V_{0}, a \rangle$$

(7)

to the factorized matrix element $V_{0} \equiv \langle Q_{0V}' \rangle_{\text{fact}} = 2 f_{J/\psi} m_{b} p_{cm} F_{1}^{B \rightarrow K}(m_{b}^{2}) = (4.26 \pm 0.16) \text{GeV}^{3}$. The uncertainty stems from the form factor $F_{1}^{B \rightarrow K}(m_{b}^{2}) = 0.586 \pm 0.021$ [8] and the $J/\psi$ decay constant $f_{J/\psi} = (0.405 \pm 0.005) \text{GeV}$. $m_{b} = 5.28 \text{GeV}$ and $p_{cm}$ is $1.68 \text{GeV}$ are the $B_d$ mass and the magnitude of the three-momentum of the $J/\psi$ or $K_S$ in the $B_d$ rest frame.

$v_{0.8}, a_{0.8}$ depend on $\mu$ in such a way that the $\mu$-dependence of $C_0, C_8$ cancels from physical quantities. When we quote numerical values we refer to the choice $\mu = m_{\psi}$. The large-$N_c$ counting of our (complex) hadronic parameters is $v_0 = 1 + O(1/N_c)$, $v_{0.8} = O(1/N_c)$, and $a_0 = O(1/N_c)$. Normalizing the branching ratio to the experimental value we find

$$B(B_d \rightarrow J/\psi K_S)_{\exp} = \left[1 \pm 0.08\right] \left|0.47 v_{0} + 7.8(v_{8} - a_{8})\right|^2.$$ (8)

Varying the phase of $v_{8} - a_{8}$ between $-\pi$ and $\pi$ one finds the correct branching ratio for $0.07 \leq |v_8 - a_8| \leq 0.19$ if $v_0$ is set to 1. Thus, there is no mystery with the branching ratio and the hadronic parameters obey the hierarchy expected from $1/N_c$ counting. The terms involving $a_0$ are negligible in view of other uncertainties and are omitted throughout this paper.

$P_f \in \text{Eq. (3)}$ receives contributions from $Q_{1,2}^{u,v}$ and the penguin operators $Q_{j}, j \geq 3$. The matrix elements of the latter can be trivially expressed in terms of the operators in Eq. (6). Therefore, this contribution to $P_f / T_f$ only depends on $v_{8}/a_{8}$ and $a_{0}/v_{0}$. Below we will see that the magnitudes of these ratios are under control thanks to the $1/N_c$ hierarchy of $v_{0}, v_{8}, a_{8}$ and the information from $B(B_d \rightarrow J/\psi K_S)_{\exp}$. By varying the parameters in the allowed ranges we can then find the maximal contribution of the penguin operators to $|\Delta \phi|$. In order to apply the same strategy to $Q_{1,2}^{u}$ we must first express the up-quark penguin depicted in Fig. 1a in terms of matrix elements of the local operators in Eq. (6). In Ref. [10] it is argued that a penguin loop flown through by a hard momentum $q$ (in our case $q^2 \sim m_{\psi}^2 = (3.1 \text{GeV})^2$) can be calculated in perturbation theory ("BSS mechanism"). In Ref. [11] this idea is used to find an estimate of $\langle Q_{1}^{u} \rangle$ which leads to an upper bound on $|\Delta \phi|$ which is smaller than the values found by SU(3)$_f$ arguments [6]. In this paper, we turn the BSS idea into a rigorous field-theoretic method by proving an operator product expansion (OPE)

$$\langle J/\psi K_S | Q_{1}^{u} | B_d \rangle = \sum_{k} \tilde{C}_{j,k} \langle J/\psi K_S | Q_{k} | B_d \rangle + \ldots$$ (9)

with $k$ running over $k = 0V, 0A, 8V, 8A$ and the dots representing terms suppressed by higher powers of $\Lambda_{\text{QCD}} / \sqrt{q^2}$. The Wilson coefficients $\tilde{C}_{j,k} = \tilde{C}_{j,k}^{(0)} + (\alpha_s(\mu) / 4\pi) \tilde{C}_{j,k}^{(1)} + \ldots$ are calculated in perturbation theory to the desired order in $\alpha_s(\mu)$, with the renormalization scale $\mu = \mathcal{O}(m_{\psi}, m_{b})$. A similar OPE has been derived to calculate charm-loop effects in the rare semileptonic decays $B \rightarrow K^{(\ast)} l^{\pm} \nu$ [12]. Since leptons carry no color charges, this application involves no four-quark operators like those in Eqs. (6) and (9). From the LO diagrams (see Fig. 1a) one finds $\tilde{C}_{j,k}^{(0)} = 0$ except for $\tilde{C}_{8G,8V}^{(0)} = - \frac{m_{\psi}^2}{q^2}$ and $\tilde{C}_{2,8V}^{(0)} = P(q^2)$ with the penguin function

$$P(q^2) = \frac{2}{3} \frac{\alpha_s}{4\pi} \left[ \ln \left( \frac{q^2}{\mu^2} \right) - i\pi - \frac{2}{3} \right].$$ (10)

PROOF OF FACTORIZATION

In order to establish Eq. (9) we must prove that the coefficients $\tilde{C}_{j,k}$ are infrared (IR) safe. To this end we analyze i) the soft IR divergences of the two-loop diagrams
contributing to \( Q_s^2 \), ii) the collinear IR divergences of these diagrams, iii) spectator scattering diagrams, and iv) higher-order diagrams in which the large momentum bypasses the penguin loop (“long distance penguins”).

i) An example of a diagram with a soft divergence is shown in Fig. 1b. This soft divergence is reproduced by the corresponding diagram of the effective-theory side (i.e. RHS) of Eq. (9), depicted in Fig. 1c, so that this divergence factorizes with \( \tilde{C}_{j,k}^{(0)} \) and does not affect \( \tilde{C}_{j,k}^{(1)} \). All soft divergences are from diagrams in which the additional gluon connects two external lines and cancel from \( \tilde{C}_{j,k}^{(1)} \) in the same way.

ii) Collinear divergences occur in diagrams in which a gluon is attached to the line with the strange quark, which we treat as massless. An example is shown in Fig. 1d. If \( l \) denotes the loop momentum flowing through the gluon propagator and \( p_s \) is the momentum of the external strange quark, the collinear divergence corresponds to the region with \( l^2 = 0 \) and \( l \propto p_s \). We can then reduce the problem to the study of one-loop diagrams with an external on-shell gluon: If we sum over all possibilities to attach this gluon to one of the lines of the LO diagram in Fig. 1a, the collinear Ward identity of QCD ensures that this sum vanishes when the open Lorentz index of the gluon line is contracted with \( l^\mu \). This feature ensures that the collinear divergences of the sum of the two-loop diagrams vanish. (For a discussion in the context of QCD factorization see Refs. [2, 13, 14].) It equally holds for the effective-theory side of the OPE. On both sides of Eq. (9) the sum over diagrams needed for the cancellation involves the diagram with a strange-quark self-energy which does not contribute to the truncated matrix element. But adding these unphysical diagrams to both sides of the OPE has no effect, since these diagrams factorize with \( \tilde{C}_{j,k}^{(0)} \). The cancellation of collinear divergences is conceptually identical to the situation in typical processes in collider physics; it is further known to be much simpler (with fewer diagrams to be discussed) if a physical gauge (with only two propagating gluon degrees of freedom) is adopted.

iii) Next we discuss the spectator scattering contributions: diagrams in which the gluon connects the \( b \) or \( s \) line with the spectator quark line trivially factorize with the corresponding diagrams on the effective side. If the gluon connects the spectator with the gluon line or a charm or up line, we have to take into account that the squared momentum in the penguin loop is \( (q + l)^2 \) instead of \( q^2 \). If the gluon is soft, \( l^\mu \sim \Lambda_{QCD} \), the expansion of the loop function \( P \) around \( q^2 \) reproduces a term which correctly factorizes with \( \tilde{C}_{j,k}^{(0)} \) up to term suppressed by \( \Lambda_{QCD}/\sqrt{q^2} \). If the gluon is hard-collinear, with virtuality \( l^2 \sim p_{cm} \Lambda_{QCD} \), the situation is more subtle: the LO diagram is suppressed by \( \Lambda_{QCD}/p_{cm} \), because the momentum of the spectator quark changes from zero to \( O(p_{cm}) \) in the decay, which is penalized by the light-cone distribution amplitude (LCDA) of the kaon [2]. The asymptotic form of the kaon LCDA, \( \Phi(x) = 6x(1-x) \), where \( x \) and \( 1-x \) are the fractions of the kaon momentum carried by the \( \pi \) and \( d \) quarks, favors momentum configurations in which the kaon momentum is roughly equally shared between the two valence quarks. While the propagator of the scattered hard-collinear gluon is suppressed as \( \sim 1/(\Lambda_{QCD} p_{cm}) \), the suppression of the LO diagram is lifted, because the spectator momentum is in the region \( x \sim 1/2 \) favored by the kaon LCDA. To identify further suppression factors we first discuss the case that the gluon connect a charm line with the spectator: counting \( q^2 \sim m_C^2 \) and the energies of \( \pi \) and spectator-\( d \) quarks as \( p_{cm}/2 \), the penguin loop gives \( P((q + l)^2) \approx P(q^2) + \frac{p_{cm}}{m_\psi} P(q^2) \). The non-factorizing piece involving the derivative \( P'(q^2) \) comes with a factor of \( p_{cm}/m_\psi \). The virtuality of the (anti-)charm propagator is around \( p_{cm} \) entailing a suppression factor of \( \Lambda_{QCD}/p_{cm} \). Thus, in total spectator scattering from the charm lines obeys Eq. (9) up to terms of order \( \Lambda_{QCD}/m_\psi \). Next we discuss the spectator scattering from the up line, with a sample diagram depicted in Fig. 1e. We find that these diagrams are power-suppressed by \( \Lambda_{QCD}/m_\psi \). In this respect these spectator diagrams differ from the similar photon penguins calculated in Ref. [15], which involve

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FIG. 1: The soft IR divergence of the diagram (b) factorizes with the corresponding diagram of the effective-theory side shown in (c). The diagram (d) is an example of a diagram with a collinear IR divergence.
adding a contribution of order $\Lambda_{\text{QCD}}$ to the
it may also be possible that the hard momentum trans-

doing away with the choice of $q^2$ associated with the choice of
where the dots represent the terms with $k$ of order $\Lambda_{\text{QCD}}$ and $m_\psi$. These diagrams, in which the
whole weak decay process occurs with small momentum transfers, have a suppression factor $(\Lambda_{\text{QCD}}/\sqrt{q^2})^3$ stemming from the hard gluon propagator and an off-shell $b$
quark propagator (or $s$ quark propagator).

In our discussion above we have considered the effect of $Q^2$. The proof equally holds for the contribution from $Q_{SG}$, which contributes to $\tilde{C}_{SG,SV}^8$ through a tree-level diagram.

In our power counting in i)-iv) we have treated $p_{cm}$ as an intermediate scale between $\Lambda_{\text{QCD}}$ and $m_\psi$ and have found no non-factorizable non-perturbative effects of order $p_{cm}/m_\psi$. While $p_{cm}$ enters two-loop diagrams through $p_{\mu} \cdot p_\nu \sim m_\psi p_{cm}$, such terms do not come with IR divergences and end up in the NLO corrections to the coefficients $C_{j,k}$. We find that the counting rule for $p_{cm}$ is irrelevant, one can reproduce our results above as well for the limiting cases $p_{cm} \sim \Lambda_{\text{QCD}}$ and $p_{cm} \sim \sqrt{q^2}$. The choice $q^2 = m_\psi^2$ for $P(q^2)$ may be altered by adding a contribution of order $\Lambda_{\text{QCD}}$ to $\sqrt{q^2}$. This shuffles a piece proportional to $(\Lambda_{\text{QCD}}/m_\psi)P(m_\psi^2)$ into the coefficient of the sub-leading operator $\bar{\psi}_\mu(1 - \gamma_5)T^a b \left[\begin{array}{c} \Box - m_\psi^2 \end{array}\right] \gamma^\mu T^a c$, which removes the ambiguity associated with the choice of $q^2$. At NLO in $\alpha_s$ one generates non-zero coefficients $\tilde{C}_{j,k}^{(1)}$ also for $j = 1$ or $k = 0, A, SA$.

**PHENOMENOLOGY**

The penguin amplitude depends on the Wilson coefficients as

$$P_f = V_0 \left(2C_4 + 2C_6 + 2C_2 \tilde{C}_{2,SV}^{(0)} + C_{SG} \tilde{C}_{SG,SV}^{(0)}\right) v_8 + \cdots$$

where the dots represent the terms with $v_0$ and $a_8$ which have much smaller coefficients. The dependence of $\tilde{C}_{2,SV}^{(0)}$ (calculated from Fig. 1a) on the renormalization scheme cancels with the scheme dependence of $C_4 + C_6$ in Eq. (11). In the NDR scheme adopted by us these penguin coefficients give a larger contribution to $P_f$ than the $u$-penguin loop contained in $\tilde{C}_{2,SV}^{(0)}$.

For the prediction of $P_f/T_f$ we implement the constraint from $B(B \to f)$ exemplified for $f = J/\psi K_S$ in Eq. (8) in the following way: adapting a phase convention in which $A_f$ in Eq. (1) is real and positive, we determine $a_8$ in terms of $V_0$, $v_0$, $v_8$, and the measured $B(B \to f)$ [16]. Then we use this to eliminate $a_8$ from $P_f/T_f$. For example, we find

$$P_{J/\psi K_S} \over T_{J/\psi K_S} = 0.01 - 0.02v_0 - (0.71 + 0.33i)v_8$$

for the central values of $V_0$ (quoted after Eq. (7)) and $B(B_d \to J/\psi K_S)$. We vary $v_0$ and $v_8$ in their allowed ranges $|v_0| < 1/3$ and $|v_8| = 1 \pm 0.15$ with the constraint that $|a_8| \leq 1/3$ must be obeyed. The allowed ranges for $\Delta \phi_7$, $C_f$, and $\Delta S_f \equiv S_f + \eta_7 \sin \phi_7$ are almost symmetric around zero. We list the upper bounds on their magnitudes for several decay modes in Tabs. I and II. The results include the uncertainties from $V_0$, the branching ratios, CKM parameters [17], and higher-order terms in our OPE. For the $b \to \tau \ell d$ decay modes with Cabibbo-unsuppressed $P_f/T_f$ the expansion in Eq. (5) has been replaced by the exact formula (see e.g. Ref. [5, 6]). Our bounds are conservative, as the considered ranges for $v_8$ and $a_8$ are wide (permitting even sizable cancellations in Eq. (8)). From Eqs. (8) and (12) one verifies that any additional information on magnitude or phase of one of these parameters will substantially reduce the ranges quoted in Tabs. I and II. Our results for $B_d \to J/\psi \pi^0$ favor the Belle measurement $C_{J/\psi \pi^0} = -0.08 \pm 0.17$, $S_{J/\psi \pi^0} = -0.65 \pm 0.22$ [18] over the BaBar result $C_{J/\psi \pi^0} = -0.20 \pm 0.19$, $S_{J/\psi \pi^0} = -1.23 \pm 0.21$ [19]. (In the absence of penguin pollution $C_{J/\psi \pi^0} = 0$ and $S_{J/\psi \pi^0} = -\sin(2\beta) = -0.69 \pm 0.02$.) In the case of a more precise and non-vanishing measurement of $C_{J/\psi \pi^0}$, for example, $C_{J/\psi \pi^0} = -0.10 \pm 0.01$, which corresponds to the current world average with a ten times smaller error, we can also put stronger restrictions on the shift of the mixing-induced CP violation $|\Delta S_{J/\psi \pi^0}| \leq 0.13$. A measurement of $C_{J/\psi \pi^0}$ that is consistent with zero, however, does not improve the bound. This feature occurs in all decay modes with Cabibbo-unsuppressed $P_f/T_f$. The recent measurements of $S_f$ and $C_f$ for the $B_d \to J/\psi \pi^0$ polarization amplitudes [20] comply with the ranges in Tab. I.

**CONCLUSIONS**

We have established a factorization formula (to leading power in $\Lambda_{\text{QCD}}/m_\psi$) for the penguin contribution to the CP-violating coefficients $S_f$ and $C_f$ in $A_{CF,1}^{B_{CP,f}}(t)$ for final states $f$ containing charmonium. As a crucial result the penguin contributions involve the same hadronic matrix elements as the tree amplitude. This allows us to constrain $P_f/T_f$ and e.g. find $|\Delta \phi_2| \leq 0.68^\circ$ for $B_d \to J/\psi K_S$ and $|\Delta \phi_2^+| \leq 0.95^\circ$ for $B_d \to J/\psi \phi$, representing bounds that were thought to be uncalculable from first principles. Novel territory are our predictions for $S_f$ and $C_f$ in $b \to \tau d$ decays, in which $P_f/T_f$
is Cabibbo-unsuppressed. Our results do not depend on any properties of the charmonium wave functions.

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TABLE I: The maximal phase shift of $\phi_d$ due to penguin pollution and limits for the $CP$ violation observables $S_f$ and $C_f$ in various $B_d \to f$ decays. Decays into two vector mesons involve different polarization amplitudes, indicated by $0$, $\|$ and $\perp$ [21].

<table>
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<th>Final State</th>
<th>$J/\psi\KS$</th>
<th>$J/\psi\pi^0$</th>
<th>$J/\psi\pi^0$</th>
<th>$J/\psi\rho^0$</th>
<th>$J/\psi\rho^0$</th>
<th>$J/\psi\K^*$</th>
<th>$J/\psi\K^*$</th>
<th>$J/\psi\K^*$</th>
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<tr>
<td>$\Delta\phi_d$</td>
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<td>0.74</td>
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<td>n.a.</td>
<td>n.a.</td>
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<tr>
<td>$\Delta\phi_f$</td>
<td>0.86</td>
<td>0.94</td>
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<td>22</td>
<td>27</td>
<td>22</td>
<td>1.11</td>
<td>1.64</td>
</tr>
<tr>
<td>$C_f$</td>
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<td>29</td>
<td>35</td>
<td>41</td>
<td>46</td>
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<td>2.47</td>
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TABLE II: Same as Tab. I for $B_s \to f$ decays.

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<th>$(J/\psi\phi)^0$</th>
<th>$(J/\psi\phi)^\perp$</th>
<th>$(J/\psi\K^*)^0$</th>
<th>$(J/\psi\K^*)^0$</th>
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<tr>
<td>$\Delta\phi_f$</td>
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<td>1.65</td>
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<td>$C_f$</td>
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<td>2.01</td>
<td>2.44</td>
<td>1.84</td>
<td>42</td>
<td>59</td>
<td>24</td>
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We modify the experimental $B(B_s \to J/\psi K_S)$ and $B(B_s \to J/\psi)$ branching ratios according to: K. De Bruyn, R. Fleischer, R. Knegjens, P. Koppenburg, M. Merk and N. Tuning, Phys. Rev. D 86, 014027 (2012) [arXiv:1204.1735 [hep-ph]].


