Updated NNLO QCD predictions for the weak radiative $B$-meson decays

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With the non-perturbative contribution $\Gamma_{\text{nonp}}$ included, one needs to determine $\Gamma_{\text{nonp}}$ at $E_e = 1.6$ GeV. Perturbative contributions to $\Gamma_{\text{nonp}}$ are potentially larger than the so-large determined ones, and induce around ±5% uncertainty in $B_{s\gamma}$ at $E_e = 1.6$ GeV. Non-perturbative uncertainties in $B_{s\gamma}$ receive additional sizeable contributions [13] due to collinear photon emission in the $b \to d\gamma$ process whose Cabibbo-Kobayashi-Maskawa (CKM) factor is only a few times smaller than the one in the leading term.

In the present paper, we provide an updated prediction for $B_{s\gamma}$, including all the contributions and estimates
worked out after 2006. They are listed in Sec. II where the necessary definitions are introduced. The interpolation in $m_c$ is still being applied. However, the $m_c = 0$ boundary condition is no longer a BLM-based estimate but rather comes from an explicit calculation [18].

The paper is organized as follows. After discussing $B_{\gamma\gamma}$ in Sec. II, our NNLO analysis is extended to $B_{d\gamma}$ in Sec. III. Next, in Sec. IV, we consider $R_\gamma \equiv \left( B_{\gamma\gamma} + B_{d\gamma} \right) / B_{d\gamma}$ which may sometimes be more convenient than $B_{\gamma\gamma}$ for deriving constraints on new physics. Sec. V is devoted to presenting a generic expression for beyond-SM contributions, as well as an updated bound for the charged Higgs boson mass in the two-Higgs-doublet-model II (THDM II). We conclude in Sec. VI.

II. $B_{\gamma\gamma}$ in the SM

Radiative $B$-meson decays are most conveniently described in the framework of an effective theory that arises after decoupling of the $W$ boson and heavier particles. Flavor-changing weak interactions that are relevant for $\Gamma(b \rightarrow X_{\ell}^{0} \gamma)$ with $q = s, d$ are given by

$$L_{\text{eff}} \sim V_{ts}^{*} V_{tb} \left[ \sum_{i=1}^{8} C_{i} Q_{i} + \kappa_{s} \sum_{i=1}^{2} C_{i}(Q_{i} - Q_{i}^{\ast}) \right].$$

(4)

Explicit expressions for the current-current ($Q_{1,2}$), four-quark penguin ($Q_{3,\ldots,6}$), photonic dipole ($Q_{7}$) and gluonic dipole ($Q_{8}$) operators can be found, e.g., in Eq. (2.5) of Ref. [15]. The CKM element ratio $\kappa_{s} = (V_{ts}^{*} V_{tb}) / (V_{tb}^{*} V_{tb})$ is small for $q = s$, and it affects $B_{\gamma\gamma}$ by less than 0.3%. Barring this effect and the higher-order electroweak ones, $\Gamma(b \rightarrow X_{\ell}^{0} \gamma)$ in the SM is given by a quadratic polynomial in the real Wilson coefficients $C_{i}$

$$\Gamma(b \rightarrow X_{\ell}^{0} \gamma) \sim \sum_{i,j=1}^{8} C_{i} C_{j} G_{ij}. 

(5)$$

A series of contributions to the above expression from our calculations in Refs. [18–27] makes the current analysis significantly improved with respect to the one in Ref. [14]. In particular, the NNLO Wilson coefficient calculation becomes complete after including the four-loop anomalous dimensions that describe $Q_{1,\ldots,6} \rightarrow Q_{8}$ mixing under renormalization [19]. Effects of the charm and bottom quark masses in loops on the gluon lines in $G_{77}$ [20], $G_{78}$ [21] and $G_{(1,2)7}$ [22], as well as a complete calculation of $G_{78}$ [23] are now available. Three- and four-body final-state contributions to $G_{88}$ [24, 25] and $G_{(1,2)8}$ [25] are included in the BLM approximation. Four-body final-state contributions involving the penguin and $Q_{1,2}^{\ast}$ operators are taken into account at the leading order (LO) [26] and next-to-leading order (NLO) [27]. Last but not least, the complete NNLO calculation [18] of $G_{17}$ and $G_{27}$ at $m_c = 0$ is used as a boundary for interpolating their unknown parts in $m_c$.

Following the algorithm described in detail in Ref. [18], taking into account new non-perturbative effects [12, 28, 29], as well as the previously omitted parts of the NNLO BLM corrections [31], we arrive at the following SM prediction

$$B_{\gamma\gamma}^{\text{SM}} = (3.36 \pm 0.23) \times 10^{-4} \quad \text{for} \quad E_{0} = 1.6 \text{GeV}. \quad (6)$$

Individual contributions to the total uncertainty are of non-perturbative (±5%), higher-order (±3%), interpolation (±3%) and parametric (±2%) origin. They are combined in quadrature. The parametric one gets reduced with respect to Ref. [14], which becomes possible thanks to the new semileptonic fits of Ref. [30]. Unfortunately, the interpolation uncertainty cannot be reduced because the interpolated parts of the $O(\alpha_{s}^{2})$ non-BLM contributions to $G_{(1,2)7}$ turn out to be sizeable. Their effect on $B_{\gamma\gamma}^{\text{SM}}$ grows from 0 to around 5% when $m_c$ changes from 0 up to the measured value.

III. $B_{d\gamma}$ in the SM

Extending our NNLO calculation to the $B_{d\gamma}$ case begins with inserting the proper CKM factors in Eq. (4). Contrary to $\kappa_{s}$, the ratio $\kappa_{d}$ is not numerically small. Using the CKM fits of Ref. [32], one finds

$$\kappa_{d} = \left( 0.007^{+0.015}_{-0.011} \right) + i \left( -0.404^{+0.012}_{-0.011} \right). \quad (7)$$

The small real part implies that the effects of $\kappa_{d}$ on the CP-averaged $B_{d\gamma}$ are dominated by those proportional to $|\kappa_{d}|^{2}$. In such terms, perturbative two- and three-body final state contributions arise only at the NNLO and NLO, respectively. They vanish in the $m_c = m_{b}$ limit, which effectively makes them suppressed by $m_{c}^{2} / m_{b}^{2} \lesssim 0.1$. In consequence, the main $\kappa_{d}$-effect comes from $b \rightarrow d\bar{u}\gamma$ at the LO, where phase-space suppression is partially compensated by the collinear logarithms.

In the first (rough) approximation, one evaluates the tree-level $b \rightarrow d\bar{u}\gamma$ diagrams retaining a common light-quark mass $m_{q}$ inside the collinear logarithms [25], and varying $m_{b} / m_{q}$ between $10 \sim m_{b} / m_{c}$ and $50 \sim m_{b} / m_{c}$ to estimate the uncertainty. The considered effect varies then from 2% to 11% of $B_{d\gamma}$. A more involved analysis with the help of fragmentation functions gives a very similar range [13]. Including this contribution in our evaluation of the entire $B_{d\gamma}$ from Eq. (4), we find

$$B_{d\gamma}^{\text{SM}} = (1.73^{+0.12}_{-0.22}) \times 10^{-5} \quad \text{for} \quad E_{0} = 1.6 \text{GeV}, \quad (8)$$

where the central value corresponds to $m_{b} / m_{q} = 50$. Our result is about 12% larger than the one given in Ref. [10] where the $b \rightarrow d\bar{u}\gamma$ contributions were neglected. The uncertainty estimate in Eq. (8) improves with respect to Ref. [10] thanks to including the NNLO QCD corrections and using the updated CKM fit [32]. Interestingly, the parametric uncertainty due to the CKM input amounts to ±2.5% only.
The collinear logarithm problem might seem artificial because isolated photons are required in the experimental signal sample. Unfortunately, requiring photon isolation on the perturbative side would necessitate introducing an infrared cutoff on the gluon energies, e.g., in the NLO corrections to the dominant $G_{77}$ term. Without a dedicated analysis (which is beyond the scope of the present paper), it is hard to verify whether such an approach would enhance or suppress the uncertainty in $B_{dγ}$.

Another question concerning the $|κ_d|^2$-terms is whether the off-shell light vector meson conversion to photons can be assumed to be included in our overall ±5% non-perturbative uncertainty. Much smaller effects found in the vector-meson-dominance analysis of Ref. [33] imply that it is likely to be the case.

IV. THE RATIO $R_γ$

In the fully inclusive measurements of radiative $B$-meson decays [1, 3–5], the final hadronic state strangeness is not verified. The actually measured quantity is $B_{sγ} + B_{τγ}$. Next, the result is divided by $(1 + |(V_{ud}V_{tb})/(V_{ub}V_{tb})|^2)$ to obtain $B_{γγ}$. To avoid such a complication, we provide here our SM prediction for $B_{sγ} + B_{τγ}$ with all the correlated uncertainties properly taken into account. Moreover, we normalize it to the CP- and isospin-averaged inclusive semileptonic branching ratio $B_{dγ}$.

In the $B_{γγ}$ case, such a normalization reduces the parametric uncertainty from ±2.0% to {+1.2, −1.4}%. It may also be useful on the experimental side because the inclusive semileptonic events can serve for determining the $B$-meson yield. Proceeding as in the previous sections, we obtain for $E_γ = 1.6$ GeV

$$R_{γ}^{SM} \equiv \frac{(B_{sγ} + B_{τγ})}{B_{dγ}} = (3.31 \pm 0.22) \times 10^{-3}. \quad (9)$$

The relative uncertainties are identical to those in $B_{sγ}$ (as given below Eq. (6)), except for the parametric one which amounts to {+1.2, −1.7}% including the effect of $m_τ/m_τ$. The gain in the overall theory uncertainty is hardly noticeable, but this may change with the future progress in determining the perturbative and non-perturbative corrections.

V. BEYOND-SM EFFECTS

In most of the new-physics scenarios considered in the literature, beyond-SM effects on $B_{sγ}$ are driven by new additive contributions to the Wilson coefficients of the dipole operators at the matching scale $μ_0$ where the heavy particles $(t, W, Z, H^0, \ldots)$ are decoupled. Denoting such contributions by $\Delta C_{7,8}$ and setting $μ_0$ to 160 GeV, we find

$$B_{sγ} \times 10^4 = (3.36 \pm 0.23) - 8.22 \Delta C_7 - 1.99 \Delta C_8,$$

$$R_γ \times 10^3 = (3.31 \pm 0.22) - 8.05 \Delta C_7 - 1.94 \Delta C_8. \quad (10)$$

The above expressions are linearized, i.e. it is assumed that the quadratic terms in $\Delta C_{7,8}$ are negligible when they enter with $O(1)$ coefficients into the above equations. If they are not, a detailed analysis of QCD corrections in the considered beyond-SM scenario is necessary.

Such an analysis is available in the THDM II [34] for which the NLO [35–37] and NNLO [38] corrections to $\Delta C_{7,8}$ are known. They are always negative and remain practically independent of the vacuum expectation value ratio $tan β$ when $tan β \gtrsim 2$. Sending $tan β$ to infinity in the expressions for $\Delta C_{7,8}$, we find the following updated bounds from $B_{sγ}$ on the charged Higgs boson mass in this model

$$M_{H^±} > 480 \text{ GeV} \text{ at } 95\% \text{ C.L.},$$

$$M_{H^±} > 358 \text{ GeV} \text{ at } 99\% \text{ C.L.}. \quad (11)$$

For $tan β \lesssim 2$ the bounds become considerably stronger, but at the same time other observables provide competitive limits [39]. In the supersymmetric case, in which the charged scalar and the neutral pseudoscalar tend to be almost degenerate, the current direct search bounds [40, 41] exceed 500 GeV for $tan β \gtrsim 20$.

VI. SUMMARY

We presented an updated prediction for $B_{sγ}$ in the SM taking into account all the perturbative and non-perturbative effects worked out after the 2006 publication [14] of the first NNLO estimate for this quantity. Some of the $O(α_s^2)$ corrections are still interpolated in $m_τ$, but the $m_τ = 0$ boundary condition now comes from an explicit calculation. Despite this improvement, the interpolation uncertainty cannot be reduced because the interpolated correction is sizeable. Future progress requires extending the calculation of $G_{1,2,7}$ to arbitrary $m_τ$, which is considered a difficult but manageable task. In parallel, one should investigate whether non-perturbative uncertainties can be suppressed by combining lattice inputs with measurements of observables like the CP- or isospin asymmetries in $B \rightarrow X_τ γ$.

The main outcome of the current update is an upwards shift by around 6.4% in the central value of $B_{sγ}^{SM}$. It originates mainly from fixing the $m_τ = 0$ boundary (+3%) and including the complete NNLO BLM corrections to the three- and four-body final state channels (+2%). Since $B_{sγ}^{SM}$ is now closer to $B_{τγ}^{SM}$ (but still $B_{sγ}^{SM} < B_{τγ}^{SM}$), the bound on $M_{H^±}$ in the THDM II becomes significantly stronger.

We supplemented our analysis with a prediction for $B_{dγ}$ as well as the ratio $R_γ = (B_{sγ} + B_{τγ})/B_{dγ}$, where correlated uncertainties are treated in a consistent manner. The ratio $R_γ$ may serve in the future as a more convenient observable for testing beyond-SM theories with minimal flavor violation.
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[2] K. Abe et al. (Belle Collaboration), Phys. Lett. B 511, 151 (2001) [hep-ex/0103042]. This measurement has recently been superseded by a new one in Ref. [42], which is not yet taken into account in the world average of Ref. [8].