Precise Charm and Bottom Quark Masses[1]

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1. Method

Exploiting the analyticity of the vacuum polarization function $\Pi(q^2)$ around $q^2 = 0$ and using dispersion relations, the derivatives at $q^2 = 0$ can be expressed as weighted integrals over the imaginary part of $\Pi(q^2)$, which in turn is given by the cross section for electron-positron annihilation into hadrons. Let us denote the normalised cross section for heavy quark production as $R_Q(s) \equiv \sigma_Q(s)/\sigma_{\rm point}(s)$. The moments of R_Q, defined as

$$\mathcal{M}_n^{\rm exp} \equiv \int \frac{{\rm d}s}{s^{n+1}} R_Q(s)\,, \tag{1}$$

can be directly related to the perturbatively calculated Taylor coefficients. In total one thus obtains the $\overline{MS}$ quark mass in terms of experimentally weighted integrals of R_Q and the perturbatively calculable coefficients $\bar{C}_n$,

$$m_Q(\mu) = \frac{1}{2}\left(\frac{9Q_Q^2\bar{C}_n}{4\mathcal{M}_n^{\rm exp}}\right)^{1/(2n)}\,. \tag{2}$$

This strategy has been suggested originally in Ref. [1] and applied to a precise charm and bottom mass determination in Ref. [2] once the three-loop results had become available. A significantly improved reanalysis based on four-loop moments as obtained in Refs. [3, 4, 5, 6, 7, 8] and with new data has been performed in Ref. [9], additional updates and improvements from new data and the precise analytical evaluation of the perturbative moments can be found in Refs. [10, 11]. For the extraction of R_Q from the data the issue of singlet contributions and secondary radiation of heavy quarks has been discussed in some detail in Ref. [9]. Furthermore, the potential influence of a non-vanishing gluon condensate $\langle \frac{\alpha_s}{\pi} GG\rangle = 0.006 \pm 0.012\,{\rm GeV}^4$ on the charm mass determination has been analysed [9, 11] and found to be small.

2. Results

Let us now present the experimental results for the moments, first for charm, later for bottom. For charm the integration region is split into one covering the narrow resonances J/ψ and ψ', the "threshold region" between $2m_D$ and 4.8 GeV and the perturbative continuum above. Note that we assume the validity of pQCD in this region with high precision, an

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n	$\mathcal{M}_n^{\text{res}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{thresh}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{cont}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{exp}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{np}}(\text{NLO})$ $\times 10^{(n-1)}$
1	0.1201(25)	0.0318(15)	0.0646(11)	0.2166(31)	−0.0002(5)
2	0.1176(25)	0.0178(8)	0.0144(3)	0.1497(27)	−0.0005(10)
3	0.1169(26)	0.0101(5)	0.0042(1)	0.1312(27)	−0.0008(16)
4	0.1177(27)	0.0058(3)	0.0014(0)	0.1249(27)	−0.0013(25)

Table 1: Experimental moments in $(\text{GeV})^{-2n}$ as defined in Eq. (1), separated according to the contributions from the narrow resonances, the charm threshold region and the continuum region above $\sqrt{s} = 4.8$ GeV. In the last column the NLO contribution from the gluon condensate is shown.

n	$m_c(3\text{ GeV})$	exp	α_s	μ	np_{NLO}	total
1	0.986	0.009	0.009	0.002	0.001	0.013
2	0.975	0.006	0.014	0.005	0.002	0.016
3	0.975	0.005	0.015	0.007	0.003	0.017
4	0.999	0.003	0.009	0.031	0.003	0.032

Table 2: Results for $m_c(3\,\text{GeV})$ in GeV. The errors are from experiment, α_s, variation of μ and the gluon condensate.

assumption that is well consistent with present measurements but for the moment remains an assumption, to be verified e.g. by future BESS experiments (for charm) and Belle (for bottom).

The results for the moments from one to four and the error budget are listed in Table 1, those for the quark mass in Table 2. The moment with $n = 1$ is most robust from the theory side, as evident from the relatively smaller coefficient in the perturbative series. In view of the smallest sensitivity to α_s and to the choice of the renormalisation scale μ we adopt the value as derived from $n = 1$ as our final result:

$$m_c(3\,\text{GeV}) = 986(13)\,\text{MeV}. \tag{3}$$

Tables 1 and 2 also illustrate the path to a further reduction of the error. For $n = 1$ important contributions arise from all three regions. Improved determinations of $\Gamma_e(J/\psi)$ would reduce the errors of all three moments. Improved measurements of R_Q in the threshold region and at 4.8 GeV would have a strong impact on $n = 1$ and strengthen our confidence in the validity of pQCD close to, but above 4.8 GeV. Another interesting option would be a simultaneous fit to all three moments, taking the proper experimental correlations into account.

Similar statements can be made for the determination of the bottom quark mass. A recent measurement of the cross section in the threshold region between 10.6 GeV and 11.2 GeV was employed in Ref. [10] and has lead to a significant reduction of the experimental error on m_b. Still, additional measurements in the region around and above 11 GeV would be welcome in order to confirm the validity of perturbative QCD relatively close to threshold. The result for the second moment has been adopted as our final answer

$$m_b(10\,\text{GeV}) = 3610(16)\,\text{MeV} \tag{4}$$

and corresponds to $m_b(m_b) = 3610(16)\,\text{MeV}$.

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