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$B_{s,d} \to \ell^+\ell^-$ in the Standard Model

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We combine our new results for the $\mathcal{O}(\alpha_{em})$ and $\mathcal{O}(\alpha_s^2)$ corrections to $B_{s,d} \to \ell^+\ell^-$, and present updated branching ratio predictions for these decays in the standard model. Inclusion of the new corrections removes major theoretical uncertainties of perturbative origin that have just begun to dominate over the parametric ones. For the recently observed muonic decay of the B_s meson, our calculation gives $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = (3.65 \pm 0.23) \times 10^{-9}$.

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Rare leptonic decays of the neutral B mesons are known to provide important constraints on extensions of the standard model (SM). Their average time-integrated branching ratios $\overline{\mathcal{B}}_{q\ell} \equiv \overline{\mathcal{B}}[B_q \to \ell^+\ell^-]$ ($q = s, d$; $\ell = e, \mu, \tau$) depend on details of $B_q\bar{B}_q$ mixing [1]. A simple relation $\overline{\mathcal{B}}_{q\ell} = \Gamma[B_q \to \ell^+\ell^-]/\Gamma^q_H$ holds in the SM to a very good approximation, with Γ^q_H denoting the heavier mass-eigenstate total width. For $\ell = \mu$, the current experimental world averages read [2]

$$\overline{\mathcal{B}}_{s\mu} = (2.9{\pm}0.7){\times}10^{-9}, \quad \overline{\mathcal{B}}_{d\mu} = \left(3.6^{+1.6}_{-1.4}\right){\times}10^{-10}. \quad (1)$$

They have been obtained by combining the recent measurements of CMS [3] and LHCb [4]. In the $\overline{\mathcal{B}}_{s\mu}$ case, reduction of uncertainties to a few percent level is expected in the forthcoming decade. To match such an accuracy, theoretical calculations must include the next-to-leading order (NLO) corrections of electroweak (EW) origin, as well as QCD corrections up to the next-to-next-to-leading order (NNLO). In the present paper, we combine our new calculations of the NLO EW [5] and NNLO QCD [6] corrections to the relevant coupling constant (Wilson coefficient) C_A, and present updated SM predictions for all the $\overline{\mathcal{B}}_{q\ell}$ branching ratios.

A convenient framework for describing the considered processes is an effective theory derived from the SM by decoupling the top quark, the Higgs boson, and the heavy electroweak bosons W and Z. The effective weak interaction Lagrangian relevant for $B_q \to \ell^+\ell^-$ reads

$$\mathcal{L}_{\rm weak} = N\, C_A(\mu_b)\, (\bar{b}\gamma_\alpha\gamma_5 q)(\bar{\ell}\gamma^\alpha\gamma_5\ell) + \ldots\,, \quad (2)$$

where C_A is the $\overline{\rm MS}$-renormalized Wilson coefficient at the scale $\mu_b \sim m_b$. The ellipses stand for other, subleading weak interaction terms (operators) which we discuss below. The normalization constant $N = V^\star_{tb}V_{tq}\, G_F^2 M_W^2/\pi^2$ is given in terms of the Fermi constant G_F (extracted from the muon decay), the W-boson on-shell mass M_W, and the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{ij}.

Once $C_A(\mu_b)$ is determined to sufficient accuracy, the branching ratio is easily expressed in terms of the lepton mass m_ℓ, the B_q-meson mass M_{B_q} and its decay constant f_{B_q}. The latter is defined by the QCD matrix element $\langle 0|\bar{b}\gamma^\alpha\gamma_5 q|B_q(p)\rangle = ip^\alpha f_{B_q}$. One finds

$$\overline{\mathcal{B}}_{q\ell} \;=\; \frac{|N|^2 M^3_{B_q} f^2_{B_q}}{8\pi\,\Gamma^q_H}\, \beta_{q\ell}\, r^2_{q\ell}\, |C_A(\mu_b)|^2 \,+\, \mathcal{O}(\alpha_{em}), \quad (3)$$

where $r_{q\ell} = 2m_\ell/M_{B_q}$ and $\beta_{q\ell} = \sqrt{1 - r^2_{q\ell}}$. Eq. (3) holds at the leading order in flavor-changing weak interactions and in $M^2_{B_q}/M^2_W$, which is accurate up to permille-level corrections. In particular, operators like $(\bar{b}\gamma_5 q)(\bar{\ell}\ell)$ from the Higgs boson exchanges give rise to $\mathcal{O}(M^2_{B_q}/M^2_W)$ effects only. Thus, one neglects such operators in the SM. However, they often matter in beyond-SM theories.

As far as the $\mathcal{O}(\alpha_{em})$ term in Eq. (3) is concerned, it requires more explanation because we are going to neglect it while including complete corrections of this order to $C_A(\mu_b)$. The first observation to make is that some of the $\mathcal{O}(\alpha_{em})$ corrections to $C_A(\mu_b)$ get enhanced by $1/\sin^2\theta_W$, powers of m_t^2/M_W^2 or logarithms $\ln^2 M_W^2/\mu_b^2$, as explained in Ref. [5]. None of these enhancements is possible for the $\mathcal{O}(\alpha_{em})$ term in Eq. (3) once $\mu_b \sim m_b$. This term is μ_b-dependent and contains contributions from operators like $(\bar{b}\gamma_\alpha\gamma_5 q)(\bar{\ell}\gamma^\alpha\ell)$ or $(\bar{b}\gamma_\alpha P_L c)(\bar{c}\gamma^\alpha P_L s)$, with photons connecting the quark and lepton lines. It depends on non-perturbative QCD in a way that is not described by f_{B_q} alone, and it must compensate the μ_b-dependence of $C_A(\mu_b)$. Since we neglect this term, scale dependence serves as one of the uncertainty estimates. When μ_b is varied from $m_b/2$ to $2m_b$, our results for $|C_A(\mu_b)|^2$ vary by about 0.3%, which corresponds to a typical size of $\mathcal{O}(\alpha_{em})$ corrections that undergo no extra enhancement. On the other hand, the NLO EW corrections to $|C_A(\mu_b)|^2$ often reach a few percent level [5].

The only other possible enhancement of QED corrections that one may worry about is related to soft pho-

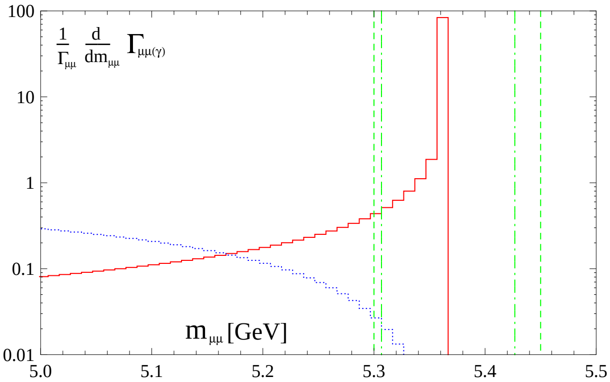

FIG. 1: Contributions to the dimuon invariant-mass spectrum in $B_s \to \mu^+\mu^-(n\gamma)$ with $n = 0, 1, 2, \ldots$ (see the text). Both of them are displayed in bins of 0.01 GeV width.

ton bremsstrahlung. For definiteness, let us consider $B_s \to \mu^+\mu^-(n\gamma)$ with $n = 0, 1, 2, \ldots$. The dimuon invariant-mass spectrum in this process is obtained by summing the two distributions shown in Fig. 1. The dotted (blue) curve corresponds to real photon emission from the quarks (Eq. (25) of Ref. [7]), while the tail of the solid (red) one is dominated by soft photon radiation from the muons (Eqs. (19)–(23) of Ref. [8]). The vertical dashed and dash-dotted (green) lines indicate the CMS [3] and LHCb [4] signal windows, respectively. In the displayed region below the windows (i.e. between 5 and 5.3 GeV), each of the two contributions integrates to around 5% of the total rate.

The determination of $\overline{\mathcal{B}}_{s\mu}$ on the experimental side includes a correction due to photon bremsstrahlung from the muons. For this purpose, LHCb [4] applies PHOTOS [9]. Such an approach is practically equivalent to extrapolating along the solid curve in Fig. 1 down to zero. In the resulting quantity, all the soft QED logarithms cancel out, and we obtain $\overline{\mathcal{B}}_{s\mu}$ as in Eq. (3), up to $\mathcal{O}(\alpha_{em})$ terms that undergo no extra enhancement [8].

The direct emission, i.e. real photon emission from the quarks is infrared safe by itself because the decaying meson is electrically neutral. It is effectively treated as background on both the experimental and theoretical sides. On the experimental side, it is neglected in the signal window (being very small there, indeed), and not included in the extrapolation. On the theory side, it is just excluded from $\overline{\mathcal{B}}_{s\mu}$ by definition. This contribution survives in the limit $m_\mu \to 0$, which explains its considerable size below the signal window in Fig. 1.

In this context, one may wonder whether the helicity suppression factor $r^2_{q\ell}$ in Eq. (3) can be relaxed at higher orders in QED. For the two-body decay it is not possible in the SM because a generic non-local interaction of B_q with massless leptons contains vector or axial-vector lepton currents contracted with the lepton momenta, which means that it vanishes on shell. On the other hand, contributions with (real or virtual) photons coupled to the quarks may survive in the $m_\ell \to 0$ limit, but they are phase-space suppressed in the signal window (cf. the dotted line in Fig. 1). In the $\overline{\mathcal{B}}_{s\mu}$ case, the phase-space suppression is at least as effective as the helicity suppression, given the applied window sizes in both experiments.

Parameter	Value	Unit	Ref.
G_F	$1.166\,379 \times 10^{-5}$	GeV^{-2}	[10]
$\alpha_s^{(5)}(M_Z)$	0.1184 (7)	–	[10]
$\alpha_{em}^{(5)}(M_Z)$	1/127.944 (14)	–	[10]
$\Delta\alpha^{(5)}_{em,\mathrm{hadr}}(M_Z)$	0.02772 (10)	–	[10]
M_Z	91.1876 (21)	GeV	[10]
M_t	173.1 (9)	GeV	[10]
M_H	125.9 (4)	GeV	[10]
M_{B_s}	5366.77 (24)	MeV	[10]
M_{B_d}	5279.58 (17)	MeV	[10]
f_{B_s}	227.7 (4.5)	MeV	[11]
f_{B_d}	190.5 (4.2)	MeV	[11]
$1/\Gamma^s_H$	1.615 (21)	ps	[17]
$2/(\Gamma^d_H + \Gamma^d_L)$	1.519 (7)	ps	[17]
$\lvert V_{cb}\rvert$	0.0424 (9)	–	[18]
$\lvert V^\star_{tb}V_{ts}/V_{cb}\rvert$	0.980 (1)	–	[19, 20]
$\lvert V^\star_{tb}V_{td}\rvert$	0.0088 (3)	–	[19, 20]

TABLE I: Numerical inputs.

We are now ready to numerically evaluate the branching ratios in Eq. (3). Our inputs are collected in Table I. The $\overline{\mathrm{MS}}$-renormalized coupling constants $\alpha_s^{(5)}(M_Z)$ and $\alpha_{em}^{(5)}(M_Z)$ are defined in the SM with decoupled top quark. Hadronic contributions to the evolution of α_{em} are given by $\Delta\alpha^{(5)}_{em,\mathrm{hadr}}$. This quantity is used to evaluate the W-boson pole mass according to the fit formula in Eqs. (6) and (9) of Ref. [21], which gives $M_W = 80.358\,(8)$ GeV, consistently with the direct measurement $M_W = 80.385\,(15)$ GeV [10]. All the masses in Table I are interpreted as the on-shell ones. In the top-quark case, this is equivalent to assuming that the so-called color reconnection effects are included in the uncertainty. Converting M_t to the $\overline{\mathrm{MS}}$-renormalized mass with respect to QCD (but still on-shell with respect to EW interactions), we get $m_t \equiv m_t(m_t) = 163.5$ GeV.

The decay constants f_{B_q} are adopted from the most recent update of the $N_f = (2+1)$ FLAG compilation [11] which averages the $N_f = 2 + 1$ results of Refs. [12–14]. More recent calculations with $N_f = 2 + 1 + 1$ [15] and $N_f = 2$ [16] are consistent with these averages. As far as the lifetimes are concerned, using the explicit result for $\tau^s_H \equiv 1/\Gamma^s_H$ from Ref. [17] allows to avoid considering correlations between the decay width difference and the average lifetime. In the case of B_d, we can safely set $1/\Gamma^d_H \simeq 2/(\Gamma^d_H + \Gamma^d_L) \equiv \tau^d_{\mathrm{av}}$ given the tiny SM expectation for $(\Gamma^d_L - \Gamma^d_H)/(\Gamma^d_L + \Gamma^d_H) \equiv \Delta\Gamma^d/(2\Gamma^d_{\mathrm{av}}) = 0.0021\,(4)$ [22].

The CKM matrix element $|V_{cb}|$ is treated in a special manner, as it is now responsible for the largest parametric uncertainty in $\overline{\mathcal{B}}_{s\mu}$. One should be aware of a long-lasting tension between its determinations from the inclusive and exclusive semileptonic decays [11]. Here, we adopt the recent inclusive fit from Ref. [18]. It is the first one where both the semileptonic data and the precise quark mass determinations from flavor-conserving processes have been taken into account. Once $|V_{cb}|$ is fixed, we evaluate $|V_{tb}^\star V_{ts}|$ using the accurately known ratio $|V_{tb}^\star V_{ts}/V_{cb}|$.

Apart from the parameters listed in Table I, our results depend on two renormalization scales $\mu_0 \sim M_t$ and $\mu_b \sim m_b$ used in the calculation of the Wilson coefficient C_A. This dependence is very weak thanks to our new calculations of the NLO EW and NNLO QCD corrections. Since this issue is discussed at length in the parallel articles [5, 6], we just fix here these scales to $\mu_0 = 160\,\mathrm{GeV}$ and $\mu_b = 5\,\mathrm{GeV}$. Our results for the Wilson coefficient C_A are then functions of the first seven parameters in Table I. Allowing only the top-quark mass and the strong coupling constant to deviate from their central values, we find the following fits for C_A

$$\begin{aligned} C_A(\mu_b) &= 0.4802\, R_t^{1.52}\, R_\alpha^{-0.09} \; - 0.0112\, R_t^{0.89}\, R_\alpha^{-0.09} \\ &= 0.4690\, R_t^{1.53}\, R_\alpha^{-0.09}\,, \end{aligned} \tag{4}$$

$$\begin{aligned} C_A(\mu_b) &= 0.4802\, \widetilde{R}_t^{1.50}\, R_\alpha^{0.015} \; - 0.0112\, \widetilde{R}_t^{0.86}\, R_\alpha^{-0.031} \\ &= 0.4690\, \widetilde{R}_t^{1.51}\, R_\alpha^{0.016}\,, \end{aligned} \tag{5}$$

where $R_\alpha = \alpha_s(M_Z)/0.1184$, $R_t = M_t/(173.1\,\mathrm{GeV})$ and $\widetilde{R}_t = m_t/(163.5\,\mathrm{GeV})$. The fits are accurate to better than 0.1% in C_A for $\alpha_s(M_Z) \in [0.11, 0.13]$, $M_t \in [170, 175]\,\mathrm{GeV}$, and $m_t \in [160, 165]\,\mathrm{GeV}$.

In the first lines of Eqs. (4) and (5), C_A is given as as a sum of two terms. The first one corresponds to the leading order EW but NNLO QCD matching calculation [6]. The second one accounts for the NLO EW matching corrections [5] at the scale μ_0, as well as for the logarithmically enhanced QED corrections that originate from the renormalization group evolution between μ_0 and μ_b [23, 24].

Inserting Eq. (4) into Eq. (3), we obtain for $\overline{\mathcal{B}}_{s\mu}$

$$\overline{\mathcal{B}}_{s\mu} \times 10^9 = (3.65 \pm 0.06)\, R_{t\alpha}\, R_s = (3.65 \pm 0.23), \tag{6}$$

where $R_{t\alpha} = R_t^{3.06}\, R_\alpha^{-0.18} = \widetilde{R}_t^{3.02}\, R_\alpha^{0.032}$ and

$$R_s = \left(\frac{f_{B_s}[\mathrm{MeV}]}{227.7}\right)^2 \left(\frac{|V_{cb}|}{0.0424}\right)^2 \left(\frac{|V_{tb}^\star V_{ts}/V_{cb}|}{0.980}\right)^2 \frac{\tau_H^s\,[\mathrm{ps}]}{1.615}\,.$$

Correlations between f_{B_s} and α_s have been ignored above. Uncertainties due to parameters that do not occur in the quantities R_α, R_t and R_s have been absorbed into the residual error in the middle term of Eq. (6). This residual error is actually dominated by a non-parametric uncertainty, which we set to 1.5% of the branching ratio. Such an estimate of the non-parametric uncertainty is supposed to include:

- Effects of the neglected $\mathcal{O}(\alpha_{em})$ term in Eq. (3). They account for the fact that $|C_A(\mu_b)|^2$ changes by around 0.3% when μ_b is varied between $m_b/2$ and $2\,m_b$. Such a dependence on μ_b must cancel order-by-order in perturbation theory.

- Higher-order $\mathcal{O}(\alpha_s^3, \alpha_{em}^2, \alpha_s\alpha_{em})$ matching corrections to C_A at the electroweak scale μ_0. Such corrections must remove the residual μ_0-dependence of $C_A(\mu_b)$. When μ_0 is varied between $m_t/2$ and $2\,m_t$, the variation of $|C_A(\mu_b)|^2$ due to EW and QCD interactions amounts to around 0.2% in each case [5, 6]. Effects of similar size in the branching ratio are observed in Ref. [5] when comparing several EW renormalization schemes.

- Higher-order $\mathcal{O}(M_{B_q}^2/M_W^2)$ power corrections.

- Uncertainties due to evaluation of m_t from the experimentally determined M_t using a three-loop relation. Note that half of the three-loop correction shifts m_t by about 200 MeV, which affects $\overline{\mathcal{B}}_{s\mu}$ by around 0.3%. Non-perturbative uncertainties at this point (renormalons, color reconnection) are expected to be of the same order of magnitude.

- Tiny $\mathcal{O}(\Delta\Gamma^q/\Gamma^q)$ corrections due to deviations from the relation $\overline{\mathcal{B}}_{q\ell} = \Gamma[B_q \to \ell^+\ell^-]/\Gamma_H^q$, i.e. due to decays of the lighter mass eigenstate in the $B_q\bar{B}_q$ system. At the leading order in α_{em} and $M_{B_q}^2/M_W^2$, such corrections are non-vanishing only because of CP-violation in the absorptive part of the $B_q\bar{B}_q$ mixing matrix. Apart from being suppressed by $\Delta\Gamma^q/\Gamma^q$, they vanish in the limit $m_c \to m_u$, and receive additional CKM suppression in the B_s case. Beyond the leading order in α_{em} or $M_{B_q}^2/M_W^2$, the lighter eigenstate can decay to leptons also in the CP-conserving limit of the SM.

	f_{B_q}	CKM	τ_H^q	M_t	α_s	other param.	non-param.	$\sum$
$\overline{\mathcal{B}}_{s\ell}$	4.0%	4.3%	1.3%	1.6%	0.1%	$< 0.1\%$	1.5%	6.4%
$\overline{\mathcal{B}}_{d\ell}$	4.5%	6.9%	0.5%	1.6%	0.1%	$< 0.1\%$	1.5%	8.5%

TABLE II: Relative uncertainties from various sources in $\overline{\mathcal{B}}_{s\ell}$ and $\overline{\mathcal{B}}_{d\ell}$. In the last column they are added in quadrature.

All the other $\overline{\mathcal{B}}_{q\ell}$ branching ratios are calculated along the same lines. We find

$$\begin{aligned} \overline{\mathcal{B}}_{se} \times 10^{14} &= (8.54 \pm 0.13)\, R_{t\alpha}\, R_s = 8.54 \pm 0.55, \\ \overline{\mathcal{B}}_{s\tau} \times 10^{7} &= (7.73 \pm 0.12)\, R_{t\alpha}\, R_s = 7.73 \pm 0.49, \\ \overline{\mathcal{B}}_{de} \times 10^{15} &= (2.48 \pm 0.04)\, R_{t\alpha}\, R_d = 2.48 \pm 0.21, \\ \overline{\mathcal{B}}_{d\mu} \times 10^{10} &= (1.06 \pm 0.02)\, R_{t\alpha}\, R_d = 1.06 \pm 0.09, \\ \overline{\mathcal{B}}_{d\tau} \times 10^{8} &= (2.22 \pm 0.04)\, R_{t\alpha}\, R_d = 2.22 \pm 0.19, \end{aligned} \tag{7}$$

with

$$R_d = \left(\frac{f_{B_d}[\mathrm{MeV}]}{190.5}\right)^2 \left(\frac{|V_{tb}^\star V_{td}|}{0.0088}\right)^2 \frac{\tau_d^{\mathrm{av}}\,[\mathrm{ps}]}{1.519}\,.$$

A summary of the error budgets for $\overline{\mathcal{B}}_{s\ell}$ and $\overline{\mathcal{B}}_{d\ell}$ is presented in Table II. It is clear that the main parametric uncertainties come from f_{B_q} and the CKM angles.

To get rid of such uncertainties, one may take advantage [25] of their cancellation in ratios like

$$\kappa_{q\ell} \equiv \frac{\overline{\mathcal{B}}_{q\ell}\,\Gamma_H^q\,\Delta M_{B_q}^{-1}}{(G_F M_W m_\ell)^2 \beta_{q\ell}} \overset{\rm SM}{\simeq} \frac{3\,|C_A(\mu_b)|^2}{\pi^3\,C_{LL}(\mu_b)\,B_{B_q}(\mu_b)}\,, \quad (8)$$

where ΔM_{B_q} is the mass difference in the $B_q\bar{B}_q$ system, and C_{LL} enters through the $\Delta B = 2$ term in $\mathcal{L}_{\rm weak}$, namely $-\frac{1}{4}N\,V_{tb}^\star V_{tq}\,C_{LL}(\bar{b}\gamma_\alpha P_L q)(\bar{b}\gamma^\alpha P_L q)$. The bag parameters B_{B_q} are defined by the QCD matrix elements $\langle\bar{B}_q|(\bar{b}\gamma_\alpha P_L q)(\bar{b}\gamma^\alpha P_L q)|B_q\rangle = \frac{2}{3} f_{B_q}^2 B_{B_q} M_{B_q}^2$.

Following FLAG [11], we take $\hat{B}_{B_s} = 1.33(6)$ and $\hat{B}_{B_d} = 1.27(10)$ [26]. For the Wilson coefficient C_{LL}, including the NLO QCD [27] and NLO EW [28] corrections, we find $\hat{C}_{LL} \equiv C_{LL}(\mu_b)\,B_{B_q}(\mu_b)/\hat{B}_{B_q} = 1.27\,R_t^{1.51}$ for $\alpha_s^{(5)}(M_Z) = 0.1184$ and $\mu_b = 5\,$GeV. The r.h.s. of Eq. (8) gives then $\kappa_{s\ell} = 0.0126(7)$ and $\kappa_{d\ell} = 0.0132(12)$. It follows that the overall theory uncertainties in $\kappa_{q\ell}$ and $\overline{\mathcal{B}}_{q\ell}$ are quite similar at present. The l.h.s. of Eq. (8) together with Eq. (1) give $\kappa_{s\mu}^{\rm exp} = 0.0104(25)$ and $\kappa_{d\mu}^{\rm exp} = 0.047(20)$, which is consistent with the SM predictions.

To conclude, we have presented updated SM predictions for all the $\overline{\mathcal{B}}_{q\ell}$ branching ratios. Thanks to our new results on the NLO EW [5] and NNLO QCD [6] matching corrections, a significant reduction of the non-parametric uncertainties has been achieved. Such uncertainties are now estimated at the level of around 1.5% of the branching ratios. As far as the parametric ones are concerned, their reduction will depend on progress in the lattice determinations of f_{B_q} and B_{B_q} in the cases of $\overline{\mathcal{B}}_{q\ell}$ and $\kappa_{q\ell}$, respectively. For $\overline{\mathcal{B}}_{q\ell}$, the CKM uncertainties are now equally important, with $|V_{cb}|$ being one of the main limiting factors in the precise determination of $\overline{\mathcal{B}}_{s\ell}$.

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