TTP13-018, SFB/CPP-13-34, CP3-Origins-2013-017, DIAS-2013-17, SISSA 22/2013/FISI

A framework for baryonic R-parity violation in grand unified theories

Luca Di Luzio[a], Marco Nardecchia[b], and Andrea Romanino[c]
[a] *Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology (KIT), D-76128 Karlsruhe, Germany*
[b] *CP3-Origins and Danish Institute for Advanced Study (DIAS), University of Southern Denmark, Campusvej 55, DK-5230, OdenseM, Denmark and*
[c] *SISSA/ISAS and INFN, I–34136 Trieste, Italy*

We investigate the possibility of obtaining sizeable R-parity breaking interactions violating baryon number but not lepton number within supersymmetric grand unified theories. Such a possibility allows to ameliorate the naturalness status of supersymmetry while maintaining successful gauge coupling unification, one of its main phenomenological motivations. We show that this can be achieved without fine-tuning or the need of large representations in simple SO(10) models.

PACS numbers: 12.10.Dm,12.60.Jv

I. INTRODUCTION

Supersymmetric scenarios without R-parity [1–6] have received a renewed interest after the negative results of supersymmetry (SUSY) searches at the LHC. R-parity accounts for the stability of the lightest supersymmetric particle (LSP), whose escape from the detector gives rise to the prototypical supersymmetry signal: missing energy. R-parity violation (RPV) may allow supersymmetric particles to evade the latter, stringent searches. In particular, it has been argued that scenarios in which R-parity is violated through baryon-number-violating interactions could be particularly suited to hide supersymmetric signals into QCD backgrounds, thus implying a significant reduction of the current LHC lower bounds on the mass of the superpartners. Hence the intense research activity on the subject in the recent years [7–29].

In order for baryon number violating RPV operators to be sizeable enough to hide supersymmetric particles, lepton number violating operators should be very suppressed, possibly absent. The simultaneous presence of $\Delta B \neq 0$ and $\Delta L \neq 0$ interactions is in fact extremely constrained by matter stability. Indeed, R-parity was originally introduced in order to obtain (accidental) lepton and baryon number conservation in the minimal supersymmetric standard model (MSSM), thus protecting it from renormalizable sources of potentially way too large proton decay rate and neutrino masses. However, it is known that it suffices to assume the absence of R-parity lepton number violating operators, by means of a "leptonic R-parity", to get rid of such sources [4, 5].

Introducing baryonic RPV is therefore relatively safe if leptonic RPV is absent. On the other hand, one can wonder whether such an asymmetry between lepton and baryon number violating operators is compatible with grand unified theories (GUTs). After all, one of the motivations to persist on supersymmetric models despite the lack of signals is the very success of supersymmetric grand unification. This is the issue we would like to address in this paper.

In the presence of grand unification, the natural expectation is that baryonic and leptonic RPV couplings are either absent or simultaneously present, as quarks and leptons share the same grand-unified multiplets [30–35]. Indeed, exact SU(5) invariance forces baryonic RPV to be accompanied by leptonic RPV. However, a source of asymmetry between the two types of RPV can be generated by SU(5) breaking.

To be more specific let us state our problem in the following terms: we would like to find a supersymmetric GUT whose low-energy limit, well below the unification scale M_G, is described by the MSSM field content and gauge group and by a superpotential whose renormalizable part is given by

$$W_{\text{ren}} = W_{\text{MSSM}} + \lambda''_{ijk} u^c_i d^c_j d^c_k, \tag{1}$$

where λ''_{ijk} is antisymmetric in the flavour indices j,k. The extra operator violates R-parity and baryon number ($\Delta B = -1$). Since grand unified gauge groups transforms leptons into baryons (preserving $B-L$ in the minimal case of SU(5)), one would expect that operator to be accompanied by RPV and lepton-number violating ($\Delta L = 1$) operators such as $\lambda_{ijk} e^c_i l_j l_k$ and $\lambda'_{ijk} q_i d^c_j l_k$. Indeed, in minimal SU(5) grand unification d^c_i and l_i are unified in a $\overline{5}_i$ and q_i, u^c_i, e^c_i are unified in a 10_i and the three above operators all come from $\Lambda_{ijk} 10_i \overline{5}_j \overline{5}_k$, which gives $\lambda_{ijk} = \frac{1}{2}\lambda'_{ijk} = \lambda''_{ijk} = \Lambda_{ijk}$. In this case, the bounds from matter stability require Λ_{ijk} to be smaller than at least 10^{-10} for any value of i,j,k and for superpartners around the TeV scale [36]. Such tiny couplings would be irrelevant for collider physics since the LSP would be stable on the scale of the detector size. We then need to find a way to obtain sizeable λ'' couplings together with vanishing λ, λ'.

While leptonic RPV in GUTs has been investigated in a number of papers, see e.g. [4, 36–44], to our knowledge, such a problem was only considered in the context of SU(5) by Smirnov and Vissani [36] and by Tamvakis [45].[1] In [36], the vanishing of λ and λ' was achieved

[1] There also exist models of baryonic R-parity violation in Flipped-SU(5) [42, 45] and SU(5) $\otimes$ SU(3) [23].

through the fine-tuning of independent parameters, similar to the one necessary to achieve doublet-triplet splitting in the Higgs sector. In ref. [45], a mechanism similar to the missing-partner solution of the 2–3 splitting in SU(5) [46, 47] was considered, at the price of introducing a number of relatively large representations. In this paper we will show that the superpotential in Eq. (1) can be obtained without the need of fine-tuning in a relatively simple SO(10) model involving only fundamental, spinorial, and adjoint representations, thanks to the vacuum expectation value (vev) of an adjoint aligned along the T_{3R} or T_{B-L} direction.

II. THE FRAMEWORK

In this section, we define the rules of the game and systematically explore the options available in SU(5) and SO(10) to generate the superpotential in Eq. (1). The reader interested to specific models can jump to section IV. The main assumptions will be i) the use of representations that can arise in perturbative string theory [48], ii) a renormalizable origin of the extra term in Eq. (1), and iii) the absence of fine-tuning.

A. SU(5)

The case of SU(5) turns out not to offer any viable option. Still, it is useful to review it in order to illustrate the logic we will follow in this section, to find results that we will use in the next subsection, and to demonstrate that the fine-tuned method used in [36] is the only way to obtain Eq. (1) using only the representations 5, 10, 15, 24 (and conjugated, where relevant) available according to our assumptions.

In order to identify the renormalizable SU(5) origin of the operator $u^c_i d^c_j d^c_k$ (i, j, k fixed and $j \neq k$), let us first observe that the light u^c_i field must be contained in a 10 of SU(5), while d^c_j and d^c_k must be contained into two different $\overline{5}$, $\overline{5}'$ of SU(5), so that $u^c_i d^c_j d^c_k$ originates from the SU(5) operator $10\,\overline{5}\,\overline{5}'$.

Let us denote by L, L' the SU(5) partners of d^c_j, d^c_k in $\overline{5}$, $\overline{5}'$ respectively and by E^c, Q the SU(5) partners of u^c_i in 10. Then

$$10\,\overline{5}\,\overline{5}' = u^c_i d^c_j d^c_k + E^c L L' + Q d^c_j L' + Q L d^c_k. \quad (2)$$

In order for lepton number violating operators involving light fields not to be generated at the renormalizable level, at least two out of the four fields L, L', E^c, Q should not be light or partially light, in the sense that they should not contain the light fields l_i, q_i, e^c_i even as a component. A splitting, analogous to the doublet-triplet splitting in the Higgs sector, must occur in either $\overline{5}$ or $\overline{5}'$ or 10.

Let us first consider the case in which one of the two leptonic fields is heavy, say L for definitess, and denote by $\overline{5}_a$ the additional SU(5) representation containing the light lepton doublet l_a, $a = 1, 2, 3$. Note that extra matter representations (four antifundamentals overall, $\overline{5}_1, \overline{5}_2, \overline{5}_3, \overline{5}$) are needed to realize a split embedding of the SM fermions. In order to preserve the Standard Model (SM) chirality content, one fundamental, 5, must also be present, to compensate the extra $\overline{5}$. A super-heavy mass term is then allowed in the form

$$5(\mu_a + \alpha_a \langle 24_H \rangle)\overline{5}_a, \quad (3)$$

where the 24_H is an SU(5) adjoint getting vev along the hypercharge generator, $\langle 24_H \rangle = V Y$. Now, our definitions and assumptions require d^c_j to have a component in $\overline{5}$ and the doublets l_a to be light. In order for the light d^c_j to have a component in $\overline{5}$, the mass term arising from Eq. (3) must be non-zero for some $a = 1, 2, 3$,

$$\mu_a + \frac{\alpha_a}{3} V \neq 0\,, \quad (4)$$

otherwise the d^c_a would also be fully contained in the $\overline{5}_a$. As a consequence, at least one of the two vectors $(\mu_a)_{a=1,2,3}$ and $(\alpha_a)_{a=1,2,3}$ should be non-vanishing. On the other hand, in order for the doublets l_a to be light, with no heavy component, the leptonic mass term arising from Eq. (3) must vanish,

$$\mu_a - \frac{\alpha_a}{2} V = 0\,. \quad (5)$$

The two above relations imply a fine-tuning in the necessary alignment of the two non-vanishing vectors $(\mu_a)_{a=1,2,3}$ and $(\alpha_a)_{a=1,2,3}$, and in the determination of the vev V. The argument easily generalizes to the case of more than two extra $5 \oplus \overline{5}$, or more than an adjoint getting vev.

The argument above also applies to the case in which neither L nor L' are fully heavy. In such a case, Q and E^c should both be, in order to prevent lepton number violating operators involving light fields to be generated. And again a splitting must be arranged between u^c_i and its SU(5) partners, Q and E^c, such that u^c_i ends up having a vanishing mass. Since the only source of SU(5) breaking available, the vev of the SU(5) adjoints, never vanishes on the L, L', E^c, Q fields, a fine-tuned cancellation with another mass term must be invoked.[2]

The above discussion identifies two important ingredients to obtain baryonic RPV in a natural way: i) a source of SU(5) breaking splitting the mass of some unified multiplets in such a way that a component remains massless, i.e. a source of SU(5) breaking projecting out some components of a unified multiplet; and ii) additional (vector-like) matter, in order to be able to realize a split embedding of the SM fermions. SU(5) misses the first ingredient, which is however available in SO(10).

[2] In principle such a cancellation could be forced to arise dynamically, as in the sliding singlet solution of the 2-3 splitting problem [49], but this does not seem to be trivially possible in SU(5).

B. SO(10)

In the case of SO(10), the available non-trivial representations are 10, 16, $\overline{16}$, 45, 54. The fields u^c can be contained in the representations 16 and 45, while the fields d^c can be contained in the representations 16 and 10. Therefore, the only SO(10)-invariant renormalizable origins of the operator $u^c_i d^c_j d^c_k$ are $16\,16'10$ (where 16 and $16'$ can coincide) and $45\,10 10'$ (where 10 and $10'$ must be different).

In both cases, the embedding of u^c_i proceeds through a 10 of SU(5) and the embedding of d^c_j and d^c_k proceeds through a $\overline{5}$ and $\overline{5}'$ of SU(5) respectively. The operator $u^c_i d^c_j d^c_k$ then again arises from the SU(5) operator $10\,\overline{5}\,\overline{5}'$ appearing in the decomposition of both $16\,16'10$ and $45\,10\,10'$. We can then conclude that in both cases the decomposition of the SO(10) operator will contain the RHS of Eq. (2), where we have denoted with L, L', E^c, Q the SU(5) partners of d^c_j, d^c_k, u^c_i in $\overline{5}$, $\overline{5}'$, 10, as before. Again, at least two out of the fields L, L', E^c, Q must not contain a light component.

Let us again first suppose that one of the two heavy fields is a lepton doublet, say L for definiteness. Then the light (SM) leptons l_a, $a = 1, 2, 3$, should be contained in three $\overline{5}_a$ independent of $\overline{5}$. We then have at least four anti-fundamentals of SU(5), which means that at least one fundamental of SU(5), 5, must exist as well, with the mass mixing $5\,\overline{5}_a$ non vanishing for the coloured components (otherwise the light d^c_a would be entirely contained in the $\overline{5}_a$, with no component in the $\overline{5}$) but vanishing for the lepton components (because the l_a must be entirely contained in the $\overline{5}_a$, with no component in the $\overline{5}$).

Unlike SU(5), SO(10) offers the possibility to achieve such a splitting without fine-tuning. As argued, a source of SU(5) breaking vanishing on the lepton components is needed. With the available field content, such a source can only be provided by the appropriately oriented vev of an adjoint. More precisely, there are two options, depending on the SO(10) operator from which the mass mixing $5\,\overline{5}_a$ arises (which for simplicity we assume to be the same for the three families).

- If the operator originates from the SU(5) fundamental and antifundamental components of a $\overline{16}$ and three 16_a, a mass term mixing the coloured components of 5 and $\overline{5}_a$, but not the lepton ones, can be obtained through the SO(10) interaction

$$\alpha_a \overline{16}\, 45_H 16_a \,, \tag{6}$$

  with the SO(10) adjoint 45_H getting a vev $\langle 45_H\rangle = V_{45} T_{3R}$ along the 3R direction. Such a vev can be obtained without fine-tuning in a number of ways [50, 51].

- If the operator originates from the SU(5) fundamental and antifundamental of a 10 and three 10_a, a mass term mixing the coloured components of 5 and $\overline{5}_a$, but not the lepton ones, can be obtained through the SO(10) interaction

$$\alpha_a 10\, 45_H 10_a \,, \tag{7}$$

  with the SO(10) adjoint 45_H getting a vev $\langle 45_H\rangle = V_{45} T_{B-L}$ along the B-L direction. Such a vev can also be obtained without fine-tuning in a number of ways [50, 51].

In the next section, we will see that both the options can be implemented in the context of simple, minimal models.[3]

So far we have assumed that at least one of the two heavy fields among L, L', E^c, Q is a lepton doublet. Let us now assume that this is not the case. Then, both E^c and Q should be fully heavy. And the light (SM) e^c_a, q_a, $a = 1, 2, 3$ should be contained in three 10_a of SU(5), independent of the 10 containing u^c_i. We then have at least four 10 of SU(5). Which means that at least one $\overline{10}$ must exist, with the mass mixing $\overline{10}\,10_a$ vanishing for the lepton singlet and quark doublet components but non-vanishing on the quark singlet components. Unfortunately, not even SO(10) allows to achieve such a splitting without fine-tuning, independently of whether the 10_a of SU(5) are embedded in spinorial or adjoint representations of SO(10). Therefore, the cases considered above are the only relevant ones.

III. EXPLICIT MODELS

In this section we discuss simple, minimal realizations of the two basic mechanisms outlined in the previous section to obtain Eq. (1). In both cases, the RPV operator will arise from the decomposition of an SO(10) operator in the form $16\,16'10$ (where 16 and $16'$ may or may not coincide). Models in which RPV arises from an operator in the form $45\,10\,10'$ are also possible, but since they involve a larger number of fields we will not present them here.

The vev of a 45_H along the T_{3R} or T_{B-L} direction can be obtained as in [50, 51] through an SO(10) breaking sector that also generates a vev for a $16_H \oplus \overline{16}_H$ along the SM-singlet direction, as necessary to fully break SO(10) to the SM. A renormalizable superpotential W_H, also involving a 54_H and an SO(10) singlet, is sufficient to achieve such vevs. The SO(10) breaking fields above will always appear together with two "matter fields" in the rest of the superpotential, which guarantees that the supersymmetric minimum provided by W_H is not affected by the rest of the superpotential.

---

[3] In the complete models, the $\overline{5}, \overline{5}_a$ defined in the SU(5) subsection end up being superpositions of the antifundamentals in $16_a, 16$ or $10_a, 10$.

A. Adjoint vev along the T_{3R} direction

In this case, the operator relevant for the necessary splitting of leptons and baryons is $\alpha_a \overline{16}\, 45_H 16_a$, with 45_H assumed to get a vev $\langle 45_H \rangle = V_{45} T_{3R}$ in the T_{3R} direction. On top of the three 16_a needed to reproduce the SM chiral field content, the "matter" content necessarily involves a $16 \oplus \overline{16}$ and a 10 (the latter in order to be able to write a RPV source in the form $16\,16\,10$). As mentioned, the SO(10)-breaking sector must involve a $16_H \oplus \overline{16}_H$ getting vev along the SM-singlet components. The case in which the role of $16_H \oplus \overline{16}_H$ is played by $16 \oplus \overline{16}$ can be in principle considered, but here we will assume for simplicity that this is not the case. The minimal matter content relevant to our goal is then

$$16_a, 16, \overline{16}, 10 \quad 45_H, 16_H, \overline{16}_H. \tag{8}$$

The three possible sources of the RPV operator $u^c_i d^c_j d^c_k$ are $16\,16\,10$, $16_a 16\,10$, $16_a 16_b 10$. The last one is not ideal, as it generically also generates lepton number violating operators, unless a specific flavour structure is specified. On the other hand, it is relatively easy to use $16\,16\,10$ or $16_a 16\,10$. In both cases the superpotential leading, at low energy, to Eq. (1), is essentially unique.

If the RPV operator originates from $16\,16\,10$, we are lead to a superpotential in the form

$$W_1 = \lambda 16\,16\,10 + \alpha_a \overline{16}\, 45_H 16_a + \beta_a 16_H 16_a 10 + M_{16} \overline{16} 16\,. \tag{9}$$

The RPV operator arises from $16\,16\,10$ because of the mixing between 16_a, 16, 10 induced by the terms $\alpha_a \overline{16}\, 45_H 16_a$ and $\beta_a 16_H 16_a 10$ after SO(10) breaking. The first term only affects the singlet fields u^c, d^c, e^c, while the second term only affects the d^c, l fields. The light quark doublets q_a are not mixed by either operators, and therefore lie in the 16_a. One lepton doublet acquires a component in the 10 because of the $\beta_a \langle 16_H \rangle 16_a 10$ mixing. One lepton singlet and one up quark singlet acquire a component in the 16 because of the $\alpha_a \overline{16} \langle 45_H \rangle 16_a$ mixing. The down quark singlets spread in the 16_a, 16, and 10 as they are affected by both mixing terms. As a consequence, the operators $q_i d^c_j l_k$ and $e^c_i l_j l_k$ are not generated by $16\,16\,10$, while $u^c_i d^c_j d^c_k$ are. A more detailed discussion can be found in Appendix A.

Notice that the two vectors α_a and β_a need to be linearly independent in order to obtain $\lambda''_{ijk} \neq 0$. This can be seen as follows. If α_a and β_a were parallel, it would be possible to choose a basis for the 16_a such that $\alpha_{1,2} = \beta_{1,2} = 0$. In such a basis, the first two families of the light fermions are contained in $16_{1,2}$ and only the third family mixes with 16 and 10. There is therefore only a single light eigenstate d^c_l with components in both 16 and 10. The coupling λ''_{ijk} then vanishes because the antisymmetry in j,k requires two different light eigenstates to have components in 16 and 10. Another way of rephrasing this result is that λ''_{ijk} vanishes in the U(2)-symmetric limit, where U(2) acts on $16_{1,2}$ [52–55]. If the size of U(2) breaking is set by the light Yukawa couplings of the SM, baryonic RPV will necessarily end up being correspondingly suppressed.

There is no room for a light Higgs field with the spectrum in Eq. (8) and the superpotential in Eq. (9). An additional 10_H must therefore be added in order to accommodate it. The MSSM Yukawas are then generated by terms in the form $y 16 16 10_H$ or $y_a 16_a 16 10_H$ or $y_{ab} 16_a 16_b 10_H$. Doublet-triplet splitting should be accounted for separately, but all the ingredients for the Dimopoulos-Wilczek mechanism are available [50, 56–63].

In Eq. (9) we have included only interactions coupling 16_H, $\overline{16}_H$, 45_H to two matter fields, as anticipated. A mass term in the form $\overline{16} 16_a$ can be eliminated by a SU(4) rotation of the four spinorials 16, 16_a, $a = 1,2,3$. Possible $\lambda_a 16_a 16\,10$ and $\lambda_{ab} 16_a 16_b 10$ terms are not allowed as they would give rise to $q\, d^c l$ operators. On the other hand, terms such as $\overline{16}_H \overline{16}\, 10$, $\overline{16}\, 45_H 16$, $M_{10} 10^2$, would not modify our conclusions.

The second case we consider is associated to the following superpotential

$$W_2 = \lambda_a 16_a 16\,10 + \alpha_a \overline{16}\, 45_H 16_a + \beta 16_H 16 10 + \overline{\beta}\, \overline{16}_H \overline{16} 10 + M_{16} \overline{16} 16\,. \tag{10}$$

The RPV operator arises from $16_a 16\,10$ because of the mixing between $16_a, 16, 10$ induced by the terms $\alpha_a \overline{16}\, 45_H 16_a$ and $\beta 16_H 16 10$ after SO(10) breaking. The light lepton and quark doublets are fully contained in the 16_a, so that no lepton number violating operators can be generated. The two vectors α_a and λ_a need to be linearly independent in order to obtain $\lambda''_{ijk} \neq 0$.

The light Higgs could be in principle accommodated in the 10, 16_3 and $\overline{16}$ (in the basis in which $\alpha_{1,2} = 0$) and doublet-triplet splitting achieved for free if $\overline{\beta} = 0$. In such a case, however, the light down singlets would be contained in $16_{1,2}$ and 10 and no down quark Yukawa would be generated. Therefore, we need to assume $\overline{\beta} \neq 0$ (or, equivalently, a non-vanishing mass term $M_{10} 10^2$) and to add an additional 10_H to accommodate the light Higgs fields. The MSSM Yukawas are then generated by terms in the form $y 16 16 10_H$ or $y_a 16_a 16 10_H$ or $y_{ab} 16_a 16_b 10_H$.

A mass term in the form $\overline{16} 16_a$ in Eq. (10) can be eliminated by a SU(4) rotation of the four spinorials 16, 16_a, $a = 1,2,3$. Possible $\beta_a 16_H 16_a 10$ and $\lambda_{ab} 16_a 16_b 10$ terms are not allowed as they would give rise to $q\, d^c l$ operators. The presence of the terms $\lambda 16\,16\,10$, $\overline{\lambda 16}\, \overline{16}\, 10$, $\alpha \overline{16} 45_H 16$ would not affect the conclusions above.

B. Adjoint vev along the T_{B-L} direction

In this case, the operator relevant for the necessary splitting of leptons and baryons in the unified multiplets is $\alpha_a 10\, 45_H 10_a$, with 45_H assumed to get a vev $\langle 45_H \rangle = V_{45} T_{B-L}$ in the T_{B-L} direction. On top of the three 16_a needed to reproduce the SM chiral field content, the "matter" content involves a 10 and three 10_a, $a = 1, 2, 3$. The minimal matter content relevant to our goal is then

$$16_a, 10_a, 10 \quad 45_H, 16_H, \overline{16}_H. \tag{11}$$

The possible sources of the RPV operator $u^c_i d^c_j d^c_k$ are $16_a 16_b 10$, $16_a 16_b 10_c$. The latter generically also generates lepton number violating operators, unless a specific flavour structure is specified. Let us then consider the following superpotential involving the former:

$$W_3 = \lambda_{ab} 16_a 16_b 10 + \alpha_a 10\, 45_H 10_a + \alpha_{ab} 10_a 45_H 10_b + h_{ab} 16_H 16_a 10_b. \tag{12}$$

The light fields q_a, u^c_a, e^c_a are only contained in the 16_a. The operator $h_{ab} 16_H 16_a 10_b$ forces the light lepton doublets l_a to lie in the 10_a only, whereas the light d^c_i are both in the 10_a, the 16_a, and the 10 because of the mixing induced by $\alpha_a 10\, 45_H 10_a$ and $\alpha_{ab} 10_a 45_H 10_b$ (note that the second one is necessary otherwise only a single light component would appear in both 16_a and 10 and λ''_{ijk} would vanish because of the antisymmetry). Only the lepton number conserving RPV operator is thus generated.

The embedding of the l_a and part of the d^c_a in the 10_a, forced by the operator $h_{ab} 16_H 16_a 10_b$ allows to obtain positive, universal sfermion masses at the tree level, if supersymmetry is broken by the vev of a 16 [64–68]. In this context, the presence of three $16_a \oplus 10_a$ can be associated to a further stage of unification in E_6 [69]. The embedding through $16_a \oplus 10_a$ also allows to obtain a predictive framework for leptogenesis [70, 71].

The doublet-triplet splitting in the Higgs sector could be in principle obtained for free. Indeed, the doublets in the 10 are also light and could play the role of the Higgs doublets. The up Yukawa interactions would in this case provided by the very same operator generating λ''_{ijk}. However, no lepton Yukawa would be generated. Therefore, we need to add again an additional 10_H to accommodate the light Higgs fields and make the doublets in the 10 heavy by adding a $M_{10} 10^2$ mass term.

Adding the term $\lambda_{abc} 16_a 16_b 10_c$ or mass terms in the form $M_{ab} 10_a 10_b$ or $M_a 10_a 10$ would introduce lepton number violation. All other terms involving two matter fields are allowed.

C. On the naturalness of the superpotential

A comment on the flavour structure of the superpotential in Eq. (9) is in order. We achieved our goal of generating an isolated baryonic RPV operator without invoking a special structure with respect to the flavour index $a = 1, 2, 3$. On the other hand, we implicitely distinguished the three 16_a from the 16_H and 16. For example, we assumed $16_H 16_a 10$ to be present in the superpotential in Eq. (9), while $16_a 16_b 10$ is not. The question then arises whether it is possible to find a symmetry forcing the superpotential to have the desired form. The answer to this question depends on the form of W_H, which constrains the quantum numbers of 16_H, $\overline{16}_H$, 45_H. Let us consider for example the case in which W_H contains the terms $M_{45} 45^2_H$ and $X(16_H \overline{16}_H - V^2_{16})$, where X is an SO(10) singlet, as e.g. in [50, 65]. In such a case, it turns out that it is not possible to find a symmetry that allows all the terms we need and forbids the ones that should not appear. In particular, it is not possible to find any symmetry that distinguishes the fields 16_H and 16_a.[4] Nonetheless, the structure of the superpotential we need can be justified at a more fundamental level, once the origin of the flavour structure of the superpotential (and of the SM fermions) is addressed. For instance, one could envisage the presence of an $SU(3)_H$ horizontal symmetry under which the 16_a transforms as the fundamental of $SU(3)_H$, while the 16_H transforms trivially. The flavor symmetry is then formally restored in the superpotential considering the various couplings as spurions. We will illustrate this point in more detail in Sect. IV A.

IV. ANALYSIS OF A SIMPLE MODEL

In this section we study in greater detail the first model of Sect. III A. To this end we consider the superpotential in Eq. (9) augmented with a mass term for the 10, namely

$$W_{\rm RPV} = \lambda 16\, 16\, 10 + \alpha_a \overline{16}\, 45_H 16_a + \beta_a 16_H 16_a 10 + M_{16} \overline{16} 16 + \frac{M_{10}}{2} 10\, 10\,, \tag{13}$$

where the adjoint gets a vev along the 3R generator. For simplicity, we will assume in what follows all the parameters to be real.

The M_{10} mass term does not change the conclusions of Sect. III A and it allows to derive a limit where the expression of λ''_{ijk} assumes a simple form in terms of the superpotential parameters of Eq. (13). Let us consider, indeed, the limit in which the extra vector-like states $10 \oplus 16 \oplus \overline{16}$ are much heavier than the the GUT vevs,

$$M_{10}, M_{16} \gg V_{16}, V_{45}. \tag{14}$$

In such a case the light MSSM superfields are mostly contained (up to V/M corrections) in the 16_a and one can integrate out the heavy fields 10, 16 and $\overline{16}$ at the SO(10) level, thus obtaining at the leading order in $1/M$

$$10 \approx -\frac{1}{M_{10}} \left(\beta_a 16_H 16_a\right)_{10}\,, \tag{15}$$

$$16 \approx -\frac{1}{M_{16}} \left(\alpha_a 45_H 16_a\right)_{16}\,, \tag{16}$$

[4] On the other hand, is it possible to find a Z_2 symmetry which discriminates 16 from 16_H and 16_a (and $\overline{16}$ from $\overline{16}_H$ as well). An explicit example being: $Z_2(45_H, 10, 16_a, 16, 16_H, \overline{16}, \overline{16}_H, X) = (-, +, +, -, +, -, +, +)$.

where the subscripts denote the proper SO(10) contractions and the $\overline{16}$ should be set to zero at this order. Substituting the full solutions for $10, 16, \overline{16}$ into Eq. (13) and expanding at the third order in $1/M$ we get

$$W_{\rm RPV}^{\rm eff} \approx -\frac{1}{2M_{10}} \left(\beta_a 16_H 16_a\right)_{10}^2 - \frac{1}{M_{16}^2 M_{10}} \lambda \left(\alpha_a 45_H 16_a\right)_{16}^2 \left(\beta_c 16_H 16_c\right)_{10} \,. \quad (17)$$

While the first term in Eq. (17) is irrelevant for our purpose, the second one leads, upon GUT-symmetry breaking, to the $\Delta B = 1$ RPV operator $\lambda''_{abc} u_a^c d_b^c d_c^c$, with

$$\lambda''_{abc} = \frac{V_{45}^2\, V_{16}}{M_{16}^2\, M_{10}} \lambda\, \alpha_a \alpha_{[b} \beta_{c]} \,. \quad (18)$$

In the expression above the square brackets denote antisymmetrization.

The result in Eq. (18) can be derived in a number of different ways. For instance, one can directly inspect the mass matrices of the relevant fields upon GUT-symmetry breaking (cf. Eq. (A37) in Appendix A) or, from a diagrammatic point of view, compute the tree-level graph in Fig. 1.

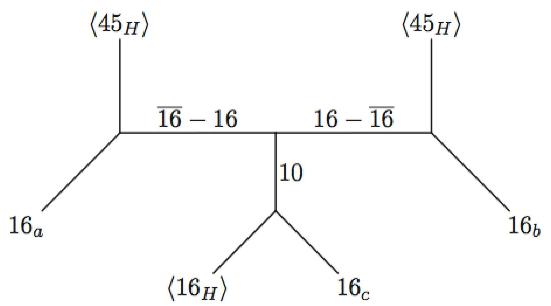

FIG. 1: SO(10) super-diagram leading to the $\Delta B = 1$ RPV operator in the effective MSSM theory. The vertices and propagators are specified by the superpotential in Eq. (13).

Note that the light fields u_a^c, d_a^c in Eq. (18) do not necessarily correspond to fermion mass eigenstates. The latter are in fact determined by the diagonalisation of the SM Yukawa couplings, which have not been specified so far. In the fermion mass eigenstate basis, in which the low-energy bounds on λ''_{ijk} are extracted, Eq. (18) becomes

$$\lambda''_{ijk} \propto (V_{u^c})_i^a (V_{d^c})_{[j}^b (V_{d^c})_{k]}^c \alpha_a \alpha_b \beta_c \,, \quad (19)$$

where V_{u^c} and V_{d^c} are the unitary transformations used to diagonalize the up and down Yukawa couplings (on the quark singlet side), determined by the SO(10) Yukawa sector.

This leads us to the discussion of the Yukawa sector. As anticipated in Sect. III A, an additional 10_H must be added in order to accommodate the Higgs field. The SM Yukawa interactions then follow from

$$W_Y = y_{ab} 16_a\, 16_b\, 10_H + y_a 16_a 16\, 10_H + y\, 16\, 16\, 10_H \,, \quad (20)$$

where the last term does not to contribute as the 16 turns out to contain only $SU(2)_L$ singlet fields (see Appendix A).

Simple expressions for the SM Yukawa matrices can be obtained at the leading order in the limit $M \gg V$ by using Eq. (16):

$$W_Y^{\rm eff} = y_{ab} 16_a\, 16_b\, 10_H - \frac{y_a}{M_{16}} 16_a \left(\alpha_b 45_H 16_b\right)_{16} 10_H \,. \quad (21)$$

Denoting the up-quark, down-quark, charged-lepton and Dirac-neutrino mass matrices by M_u, M_d, M_e and M_D respectively, Eq. (21) leads to

$$(M_u)_{ab} = (2y_{ab} + \theta\, y_a \hat{\alpha}_b) v_u \,, \quad (22)$$
$$(M_d)_{ab} = (2y_{ab} - \theta\, y_a \hat{\alpha}_b) v_d \,, \quad (23)$$
$$(M_e)_{ab} = (2y_{ab} - \theta\, y_a \hat{\alpha}_b) v_d \,, \quad (24)$$
$$(M_D)_{ab} = (2y_{ab} + \theta\, y_a \hat{\alpha}_b) v_u \,, \quad (25)$$

where y_{ab} is symmetric, $\theta \equiv \alpha V_{45}/M_{16}$, $\alpha \equiv \sqrt{\sum_a \alpha_a^2}$, and $v_{u,d}$ are the EW vevs. The above equations can reproduce the observed patter of fermion masses and mixings,[5] but the larger hierarchy of masses in the up sector and the deviations from SU(5) relations for the light down quark and charged lepton require a certain amount of fine-tuning. Moreover, the above equations do not address the origin of the fermion mass hierarchy. Both such issues can be addressed in the context of flavour models, as shown by the simple example in the next subsection.

A. Addressing flavour

So far, we did not make any assumption on the flavour structure of the couplings in Eq. (13). On the other hand, the latter is relevant for three reasons: *i*) to account at the same time for the pattern of SM fermion masses and mixings, *ii*) to distinguish different representations with the same gauge quantum number (e.g. 16_H and 16_a), thus making the superpotential in Eq. (13) technically natural, and *iii*) to relate the size of the RPV couplings to the pattern of fermion masses and mixings. In this section we analyse the consequences of having a controlled flavour structure by means of a simple flavour model.

Let us assume that the theory specified by Eq. (13) and Eq. (20) is invariant under the horizontal symmetry

[5] The relation $M_u = M_D$ implies that the neutrino sector must be extended with a Majorana mass term for $\nu^c \nu^c$. This can be achieved, for instance, by means of the effective operator $16_i 16_j \overline{16}_H \overline{16}_H / \Lambda$. In this context it is worth to recall that, due to the selection rules imposed by kinematics and Lorentz invariance, the simultaneous presence of $\Delta B = 1$ and $\Delta L = 2$ interactions do not endanger matter stability.

group $SU(3)_H$,[6] with the 16_a transforming as a triplet and all the other fields transforming trivially. Let us also assume then that the horizontal symmetry is broken by the vev of two linearly independent spurion fields A and B, which transform as anti-triplets of $SU(3)_H$ and whose absolute values are hierarchical, $|A| \gg |B|$. We neglect the masses and mixings related to the first families, which are zero in absence of a third source of $SU(3)_H$ breaking.

Being A and B the only sources of flavour symmetry breaking, we can write the parameters $\alpha_a, \beta_a, y_a, y_{ab}$ in terms of the spurions A_a and B_a, in such a way that the superpotential in Eq. (13) and Eq. (20) is formally invariant under the horizontal $SU(3)_H$:

$$
\begin{aligned}
\alpha_a &= r_\alpha A_a + s_\alpha B_a \,, && (26)\\
\beta_a &= r_\beta A_a + s_\beta B_a \,, && (27)\\
y_a &= r_z A_a + s_z B_a \,, && (28)\\
y_{ab} &= r_y A_a A_b + s_y B_a B_b + t_y \left(A_a B_b + B_a A_b\right) \,, && (29)
\end{aligned}
$$

where the coefficients $r_\#$, $s_\#$ and t_y are $\mathcal{O}(1)$ numbers, but they could be assumed to be small or vanishing without fine-tuning. For the same reason, unwanted interactions such as $\lambda_a 16_a 16\, 10$ can be assumed to be absent from Eq. (13) without fine-tuning.

In what follows it turns out to be useful to trade the vectors A_a and B_a for α_a and β_a and, by means of an $SU(3)_H$ rotation, to go in the basis $(\alpha_a) = \alpha(0, 0, 1)$ and $(\beta_a) = \beta(0, \epsilon, 1)$, where α and β are $\mathcal{O}(1)$ numbers and $\epsilon \ll 1$, as a consequence of $|B| \ll |A|$. In the latter basis the remaining parameters of the superpotential transforming non-trivially under the flavour group are

$$
\begin{aligned}
y_{33} \sim y_3 &= \mathcal{O}(1)\,, && (30)\\
y_{23} = y_{32} \sim y_2 &= \mathcal{O}(\epsilon)\,, && (31)\\
y_{22} &= \mathcal{O}(\epsilon^2)\,. && (32)
\end{aligned}
$$

For simplicity we shall factor out the appropriate ϵ dependence from the parameters in Eqs. (31)–(32), i.e. $y_{23} \to y_{23}\epsilon$, $y_2 \to y_2\epsilon$ and $y_{22} \to y_{22}\epsilon^2$, so that all the parameters of the superpotential except ϵ are $\mathcal{O}(1)$ numbers.

At this point one can inspect the mass matrices after SO(10)-symmetry breaking from Eq. (13) and find the light MSSM content of 16_a, 16, 10 (cf. Eqs. (B1)–(B5) in Appendix B). The Yukawa matrices (in the 2×2 approximation) can then be read directly from Eq. (20). We report them for completeness in Eqs. (B6)–(B8) of Appendix B. At the leading order in ϵ they yield the following relations for the physical observables:

$$m_t = (2c_\theta y_{33} + s_\theta y_3) v_u \,, \tag{33}$$

$$m_c = \epsilon^2 \left(2y_{22} - 2y_{23} \frac{2c_\theta y_{23} + s_\theta y_2}{2c_\theta y_{33} + s_\theta y_3} \right) v_u \,, \tag{34}$$

$$m_b = N \left(2c_\theta y_{33} - s_\theta y_3\right) v_d \,, \tag{35}$$

$$m_s = \epsilon^2 \left(2y_{22} - 2y_{23} \frac{2c_\theta y_{23} - s_\theta y_2}{2c_\theta y_{33} - s_\theta y_3} \right) v_d \,, \tag{36}$$

$$m_\tau = c_\phi \left(2c_\theta y_{33} - s_\theta y_3\right) v_d \,, \tag{37}$$

$$m_\mu = \epsilon^2 \left(2y_{22} - 2y_{23} \frac{2c_\theta y_{23} - s_\theta y_2}{2c_\theta y_{33} - s_\theta y_3} \right) v_d \,, \tag{38}$$

$$|V_{ts}| = |V_{cb}| = \epsilon \left| \frac{2c_\theta y_{23} + s_\theta y_2}{2c_\theta y_{33} + s_\theta y_3} - \frac{2c_\theta y_{23} - s_\theta y_2}{2c_\theta y_{33} - s_\theta y_3} \right| \,, \tag{39}$$

where we defined the quantities:

$$t_\theta \equiv \frac{V_{45}\alpha}{M_{16}} \,, \quad t_\phi \equiv \frac{V_{16}\beta}{M_{10}} \,, \quad N \equiv \left(\frac{1 + t_\theta^2}{1 + t_\theta^2 + t_\phi^2} \right)^{\frac{1}{2}} \,, \tag{40}$$

with t, s and c denoting the tan, sin and cos functions respectively.

The expression above show that the larger hierarchy in the up sector, $(m_c/m_t)_{\rm GUT} \ll (m_s/m_b)_{\rm GUT}$ at the GUT scale, can be due to $N \ll 1$ (so that a cancellation between the two terms in $2c_\theta y_{33} - s_\theta y_3$ does not need to be invoked). Moreover, $(m_b)_{\rm GUT} \approx (m_\tau)_{\rm GUT}$ follows from $N \approx c_\phi$. The two conditions are both satisfied if $t_\theta^2 \ll 1 \ll t_\phi^2$, i.e. if

$$M_{10} < V_{16}, \quad V_{45} < M_{16}, \tag{41}$$

which can be interpreted as a sign of a two-step breaking $SO(10) \to SU(5)$ at the scale $V_{16} \sim M_{16}$ followed by $SU(5) \to G_{\rm SM}$ at the lower scale $V_{45} \sim M_{10}$.

On the other hand, the expressions in Eqs. (33)–(38) show that, independent of the limit chosen, $m_\mu \approx m_s$ at the GUT scale, which is not phenomenologically viable. This conclusion can be evaded if the subleading spurion B is not SU(5) invariant (which may be associated to its being subleading). Let us then concentrate on the third family relations. In the limit in Eq. (41), the expressions for the third family fermion masses become

$$m_t \approx 2y_{33} v_u \,, \tag{42}$$

$$m_b \approx 2y_{33} \left(\frac{M_{10}}{\beta V_{16}} \right) v_d \,, \tag{43}$$

$$m_\tau \approx 2y_{33} \left(\frac{M_{10}}{\beta V_{16}} \right) v_d \,. \tag{44}$$

Let us now consider the size and the structure of the RPV couplings. The latter are obtained by projecting the $16\,16\,10$ operator in Eq. (13) onto the light components (cf. Eq. (B11) in Appendix B) and by taking into account the subsequent EW rotation matrices V_{u^c} and

[6] The horizontal $SU(3)_H$ symmetry in the context of GUTs was originally discussed in Refs. [72–74].

V_{d^c} (cf. Eqs. (B9)–(B10) in Appendix B). This yields:

$$\lambda''_{tbs} = 2\,\lambda\,\epsilon\,\frac{s_\theta t_\theta t_\phi}{(1+t_\theta^2+t_\phi^2)^{1/2}}\,, \tag{45}$$

$$\lambda''_{cbs} = -\epsilon\,\frac{2y_{23}}{2c_\theta y_{33}+s_\theta y_3}\lambda''_{tbs}\,. \tag{46}$$

The RPV couplings involving the first family vanish because, having introduced only two spurions, we have neglected the structure associated to the first family masses. Note also that the RPV coupling is proportional to the small misalignment between the 3-vectors α_a and β_a, i.e. Eqs. (45)–(46) vanish in the $\epsilon \to 0$ limit. Eventually, we obtain an hierarchical structure for the RPV couplings. In the limit in Eq. (41), the expressions above simplify to

$$\lambda''_{tbs} = 2\,\lambda\,\epsilon\,t_\theta^2\,, \tag{47}$$

$$\lambda''_{cbs} = -\epsilon\,\frac{y_{23}}{y_{33}}\lambda''_{tbs}\,. \tag{48}$$

The RPV couplings are therefore proportional to t_θ^2, which is the same parameter that controls the deviation of m_b/m_τ from 1 at the unification scale.

V. PHENOMENOLOGICAL REMARKS

The baryon number RPV interactions are subject to stringent low-energy constraints coming mainly from proton decay, di-nucleon decay, n-$\overline{n}$ oscillations and flavour violating observables. Rescaling the bounds in ref. [75] for superpartners around 500 GeV and assuming a gravitino heavier than the proton (in order to evade the constraints from proton decay) but not too heavy (in order not to enhance flavour-anarchical supergravity effects) one gets:[7]

$$|\lambda''_{uds}| < \mathcal{O}(10^{-5}) \quad [NN \to KK]\,, \tag{49}$$
$$|\lambda''_{udb}| < \mathcal{O}(10^{-2}) \quad [n-\overline{n}]\,, \tag{50}$$
$$|\lambda''_{tds}| < \mathcal{O}(10^{-1}) \quad [n-\overline{n}]\,, \tag{51}$$
$$|\lambda''_{tdb}| < \mathcal{O}(10^{-1}) \quad [n-\overline{n}]\,, \tag{52}$$

and

$$|\lambda''_{cdb}\,\lambda''_{csb}| < \mathcal{O}(10^{-3}) \quad [K-\overline{K}]\,, \tag{53}$$
$$|\lambda''_{tdb}\,\lambda''_{tsb}| < \mathcal{O}(10^{-3}) \quad [K-\overline{K}]\,, \tag{54}$$
$$|\lambda''_{ids}\,\lambda''_{idb}| < \mathcal{O}(10^{-1}) \quad [B^+ \to K^0\pi^+]\,, \tag{55}$$
$$|\lambda''_{ids}\,\lambda''_{isb}| < \mathcal{O}(10^{-3}) \quad [B^- \to \phi\pi^-]\,, \tag{56}$$

with $i = u, c, t$ for the product of two RPV couplings.

On the other hand, the RPV couplings cannot be too small, if the SUSY searches based on the missing energy signature are to be evaded. This is the case if the NLSP (we assume the LSP to be the gravitino) has a prompt decay corresponding to a decay length smaller than about 2 mm [28].[8]

This way supersymmetry can be "hidden" into QCD backgrounds and the lower bounds on superpartners can be relaxed with respect to the R-parity conserving case.

To illustrate this point, let us compare the current exclusion limits from LHC in standard MSSM scenarios to the case with bayonic RPV. In the case of the R-parity conserving MSSM the present lower bounds on the stop and gluino masses are respectively $m_{\tilde{t}} \gtrsim 700$ GeV [76, 77] and $m_{\tilde{g}} \gtrsim 1.3$ TeV [78, 79].[9] In the case of the simplified squark-gluino-neutralino model one gets $m_{\tilde{g}}, m_{\tilde{t}} \gtrsim 1.5$ TeV (with only 5.8 fb^{-1} of integrated luminosity and $\sqrt{s} = 8$ TeV) [80]. On the other hand, if we allow the light colored s-particles (gluinos and squarks) to decay promptly via the $u^c d^c d^c$ operator the bounds are much less stringent. For instance, if the stop decays directly into jets neither ATLAS nor CMS can currently place significant limits on the stop mass [17, 81–83]. The decay of the gluino can proceed either through $\tilde{g} \to \tilde{t}t$ (and consequently $\tilde{t} \to bs$ for example) or directly into jets. In the former case the bound on the gluino mass is 890 GeV (with 20.7 fb^{-1} and $\sqrt{s} = 8$ TeV) [84], while in the latter case 666 GeV (with 4.6 fb^{-1} and $\sqrt{s} = 7$ TeV) [85].

Let us quantify now the minimal amount of RPV needed in order to have a prompt vertex. As a benchmark scenario we consider the case of a right-handed squark NLSP decaying into two SM fermions. In such a case the decay length reads

$$L = 2\,\mathrm{mm}\,(\beta\gamma)\left(\frac{500\ \mathrm{GeV}}{m_{\tilde{q}^c}}\right)\left(\frac{0.9\cdot 10^{-7}}{\lambda''}\right)^2\,, \tag{57}$$

where β is the velocity of the decaying particle and γ is the Lorentz boost factor. Hence, a decay length smaller than about 2 mm, requires $\lambda'' > \mathcal{O}(10^{-7})$. Therefore, a RPV coupling in the range $10^{-7} \lesssim \lambda'' \lesssim 10^{-5}$ would give rise to a prompt decay, while also satisfying the bounds in Eqs. (49)–(56) independent of its flavour numbers.[10]

Nevertheless, it is worth to mention that the flavour structure of the GUT-induced λ''_{ijk} which emerges from Eq. (19) is of the type

$$\lambda''_{ijk} \propto \alpha_i \beta_{[j}\gamma_{k]}\,, \tag{58}$$

where α_i, β_j and γ_k are independent 3-vectors in the flavour space. This non-generic structure implies a set of

[7] The quoted bounds have a strong dependence on the spectrum of the superpartners, and large uncertainties related to the flavour structure of the soft terms and the hadronic matrix elements. However, for the purposes of our discussion an order of magnitude estimate is sufficient (see e.g. [75] and references therein).

[8] Larger decay lengths give rise to displaced vertices which require dedicated analysis. See for instance [10, 44].

[9] These conservative bounds apply in the case of a light neutralino and away from the kinematical configuration $m_{\tilde{t}} \approx m_t + m_\chi$.

[10] There could be cases where a larger λ'' is needed for a prompt decay. For example in the case of a gluino decaying in the kinematical configuration $m_{\tilde{g}} \approx m_t + m_{\tilde{t}}$ [10].

low-energy correlations among the RPV couplings. For instance, we find that the following relations

$$\frac{\lambda''_{ids}}{\lambda''_{jds}} = \frac{\lambda''_{idb}}{\lambda''_{jdb}} = \frac{\lambda''_{isb}}{\lambda''_{jsb}}\,, \tag{59}$$

must be satisfied for $i, j = u, c, t$. However, the relations in Eq. (59) are irrelevant when crossed with the low-energy bounds in Eqs. (49)–(56) whenever the absolute size of λ'' is smaller than $\mathcal{O}(10^{-5})$.

The absolute size of λ'' predicted by the GUT model is in general model dependent. Interestingly, in the case of a hierarchy such as the one in Eq. (41) or Eq. (14), the value for the couplings λ'' can be expected to lie in the $10^{-7} \lesssim \lambda'' \lesssim 10^{-5}$ window mentioned above, which satisfies all the low-energy bounds. Upper bounds coming from the requirement of not washing out a pre-existing baryon asymmetry generated above the EW scale turn out to give $\lambda'' < 3 \cdot 10^{-7}$ for sfermion masses of about 1 TeV [75].

In the presence of additional assumption on a common origin of the flavour structure of both the SM fermions and the RPV couplings, the RPV couplings also show a hierarchical pattern, as illustrated by the example in Sect. IV A. A simple consequence is that a stop will decay predominantly into $\tilde{t} \to bs$.

VI. SUMMARY

Supersymmetric models with R-parity violation have the potential to relieve some of the pressure on the naturalness of supersymmetric extensions of the SM due to the lack of signals at the LHC. This is welcome, as providing a natural framework for electroweak symmetry breaking is one of the main phenomenological motivations of supersymmetry. On the other hand, this requires baryon number violating RPV operators not to be accompanied by lepton number violating ones, which in turn may seem to require giving up another important phenomenological motivation: the possibility to explain the SM fermion gauge quantum numbers within a grand unified framework leading to a successful prediction for the unification of gauge couplings. We have shown that this is not the case. Dimension four lepton number violating interactions can vanish, despite the presence of sizeable baryon number violating interactions and the existence of a grand unified gauge symmetry relating baryon and leptons, in models in which the necessary sources of GUT-breaking split the unified multiplets and additional vector-like matter is added to the MSSM chiral content.

In particular, we have shown that this can be achieved without fine-tuning or the need of large representations in simple renormalizable SO(10) models in which the adjoint vev is aligned along the 3R or B-L generators. In this context, it is also possible to relate the size of baryonic R-parity violation to the origin of the SM fermion mass hierarchy and to the success (to some extent) of unified relations among third family fermion masses.

Acknowledgments

We thank Giorgio Arcadi, Zurab Berezhiani, Roberto Franceschini, Giovanni Villadoro, and Francesco Vissani for useful discussions. The work of L.D.L. was supported by the DFG through the SFB/TR 9 "Computational Particle Physics". The work of M.N. was partly supported by the Danish National Research Foundation under the research grant DNRF:90. The work of A.R. was partly supported by the EU Marie Curie ITN "UNILHC" (PITN-GA-2009-237920) and the ERC Advanced Grant no. 267985"DaMESyFla". M.N. and A.R. thank the Galileo Galilei Institute for Theoretical Physics for the hospitality and the INFN for partial support.

Appendix A: Details of the model in Sect. IV

In this appendix we illustrate the details of the analysis of the model specified by Eq. (13) and Eq. (20) of Sect. IV. In order to identify the light MSSM components populating the SO(10) fields 16_a, 16, and 10, one has to inspect the mass matrices stemming from Eq. (13) upon SO(10)-symmetry breaking. In particular, the piece of superpotential responsible for the non-pure embedding of the MSSM degrees of freedom into the relevant SO(10) representations reads

$$W \supset \alpha_a \overline{16}\, 45_H 16_a + \beta_a 16_H 16_a 10 + M_{16} \overline{16} 16 + \frac{M_{10}}{2} 10\, 10\,, \tag{A1}$$

where the Higgs superfields 45_H and 16_H are assumed to pick up a GUT-scale vev $\langle 45_H \rangle = V_{45} T_{3R}$ and $\langle 16_H \rangle = V_{16}$, along the $\mathrm{SU}(4)_{PS} \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_R$ and SU(5) invariant directions respectively.

The mechanism we are going to consider is based on the fact that the 45_H vev picks up the $\mathrm{SU}(2)_L$ singlet components of 16_a and the vev of the 16_H picks up the $\overline{5}_{16_a}$ and 5_{10} SU(5) components of 16_a and 10. Hence, upon SO(10)-symmetry breaking, Eq. (A1) leads to the following mass matrices involving the MSSM-like degrees of freedom:

$$\left(\overline{d}^c_{\overline{16}}\;\; \overline{d}^c_{10}\right) \begin{pmatrix} V_{45}\,\alpha_a & M_{16} & 0 \\ V_{16}\,\beta_a & 0 & M_{10} \end{pmatrix} \begin{pmatrix} d^c_{16_a} \\ d^c_{16} \\ d^c_{10} \end{pmatrix}, \tag{A2}$$

$$\left(\overline{l}_{\overline{16}}\;\; \overline{l}_{10}\right) \begin{pmatrix} 0 & M_{16} & 0 \\ V_{16}\,\beta_a & 0 & M_{10} \end{pmatrix} \begin{pmatrix} l_{16_a} \\ l_{16} \\ l_{10} \end{pmatrix}, \tag{A3}$$

$$\left(\overline{u}^c_{\overline{16}}\right) \left(-V_{45}\,\alpha_a \;\; M_{16}\right) \begin{pmatrix} u^c_{16_a} \\ u^c_{16} \end{pmatrix}, \tag{A4}$$

$$\left(\overline{e}^c_{\overline{16}}\right) \left(V_{45}\,\alpha_a \;\; M_{16}\right) \begin{pmatrix} e^c_{16_a} \\ e^c_{16} \end{pmatrix}, \tag{A5}$$

$$\left(\overline{q}_{\overline{16}}\right) \left(0 \;\; M_{16}\right) \begin{pmatrix} q_{16_a} \\ q_{16} \end{pmatrix}. \tag{A6}$$

Let us leave aside for a while the d^c-like states and focus on the others. From Eqs. (A3)–(A6) one can readily find the heavy (GUT-scale) mass eigenstates

$$L_1 = l_{16}\,, \quad (A7)$$
$$L_2 = \cos\phi\, l_{10} + \sin\phi\, \hat\beta_a l_{16_a}\,, \quad (A8)$$
$$U^c = \cos\theta\, u^c_{16} - \sin\theta\, \hat\alpha_a u^c_{16_a}\,, \quad (A9)$$
$$E^c = \cos\theta\, e^c_{16} + \sin\theta\, \hat\alpha_a e^c_{16_a}\,, \quad (A10)$$
$$Q = q_{16}\,, \quad (A11)$$

where we defined the quantities $\tan\theta \equiv V_{45}\alpha/M_{16}$ and $\tan\phi \equiv V_{16}\beta/M_{10}$, and the normalized vectors $\hat\alpha_a \equiv \alpha_a/\alpha$ and $\hat\beta_a \equiv \beta_a/\beta$, with $\alpha \equiv \sqrt{\sum_a \alpha_a^2}$ and $\beta \equiv \sqrt{\sum_a \beta_a^2}$. The light MSSM components l_a, u^c_a, e^c_a, q_a ($a = 1, 2, 3$), can be identified with the linear combinations orthogonal to those in Eqs. (A7)–(A11). A possible choice is

$$l_3 = -\sin\phi\, l_{10} + \cos\phi\, \hat\beta_a l_{16_a} \quad (A12)$$
$$u^c_3 = \sin\theta\, u^c_{16} + \cos\theta\, \hat\alpha_a u^c_{16_a}\,, \quad (A13)$$
$$e^c_3 = -\sin\theta\, e^c_{16} + \cos\theta\, \hat\alpha_a e^c_{16_a}\,, \quad (A14)$$

while the remaining light components components: l_m, u^c_m, e^c_m ($m = 1, 2$) and q_a ($a = 1, 2, 3$) are only contained in the 16_a. In particular, we are interested in the projection of the 10 and 16 fields on the light eigenstates. Inverting the transformations in Eqs. (A7)–(A14) we get

$$l_{10} \to -\sin\phi\, l_3\,, \quad (A15)$$
$$l_{16} \to 0\,, \quad (A16)$$
$$u^c_{16} \to \sin\theta\, u^c_3\,, \quad (A17)$$
$$e^c_{16} \to -\sin\theta\, e^c_3\,, \quad (A18)$$
$$q_{16} \to 0\,. \quad (A19)$$

The identification of the d^c-like light states is more involved. Therefore, let us first consider the simple limit in which the vectors (α_a) and (β_a) are orthogonal, before considering the general case. In such a case, the heavy mass eigenstates are

$$D^c_1 = \cos\theta\, d^c_{16} + \sin\theta\, \hat\alpha_a d^c_{16_a}\,, \quad (A20)$$
$$D^c_2 = \cos\phi\, d^c_{10} + \sin\phi\, \hat\beta_a d^c_{16_a}\,, \quad (A21)$$

the light d^c_a components can be chosen to be

$$d^c_3 = -\sin\theta\, d^c_{16} + \cos\theta\, \hat\alpha_a d^c_{16_a}\,, \quad (A22)$$
$$d^c_2 = -\sin\phi\, d^c_{10} + \cos\phi\, \hat\beta_a d^c_{16_a}\,, \quad (A23)$$

while d^c_1 is entirely contained in the 16_a. The projection of the 10 and 16 fields on the light d^c-like states then reads

$$d^c_{16} \to -\sin\theta\, d^c_3\,, \quad (A24)$$
$$d^c_{10} \to -\sin\phi\, d^c_2\,. \quad (A25)$$

The only renormalizable (RPV) interaction generated by the operator $\lambda\, 161610$ (cf. Eq. (13)) is therefore

$$2\lambda \sin^2\theta \sin\phi\, u^c_3 d^c_3 d^c_2\,. \quad (A26)$$

In the opposite case in which α_a and β_a are parallel, both d^c_{16} and d^c_{10} contain only one linear combination of the light fields and the baryon number violating RPV operator would vanish by antisymmetry.

Let us now consider the general case. In order to identify the light d^c eigenstates, it is useful to consider a basis in the SO(10) flavour space in which $\beta_1 = 0$, $\alpha_{1,2} = 0$, so that $(\alpha_a) = (0, 0, \alpha_3)$, $\alpha_3 > 0$, $(\beta_a) = (0, \beta_2, \beta_3)$, $\alpha = \alpha_3$, $\beta = (\beta_2^2 + \beta_3^2)^{1/2}$. In such a basis, one of the three light eigenstates is d^c_1 and the other two are linear combinations of $d^c_{16_2}$, $d^c_{16_3}$, d^c_{16}, d^c_{10} orthogonal to the heavy linear combinations (linearly independent but not orthogonal nor normalized)

$$D^c_1 = \alpha V_{45} d^c_{16_3} + M_{16} d^c_{16} \quad (A27)$$
$$D^c_2 = V_{16}(\beta_3 d^c_{16_3} + \beta_2 d^c_{16_2}) + M_{10} d^c_{10}\,. \quad (A28)$$

A possible choice of the light fields is given by the exterior products

$$d^c_2 = (D^c_1 \wedge D^c_2 \wedge d^c_{16_3})/N_2 \quad (A29)$$
$$d^c_3 = (D^c_1 \wedge D^c_2 \wedge d^c_2)/N_3\,, \quad (A30)$$

where N_2 and N_3 are normalization factors. The explicit expressions are

$$d^c_2 = \frac{d^c_{16_2} - \hat\beta_2 t_\phi\, d^c_{10}}{(1 + (\hat\beta_2 t_\phi)^2)^{1/2}} \quad (A31)$$
$$d^c_3 = \frac{(1 + (\hat\beta_2 t_\phi)^2)(d^c_{16_3} - t_\theta d^c_{16}) - \hat\beta_3 t_\phi(\hat\beta_2 t_\phi d^c_{16_2} + d^c_{10})}{(1 + (\hat\beta_2 t_\phi)^2)^{1/2}(1 + t_\theta^2 + t_\phi^2 + \hat\beta_2^2 t_\phi^2 t_\theta^2)^{1/2}}\,, \quad (A32)$$

from which we get

$$\lambda 16\, 16\, 10 = \text{heavy} + \frac{2\lambda \hat\alpha_{[3}\hat\beta_{2]} s_\theta t_\theta t_\phi}{(1 + t_\theta^2 + t_\phi^2 + (1 - (\hat\alpha \cdot \hat\beta)^2) t_\phi^2 t_\theta^2)^{1/2}} u^c_3 d^c_3 d^c_2. \quad (A33)$$

The coefficient of the RPV operator in the previous expression is independent of the choice of the two light fields d^c_3, d^c_2 made in Eqs. (A29)–(A30), provided that d^c_3, d^c_2 are orthonormal and orthogonal to $d^c_{16_1}, D^c_1, D^c_2$. The form in which it is written is independent of the basis in which the vectors (α_a) and (β_a) are written, as long as $\alpha_1 = \beta_1 = 0$.

In the $t_\theta^2 \ll 1 \ll t_\phi^2$ limit identified in Sect. IV A the coefficient of the RPV operator becomes

$$2\lambda s_\theta t_\theta \hat\alpha_{[3}\hat\beta_{2]} \approx 2\lambda \frac{V_{45}^2 \alpha^2}{M_{16}^2} \hat\alpha_{[3}\hat\beta_{2]}\,. \quad (A34)$$

We remind that Eq. (A33) should be written in terms of the fermion mass eigenstates, which are determined by the SM Yukawas after electroweak symmetry breaking.

In Sect. IV, we also considered the limit $M_{16,10} \gg V_{45,16}$. In this limit, corresponding to small angles θ and ϕ, the light d^c_a states can be obtained as perturbations of the states $d^c_{16_a}$,

$$d^c_a \approx d^c_{16_a} - \theta\, \hat\alpha_a d^c_{16} - \phi\, \hat\beta_a d^c_{10}\,. \quad (A35)$$

from which we get

$$\lambda\, 161610 \approx 2\lambda\theta^2\phi\, \hat{\alpha}_a\hat{\alpha}_b\hat{\beta}_c u^c_a d^c_b d^c_c\,, +\text{heavy} \tag{A36}$$

which yields the operator $\lambda''_{abc}u^c_a d^c_b d^c_c$, with

$$\lambda''_{abc} = \lambda\theta^2\phi\, \hat{\alpha}_a\hat{\alpha}_{[b}\hat{\beta}_{c]}\,, \tag{A37}$$

the same expression in Eq. (18), obtained by integrating out the heavy vector-like fields $16\oplus\overline{16}\oplus 10$ at the SO(10) level.

Appendix B: Details of the flavor model

In order to obtain Eqs. (33)–(39), one can follow the procedure illustrated in the previous Appendix with $(\alpha_a) = \alpha(0,0,1)$ and $(\beta_a) = \beta(0,\epsilon,1)$, and expand at the leading order in $\epsilon \ll 1$. In particular, we choose the same basis as in Eqs. (A31)–(A32) for the light d^c eigenstates. Analogously, the light l eigenstates are defined by replacing $d^c \leftrightarrow l$ and setting $t_\theta = 0$ in Eqs. (A31)–(A32). The basis for the other light states follows the conventions given in Appendix A. At the leading order in ϵ, we find the following projections for the SO(10) current eigenstates onto the light degrees of freedom:

- d^c-like states

$$\begin{aligned} d^c_{16_2} &\to d^c_2 - \epsilon N c_\theta t^2_\phi d^c_3\,, \\ d^c_{16_3} &\to N c_\theta d^c_3\,, \\ d^c_{16} &\to -N s_\theta d^c_3\,, \\ d^c_{10} &\to -\epsilon t_\phi d^c_2 - N c_\theta t_\phi d^c_3\,, \end{aligned} \tag{B1}$$

  where N is defined as in Eq. (40).

- l-like states

$$\begin{aligned} l_{16_2} &\to l_2 - \epsilon s_\phi t_\phi l_3\,, \\ l_{16_3} &\to c_\phi l_3\,, \\ l_{16} &\to 0\,, \\ l_{10} &\to -\epsilon t_\phi l_2 - s_\phi l_3\,. \end{aligned} \tag{B2}$$

  Notice that in the $t_\theta \to 0$ limit, $N \to c_\phi$.

- u^c-like states

$$\begin{aligned} u^c_{16_2} &\to u^c_2\,, \\ u^c_{16_3} &\to c_\theta u^c_3\,, \\ u^c_{16} &\to s_\theta u^c_3\,. \end{aligned} \tag{B3}$$

- e^c-like states

$$\begin{aligned} e^c_{16_2} &\to e^c_2\,, \\ e^c_{16_3} &\to c_\theta e^c_3\,, \\ e^c_{16} &\to -s_\theta e^c_3\,. \end{aligned} \tag{B4}$$

- q-like states

$$\begin{aligned} q_{16_2} &\to q_2\,, \\ q_{16_3} &\to q_3\,, \\ q_{16} &\to 0\,. \end{aligned} \tag{B5}$$

By substituting Eqs. (B1)–(B5) into the Yukawa superpotential of Eq. (20) we obtain the following Yukawa matrices for the second and third families at the leading order in ϵ:

$$Y_u = \begin{pmatrix} \epsilon^2(2y_{22}) & \epsilon(2c_\theta y_{23} + s_\theta y_2) \\ \epsilon(2y_{23}) & 2c_\theta y_{33} + s_\theta y_3 \end{pmatrix}, \tag{B6}$$

$$Y_d = \begin{pmatrix} \epsilon^2(2y_{22}) & \epsilon N(2c_\theta y_{23} - s_\theta y_2) \\ \epsilon(2y_{23}) & N(2c_\theta y_{33} - s_\theta y_3) \end{pmatrix}, \tag{B7}$$

$$Y_e = \begin{pmatrix} \epsilon^2(2y_{22}) & \epsilon(2c_\theta y_{23} - s_\theta y_2) \\ \epsilon(2c_\phi y_{23}) & c_\phi(2c_\theta y_{33} - s_\theta y_3) \end{pmatrix}, \tag{B8}$$

where the basis for $Y_{u,d,e}$ is chosen is such a way that the $SU(2)_L$ doublets act from the left. Notice that $Y_d = Y_e^T$ in the $t_\theta \to 0$ (and hence $N \to c_\phi$) limit, as expected from the fact that SU(5) is unbroken in this limit. Analogously, $Y_d = Y_e$ in the $t_\phi \to 0$ (and hence $N \to 1$) limit, as expected from the fact the Pati-Salam factor $SU(4)_{PS}$ is unbroken in this limit.

The perturbative diagonalization of Eqs. (B6)–(B8) lead to the physical masses and mixings collected in Eqs. (33)–(39) and to right-handed rotation matrices whose "2–3" sector has the following form:

$$V_{u^c} = \begin{pmatrix} 1 & -\epsilon\dfrac{2y_{23}}{2c_\theta y_{33} + s_\theta y_3} \\ \epsilon\dfrac{2y_{23}}{2c_\theta y_{33} + s_\theta y_3} & 1 \end{pmatrix}, \tag{B9}$$

$$V_{d^c} = \begin{pmatrix} 1 & -\dfrac{\epsilon}{N}\dfrac{2y_{23}}{2c_\theta y_{33} - s_\theta y_3} \\ \dfrac{\epsilon}{N}\dfrac{2y_{23}}{2c_\theta y_{33} - s_\theta y_3} & 1 \end{pmatrix}. \tag{B10}$$

Finally, the RPV coupling at low-energy is obtained by projecting the operator $16\,16\,10$ onto the light states:

$$\lambda\, 16\, 16\, 10 \to 2\,\lambda\,\epsilon\, \frac{s_\theta t_\theta t_\phi}{(1 + t^2_\theta + t^2_\phi)^{1/2}} u^c_3 d^c_3 d^c_2. \tag{B11}$$

At the leading order in ϵ, the rotations in Eqs. (B9)–(B10) do not affect the result, which can be obtained expanding Eq. (A33) at the leading order in ϵ. Eqs. (45)–(46) follow.

[1] S. Weinberg, Phys.Rev. **D26**, 287 (1982).
[2] N. Sakai and T. Yanagida, Nucl.Phys. **B197**, 533 (1982).
[3] C. Aulakh and R. N. Mohapatra, Phys.Lett. **B119**, 136 (1982).
[4] L. J. Hall and M. Suzuki, Nucl.Phys. **B231**, 419 (1984).
[5] F. Zwirner, Phys.Lett. **B132**, 103 (1983).
[6] G. G. Ross and J. Valle, Phys.Lett. **B151**, 375 (1985).
[7] C. Brust, A. Katz, S. Lawrence, and R. Sundrum, JHEP **1203**, 103 (2012), arXiv:1110.6670.
[8] C. Csaki, Y. Grossman, and B. Heidenreich, Phys.Rev. **D85**, 095009 (2012), arXiv:1111.1239.
[9] G. Arcadi, L. Di Luzio, and M. Nardecchia, JHEP **1205**, 048 (2012), arXiv:1111.3941.
[10] B. Allanach and B. Gripaios, JHEP **1205**, 062 (2012), arXiv:1202.6616.
[11] H. Dreiner, F. Staub, A. Vicente, and W. Porod, Phys.Rev. **D86**, 035021 (2012), arXiv:1205.0557.
[12] B. Allanach and K. Sridhar, Phys.Rev. **D86**, 075016 (2012), arXiv:1205.5170.
[13] B. Keren-Zur *et al.*, Nucl.Phys. **B867**, 429 (2013), arXiv:1205.5803.
[14] G. Dupuis and J. M. Cline, JHEP **1301**, 058 (2013), arXiv:1206.1845.
[15] C. Brust, A. Katz, and R. Sundrum, JHEP **1208**, 059 (2012), arXiv:1206.2353.
[16] J. T. Ruderman, T. R. Slatyer, and N. Weiner, (2012), arXiv:1207.5787.
[17] J. A. Evans and Y. Kats, JHEP **1304**, 028 (2013), arXiv:1209.0764.
[18] J. Berger, C. Csaki, Y. Grossman, and B. Heidenreich, (2012), arXiv:1209.4645.
[19] M. Asano, K. Rolbiecki, and K. Sakurai, JHEP **1301**, 128 (2013), arXiv:1209.5778.
[20] Z. Han, A. Katz, M. Son, and B. Tweedie, (2012), arXiv:1211.4025.
[21] R. Franceschini and R. Torre, (2012), arXiv:1212.3622.
[22] G. Krnjaic and D. Stolarski, JHEP **JHEP04**, 064 (2013), arXiv:1212.4860.
[23] B. Bhattacherjee, J. L. Evans, M. Ibe, S. Matsumoto, and T. T. Yanagida, (2013), arXiv:1301.2336.
[24] R. Franceschini and R. Mohapatra, JHEP **1304**, 098 (2013), arXiv:1301.3637.
[25] C. Csaki and B. Heidenreich, (2013), arXiv:1302.0004.
[26] J. Berger, M. Perelstein, M. Saelim, and P. Tanedo, JHEP **1304**, 077 (2013), arXiv:1302.2146.
[27] A. Florez, D. Restrepo, M. Velasquez, and O. Zapata, (2013), arXiv:1303.0278.
[28] G. Krnjaic and Y. Tsai, (2013), arXiv:1304.7004.
[29] A. Monteux, (2013), arXiv:1305.2921.
[30] S. P. Martin, Phys.Rev. **D46**, 2769 (1992), arXiv:hep-ph/9207218.
[31] D.-G. Lee and R. Mohapatra, Phys.Rev. **D51**, 1353 (1995), arXiv:hep-ph/9406328.
[32] R. Mohapatra, Phys.Lett. **B379**, 115 (1996), arXiv:hep-ph/9601203.
[33] R. Mohapatra, Phys.Rev. **D54**, 5728 (1996).
[34] C. S. Aulakh, A. Melfo, A. Rasin, and G. Senjanovic, Phys.Lett. **B459**, 557 (1999), arXiv:hep-ph/9902409.
[35] C. S. Aulakh, B. Bajc, A. Melfo, A. Rasin, and G. Senjanovic, Nucl.Phys. **B597**, 89 (2001), arXiv:hep-ph/0004031.
[36] A. Y. Smirnov and F. Vissani, Nucl.Phys. **B460**, 37 (1996), arXiv:hep-ph/9506416.
[37] D. E. Brahm and L. J. Hall, Phys.Rev. **D40**, 2449 (1989).
[38] K. Tamvakis, Phys.Lett. **B383**, 307 (1996), arXiv:hep-ph/9602389.
[39] F. Vissani, (1995), arXiv:hep-ph/9602395.
[40] F. Vissani, Nucl.Phys.Proc.Suppl. **52A**, 94 (1997), arXiv:hep-ph/9607423.
[41] R. Barbieri, A. Strumia, and Z. Berezhiani, Phys.Lett. **B407**, 250 (1997), arXiv:hep-ph/9704275.
[42] G. Giudice and R. Rattazzi, Phys.Lett. **B406**, 321 (1997), arXiv:hep-ph/9704339.
[43] G. Arcadi, L. Di Luzio, and M. Nardecchia, JHEP **1112**, 040 (2011), arXiv:1110.2759.
[44] P. W. Graham, D. E. Kaplan, S. Rajendran, and P. Saraswat, JHEP **1207**, 149 (2012), arXiv:1204.6038.
[45] K. Tamvakis, Phys.Lett. **B382**, 251 (1996), arXiv:hep-ph/9604343.
[46] A. Masiero, D. V. Nanopoulos, K. Tamvakis, and T. Yanagida, Phys.Lett. **B115**, 380 (1982).
[47] B. Grinstein, Nucl.Phys. **B206**, 387 (1982).
[48] K. R. Dienes, Phys.Rept. **287**, 447 (1997), arXiv:hep-th/9602045.
[49] S. M. Barr, Phys.Rev. **D57**, 190 (1998), arXiv:hep-ph/9705266.
[50] K. Babu and S. M. Barr, Phys.Rev. **D51**, 2463 (1995), arXiv:hep-ph/9409285.
[51] G. Dvali and S. Pokorski, Phys.Lett. **B379**, 126 (1996), arXiv:hep-ph/9601358.
[52] A. Pomarol and D. Tommasini, Nucl.Phys. **B466**, 3 (1996), arXiv:hep-ph/9507462.
[53] R. Barbieri, G. Dvali, and L. J. Hall, Phys.Lett. **B377**, 76 (1996), arXiv:hep-ph/9512388.
[54] R. Barbieri, L. J. Hall, S. Raby, and A. Romanino, Nucl.Phys. **B493**, 3 (1997), arXiv:hep-ph/9610449.
[55] R. Barbieri, L. J. Hall, and A. Romanino, Phys.Lett. **B401**, 47 (1997), arXiv:hep-ph/9702315.
[56] S. Dimopoulos and F. Wilczek, Print-81-0600 (SANTA BARBARA), NSF-ITP-82-07 (1981).
[57] K. Babu and S. M. Barr, Phys.Rev. **D48**, 5354 (1993), arXiv:hep-ph/9306242.
[58] K. Babu and S. M. Barr, Phys.Rev. **D50**, 3529 (1994), arXiv:hep-ph/9402291.
[59] Z. Berezhiani and Z. Tavartkiladze, Phys.Lett. **B409**, 220 (1997), arXiv:hep-ph/9612232.
[60] S. M. Barr and S. Raby, Phys.Rev.Lett. **79**, 4748 (1997), arXiv:hep-ph/9705366.
[61] Z. Chacko and R. N. Mohapatra, Phys.Rev. **D59**, 011702 (1999), arXiv:hep-ph/9808458.
[62] N. Maekawa, Prog.Theor.Phys. **106**, 401 (2001), arXiv:hep-ph/0104200.
[63] K. Babu, J. C. Pati, and Z. Tavartkiladze, JHEP **1006**, 084 (2010), arXiv:1003.2625.
[64] M. Nardecchia, A. Romanino, and R. Ziegler, JHEP **0911**, 112 (2009), arXiv:0909.3058.
[65] M. Nardecchia, A. Romanino, and R. Ziegler, JHEP **1003**, 024 (2010), arXiv:0912.5482.
[66] M. Monaco, M. Pierini, A. Romanino, and M. Spinrath, (2013), arXiv:1302.1305.
[67] F. Caracciolo and A. Romanino, JHEP **1212**, 109 (2012), arXiv:1207.5376.

[68] F. Caracciolo, (2012), arXiv:1211.2924.
[69] M. Monaco, M. Nardecchia, A. Romanino, and R. Ziegler, JHEP **1110**, 022 (2011), arXiv:1108.1706.
[70] M. Frigerio, P. Hosteins, S. Lavignac, and A. Romanino, Nucl.Phys. **B806**, 84 (2009), arXiv:0804.0801.
[71] L. Calibbi, M. Frigerio, S. Lavignac, and A. Romanino, JHEP **0912**, 057 (2009), arXiv:0910.0377.
[72] Z. Berezhiani and J. Chkareuli, Sov.J.Nucl.Phys. **37**, 618 (1983).
[73] Z. Berezhiani, Phys.Lett. **B129**, 99 (1983).
[74] Z. Berezhiani, Phys.Lett. **B150**, 177 (1985).
[75] R. Barbier *et al.*, Phys.Rept. **420**, 1 (2005), arXiv:hep-ph/0406039.
[76] ATLAS Collaboration, ATLAS-CONF-2013-024 (2013).
[77] ATLAS Collaboration, ATLAS-CONF-2013-037 (2013).
[78] CMS Collaboration, CMS-SUS-13-007 (2013).
[79] CMS Collaboration, CMS-SUS-12-024 (2012).
[80] ATLAS Collaboration, ATLAS-CONF-2012-109, ATLAS-COM-CONF-2012-140, (2012).
[81] ATLAS Collaboration, G. Aad *et al.*, Eur.Phys.J. **C71**, 1828 (2011), arXiv:1110.2693.
[82] ATLAS Collaboration, G. Aad *et al.*, Eur.Phys.J. **C73**, 2263 (2013), arXiv:1210.4826.
[83] CMS Collaboration, S. Chatrchyan *et al.*, Phys. Rev. Lett. **110**, 141802 (2013), arXiv:1302.0531.
[84] ATLAS Collaboration, ATLAS-CONF-2013-007 (2013).
[85] ATLAS Collaboration, G. Aad *et al.*, JHEP **1212**, 086 (2012), arXiv:1210.4813.