Updated NNLO QCD predictions for the weak radiative B-meson decays

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We perform an updated analysis of the inclusive weak radiative B-meson decays in the standard model, incorporating all our results for the $\mathcal{O}(\alpha_s^2)$ and lower-order perturbative corrections that have been calculated after 2006. New estimates of non-perturbative effects are taken into account, too. For the CP- and isospin-averaged branching ratios, we find $\mathcal{B}_{s\gamma} = (3.36 \pm 0.23) \times 10^{-4}$ and $\mathcal{B}_{d\gamma} = \left(1.73^{+0.12}_{-0.22}\right) \times 10^{-5}$, for $E_\gamma > 1.6\,\text{GeV}$. These results remain in agreement with the current experimental averages. Normalizing their sum to the inclusive semileptonic branching ratio, we obtain $R_\gamma \equiv (\mathcal{B}_{s\gamma} + \mathcal{B}_{d\gamma})/\mathcal{B}_{c\ell\nu} = (3.31 \pm 0.22) \times 10^{-3}$. A new bound from $\mathcal{B}_{s\gamma}$ on the charged Higgs boson mass in the two-Higgs-doublet-model II reads $M_{H^\pm} > 480\,\text{GeV}$ at 95%C.L.

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I. INTRODUCTION

The inclusive decays $\bar{B} \to X_s\gamma$ and $\bar{B} \to X_d\gamma$ are considered among the most interesting flavor changing neutral current processes. They contribute in a significant manner to current bounds on masses and interactions of possible additional Higgs bosons and/or supersymmetric particles. Measurements of the CP- and isospin-averaged $\bar{B} \to X_s\gamma$ branching ratio by CLEO [1], Belle [2, 3] and BABAR [4–7] lead to the combined result [8]

$$\mathcal{B}_{s\gamma}^{\text{exp}} = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}, \tag{1}$$

for the photon energy $E_\gamma > E_0 = 1.6\,\text{GeV}$ in the decaying meson rest frame. The combination involves an extrapolation from measurements performed at $E_0 \in [1.7, 2.0]\,\text{GeV}$. Applying the same extrapolation method to the available $\bar{B} \to X_d\gamma$ measurement [9], one finds

$$\mathcal{B}_{d\gamma}^{\text{exp}} = (1.41 \pm 0.57) \times 10^{-5} \tag{2}$$

at $E_0 = 1.6\,\text{GeV}$ [10]. More precise determinations of $\mathcal{B}_{q\gamma}^{\text{exp}}$ for $q = s, d$ are expected from Belle II [11].

Theoretical calculations of $\mathcal{B}_{q\gamma}$ have a chance to match the experimental precision only in a certain range of E_0 where the non-perturbative contribution $\delta\Gamma_{\text{nonp}}$ in the relation

$$\Gamma(\bar{B} \to X_q\gamma) \;=\; \Gamma(b \to X_q^p\gamma) \;+\; \delta\Gamma_{\text{nonp}} \tag{3}$$

remains under control. Here, $\Gamma(b \to X_q^p\gamma)$ denotes the perturbatively calculable rate of the radiative b-quark decay involving only charmless partons in the final state. Their overall strangeness vanishes for X_d^p and equals -1 for X_s^p. The analysis of Ref. [12] implies that unknown contributions to $\delta\Gamma_{\text{nonp}}$ are potentially larger than the so-far determined ones, and induce around $\pm 5\%$ uncertainty in $\mathcal{B}_{s\gamma}$ at $E_0 = 1.6\,\text{GeV}$. Non-perturbative uncertainties in $\mathcal{B}_{d\gamma}$ receive additional sizeable contributions [13] due to collinear photon emission in the $b \to du\bar{u}\gamma$ process whose Cabibbo-Kobayashi-Maskawa (CKM) factor is only a few times smaller than the one in the leading term.

Apart from possible future progress in analyzing non-perturbative effects, one needs to determine $\Gamma(b \to X_q^p\gamma)$ to a few percent accuracy. It requires evaluating next-to-next-to-leading order (NNLO) QCD corrections that involve Feynman diagrams up to four loops. The first standard model (SM) estimate of the $\bar{B} \to X_s\gamma$ branching ratio at this level was presented in Ref. [14] where all the corrections calculated up to 2006 were taken into account. A part of the $\mathcal{O}(\alpha_s^2)$ contribution was obtained via interpolation [15] in the charm quark mass between the large-m_c asymptotic expression [16] and the $m_c = 0$ boundary condition that was estimated using the Brodsky-Lepage-Mackenzie (BLM) approximation [17].

In the present paper, we provide an updated prediction for $\mathcal{B}_{s\gamma}$, including all the contributions and estimates worked out after 2006. They are listed in Sec. II where the necessary definitions are introduced. The interpolation in m_c is still being applied. However, the $m_c = 0$ boundary condition is no longer a BLM-based estimate but rather comes from an explicit calculation [18].

The paper is organized as follows. After discussing $\mathcal{B}_{s\gamma}$ in Sec. II, our NNLO analysis is extended to $\mathcal{B}_{d\gamma}$ in Sec. III. Next, in Sec. IV, we consider $R_\gamma \equiv (\mathcal{B}_{s\gamma} + \mathcal{B}_{d\gamma})/\mathcal{B}_{c\ell\nu}$ which may sometimes be more convenient than $\mathcal{B}_{s\gamma}$ for deriving constraints on new physics. Sec. V is devoted to presenting a generic expression for beyond-SM contributions, as well as an updated bound for the charged Higgs boson mass in the two-Higgs-doublet-model II (THDM II). We conclude in Sec. VI.

II. $\mathcal{B}_{s\gamma}$ IN THE SM

Radiative B-meson decays are most conveniently described in the framework of an effective theory that arises after decoupling of the W boson and heavier particles. Flavor-changing weak interactions that are relevant for $\Gamma(b \to X_q^p \gamma)$ with $q = s, d$ are given by

$$\mathcal{L}_{\rm eff} \sim V_{tq}^* V_{tb} \left[\sum_{i=1}^{8} C_i Q_i + \kappa_q \sum_{i=1}^{2} C_i (Q_i - Q_i^u) \right]. \quad (4)$$

Explicit expressions for the current-current ($Q_{1,2}$), four-quark penguin ($Q_{3,\ldots,6}$), photonic dipole (Q_7) and gluonic dipole (Q_8) operators can be found, e.g., in Eq. (2.5) of Ref. [15]. The CKM element ratio $\kappa_q = (V_{uq}^* V_{ub})/(V_{tq}^* V_{tb})$ is small for $q = s$, and it affects $\mathcal{B}_{s\gamma}$ by less than 0.3%. Barring this effect and the higher-order electroweak ones, $\Gamma(b \to X_s^p \gamma)$ in the SM is given by a quadratic polynomial in the real Wilson coefficients C_i

$$\Gamma(b \to X_s^p \gamma) \sim \sum_{i,j=1}^{8} C_i C_j \, G_{ij}. \quad (5)$$

A series of contributions to the above expression from our calculations in Refs. [18–27] makes the current analysis significantly improved with respect to the one in Ref. [14]. In particular, the NNLO Wilson coefficient calculation becomes complete after including the four-loop anomalous dimensions that describe $Q_{1,\ldots,6} \to Q_8$ mixing under renormalization [19]. Effects of the charm and bottom quark masses in loops on the gluon lines in G_{77} [20], G_{78} [21] and $G_{(1,2)7}$ [22], as well as a complete calculation of G_{78} [23] are now available. Three- and four-body final-state contributions to G_{88} [24, 25] and $G_{(1,2)8}$ [25] are included in the BLM approximation. Four-body final-state contributions involving the penguin and $Q_{1,2}^u$ operators are taken into account at the leading order (LO) [26] and next-to-leading order (NLO) [27]. Last but not least, the complete NNLO calculation [18] of G_{17} and G_{27} at $m_c = 0$ is used as a boundary for interpolating their unknown parts in m_c.

Following the algorithm described in detail in Ref. [18], taking into account new non-perturbative effects [12, 28, 29], as well as the previously omitted parts of the NNLO BLM corrections [31], we arrive at the following SM prediction

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.36 \pm 0.23) \times 10^{-4} \quad \text{for } E_0 = 1.6\,\text{GeV}. \quad (6)$$

Individual contributions to the total uncertainty are of non-perturbative (±5%), higher-order (±3%), interpolation (±3%) and parametric (±2%) origin. They are combined in quadrature. The parametric one gets reduced with respect to Ref. [14], which becomes possible thanks to the new semileptonic fits of Ref. [30]. Unfortunately, the interpolation uncertainty cannot be reduced because the interpolated parts of the $\mathcal{O}(\alpha_s^2)$ non-BLM contributions to $G_{(1,2)7}$ turn out to be sizeable. Their effect on $\mathcal{B}_{s\gamma}^{\rm SM}$ grows from 0 to around 5% when m_c changes from 0 up to the measured value.

III. $\mathcal{B}_{d\gamma}$ IN THE SM

Extending our NNLO calculation to the $\mathcal{B}_{d\gamma}$ case begins with inserting the proper CKM factors in Eq. (4). Contrary to κ_s, the ratio κ_d is not numerically small. Using the CKM fits of Ref. [32], one finds

$$\kappa_d = \left(0.007^{+0.015}_{-0.011}\right) + i\left(-0.404^{+0.012}_{-0.014}\right). \quad (7)$$

The small real part implies that the effects of κ_d on the CP-averaged $\mathcal{B}_{d\gamma}$ are dominated by those proportional to $|\kappa_d|^2$. In such terms, perturbative two- and three-body final state contributions arise only at the NNLO and NLO, respectively. They vanish in the $m_c = m_u$ limit, which effectively makes them suppressed by $m_c^2/m_b^2 \lesssim 0.1$. In consequence, the main κ_d-effect comes from $b \to d u \bar{u} \gamma$ at the LO, where phase-space suppression is partially compensated by the collinear logarithms.

In the first (rough) approximation, one evaluates the tree-level $b \to d u \bar{u} \gamma$ diagrams retaining a common light-quark mass m_q inside the collinear logarithms [25], and varying m_b/m_q between $10 \sim m_B/m_K$ and $50 \sim m_B/m_\pi$ to estimate the uncertainty. The considered effect varies then from 2% to 11% of $\mathcal{B}_{d\gamma}$. A more involved analysis with the help of fragmentation functions gives a very similar range [13]. Including this contribution in our evaluation of the entire $B_{d\gamma}$ from Eq. (4), we find

$$\mathcal{B}_{d\gamma}^{\rm SM} = \left(1.73^{+0.12}_{-0.22}\right) \times 10^{-5} \quad \text{for } E_0 = 1.6\,\text{GeV}, \quad (8)$$

where the central value corresponds to $m_b/m_q = 50$. Our result is about 12% larger than the one given in Ref. [10] where the $b \to d u \bar{u} \gamma$ contributions were neglected. The uncertainty estimate in Eq. (8) improves with respect to Ref. [10] thanks to including the NNLO QCD corrections and using the updated CKM fit [32]. Interestingly, the parametric uncertainty due to the CKM input amounts to ±2.5% only.

The collinear logarithm problem might seem artificial because isolated photons are required in the experimental signal sample. Unfortunately, requiring photon isolation on the perturbative side would necessitate introducing an infrared cutoff on the gluon energies, e.g., in the NLO corrections to the dominant G_{77} term. Without a dedicated analysis (which is beyond the scope of the present paper), it is hard to verify whether such an approach would enhance or suppress the uncertainty in $\mathcal{B}_{d\gamma}$.

Another question concerning the $|\kappa_d|^2$-terms is whether the off-shell light vector meson conversion to photons can be assumed to be included in our overall $\pm5\%$ non-perturbative uncertainty. Much smaller effects found in the vector-meson-dominance analysis of Ref. [33] imply that it is likely to be the case.

IV. THE RATIO R_γ

In the fully inclusive measurements of radiative B-meson decays [1, 3–5], the final hadronic state strangeness is not verified. The actually measured quantity is $\mathcal{B}_{s\gamma} + \mathcal{B}_{d\gamma}$. Next, the result is divided by $\left(1 + |(V^*_{td}V_{tb})/(V^*_{ts}V_{tb})|^2\right)$ to obtain $\mathcal{B}_{s\gamma}$. To avoid such a complication, we provide here our SM prediction for $\mathcal{B}_{s\gamma} + \mathcal{B}_{d\gamma}$ with all the correlated uncertainties properly taken into account. Moreover, we normalize it to the CP- and isospin-averaged inclusive semileptonic branching ratio $\mathcal{B}_{c\ell\nu}$. In the $\mathcal{B}_{s\gamma}$ case, such a normalization reduces the parametric uncertainty from $\pm2.0\%$ to $\{+1.2, -1.4\}\%$. It may also be useful on the experimental side because the inclusive semileptonic events can serve for determining the B-meson yield. Proceeding as in the previous sections, we obtain for $E_\gamma = 1.6\,\mathrm{GeV}$

$$R^{\mathrm{SM}}_\gamma \equiv \left(\mathcal{B}^{\mathrm{SM}}_{s\gamma} + \mathcal{B}^{\mathrm{SM}}_{d\gamma}\right)/\mathcal{B}_{c\ell\nu} = (3.31 \pm 0.22) \times 10^{-3}. \quad (9)$$

The relative uncertainties are identical to those in $\mathcal{B}_{s\gamma}$ (as given below Eq. (6)), except for the parametric one which amounts to $\{+1.2, -1.7\}\%$ including the effect of m_b/m_q. The gain in the overall theory uncertainty is hardly noticeable, but this may change with the future progress in determining the perturbative and non-perturbative corrections.

V. BEYOND-SM EFFECTS

In most of the new-physics scenarios considered in the literature, beyond-SM effects on $\mathcal{B}_{s\gamma}$ are driven by new additive contributions to the Wilson coefficients of the dipole operators at the matching scale μ_0 where the heavy particles (t, W, Z, H^0, ...) are decoupled. Denoting such contributions by $\Delta C_{7,8}$ and setting μ_0 to 160 GeV, we find

$$\begin{aligned} \mathcal{B}_{s\gamma} \times 10^4 &= (3.36 \pm 0.23) - 8.22\,\Delta C_7 - 1.99\,\Delta C_8, \\ R_\gamma \times 10^3 &= (3.31 \pm 0.22) - 8.05\,\Delta C_7 - 1.94\,\Delta C_8. \quad (10) \end{aligned}$$

The above expressions are linearized, i.e. it is assumed that the quadratic terms in $\Delta C_{7,8}$ are negligible when they enter with $\mathcal{O}(1)$ coefficients into the above equations. If they are not, a detailed analysis of QCD corrections in the considered beyond-SM scenario is necessary.

Such an analysis is available in the THDM II [34] for which the NLO [35–37] and NNLO [38] corrections to $\Delta C_{7,8}$ are known. They are always negative and remain practically independent of the vacuum expectation value ratio $\tan\beta$ when $\tan\beta \gtrsim 2$. Sending $\tan\beta$ to infinity in the expressions for $\Delta C_{7,8}$, we find the following updated bounds from $\mathcal{B}_{s\gamma}$ on the charged Higgs boson mass in this model

$$\begin{aligned} M_{H^\pm} &> 480\,\mathrm{GeV} \text{ at } 95\%\mathrm{C.L.}, \\ M_{H^\pm} &> 358\,\mathrm{GeV} \text{ at } 99\%\mathrm{C.L.}. \quad (11) \end{aligned}$$

For $\tan\beta \lesssim 2$ the bounds become considerably stronger, but at the same time other observables provide competitive limits [39]. In the supersymmetric case, in which the charged scalar and the neutral pseudoscalar tend to be almost degenerate, the current direct search bounds [40, 41] exceed 500 GeV for $\tan\beta \gtrsim 20$.

VI. SUMMARY

We presented an updated prediction for $\mathcal{B}_{s\gamma}$ in the SM taking into account all the perturbative and non-perturbative effects worked out after the 2006 publication [14] of the first NNLO estimate for this quantity. Some of the $\mathcal{O}(\alpha_s^2)$ corrections are still interpolated in m_c, but the $m_c = 0$ boundary condition now comes from an explicit calculation. Despite this improvement, the interpolation uncertainty cannot be reduced because the interpolated correction is sizeable. Future progress requires extending the calculation of $G_{(1,2)7}$ to arbitrary m_c, which is considered a difficult but manageable task. In parallel, one should investigate whether non-perturbative uncertainties can be suppressed by combining lattice inputs with measurements of observables like the CP- or isospin asymmetries in $\bar{B} \to X_q\gamma$.

The main outcome of the current update is an upwards shift by around 6.4% in the central value of $\mathcal{B}^{\mathrm{SM}}_{s\gamma}$. It originates mainly from fixing the $m_c = 0$ boundary (+3%) and including the complete NNLO BLM corrections to the three- and four-body final state channels (+2%). Since $\mathcal{B}^{\mathrm{SM}}_{s\gamma}$ is now closer to $\mathcal{B}^{\mathrm{exp}}_{s\gamma}$ (but still $\mathcal{B}^{\mathrm{SM}}_{s\gamma} < \mathcal{B}^{\mathrm{exp}}_{s\gamma}$), the bound on $M_{H^\pm}$ in the THDM II becomes significantly stronger.

We supplemented our analysis with a prediction for $\mathcal{B}_{d\gamma}$ as well as the ratio $R_\gamma = (\mathcal{B}_{s\gamma} + \mathcal{B}_{d\gamma})/\mathcal{B}_{c\ell\nu}$ where correlated uncertainties are treated in a consistent manner. The ratio R_γ may serve in the future as a more convenient observable for testing beyond-SM theories with minimal flavor violation.

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