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Tree-level contributions to $\bar{B} \to X_s\gamma$

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Weak radiative decay $\bar{B} \to X_s\gamma$ is known to be a loop-generated process. However, it does receive tree-level contributions from CKM-suppressed $b \to u\bar{u}s\gamma$ transitions. In the present paper, we evaluate such contributions together with similar ones from the QCD penguin operators. For a low value of the photon energy cutoff $E_0 \simeq m_b/20$ that has often been used in the literature, they can enhance the inclusive branching ratio by more than 10%. For $E_0 = 1.6$ GeV or higher, the effect does not exceed 0.4%, which is due to phase-space suppression. Our perturbative results contain collinear logarithms that depend on the light quark masses m_q $\;(q = u, d, s)$. We have allowed m_b/m_q to vary from 10 to 50, which corresponds to values of m_q that are typical for the constituent quark masses. Such a rough method of estimation may be improved in the future with the help of fragmentation functions once the considered effects begin to matter in the overall error budget for $\mathcal{B}(\bar{B} \to X_s\gamma)$.

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I. INTRODUCTION

Weak radiative decay of the B meson is an invaluable and well-established means for constraining physics beyond the Standard Model (SM). Its branching ratio has been measured to a few percent accuracy at the B-factories [1]. Theoretical calculations have acquired similar precision [2]. The decay is generated dominantly by the $b \to s\gamma$ transition that arises at one loop in the SM and its most popular extensions. However, it receives also tree-level contributions from the CKM-suppressed $b \to u\bar{u}s\gamma$ transitions. In the present paper, we evaluate such contributions together with similar ones that originate from the QCD penguin operators.

It is well known that the inclusive $\bar{B} \to X_s\gamma$ decay rate can be approximated by its perturbative counterpart

$$\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} \simeq \Gamma(b \to X_s^p\gamma)_{E_\gamma > E_0}, \tag{1.1}$$

where X_s^p stands for $s, sg, sq\bar{q}, \ldots$ partonic states (with $q = u, d, s$ only, as no charmed hadrons appear in X_s by definition). Deviations from Eq. (1.1) appear as corrections when E_0 is large ($E_0 \sim \frac{1}{2}m_b$) but not too close to the endpoint ($m_b - 2E_0 \gg \Lambda \sim \Lambda_{\rm QCD}$). For $E_0 = 1.6\,$GeV, the corresponding non-perturbative uncertainty amounts to around $\pm5\%$ [3], while the known $\mathcal{O}(\Lambda^2/m_{b,c}^2)$ corrections [4, 5] are smaller than that.

The considered process is most conveniently analyzed in the framework of an effective low-energy theory which arises after integrating out the electroweak bosons and the top quark. The relevant effective weak interaction Lagrangian reads

$$\mathcal{L}_{\rm weak} = \frac{4G_F}{\sqrt{2}}\left[V_{ts}^*V_{tb}\sum_{i=1}^{8} C_iP_i + V_{us}^*V_{ub}\sum_{i=1}^{2} C_i(P_i - P_i^u)\right], \tag{1.2}$$

where P_i denote either dipole-type or four-quark operators, and C_i stand for their Wilson coefficients. The operators are given by

$$\begin{aligned}
P_1^u &= (\bar{s}_L\gamma_\mu T^a u_L)(\bar{u}_L\gamma^\mu T^a b_L),\\
P_2^u &= (\bar{s}_L\gamma_\mu u_L)(\bar{u}_L\gamma^\mu b_L),\\
P_1 &= (\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L),\\
P_2 &= (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L),\\
P_3 &= (\bar{s}_L\gamma_\mu b_L)\textstyle\sum_q(\bar{q}\gamma^\mu q),\\
P_4 &= (\bar{s}_L\gamma_\mu T^a b_L)\textstyle\sum_q(\bar{q}\gamma^\mu T^a q),\\
P_5 &= (\bar{s}_L\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} b_L)\textstyle\sum_q(\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} q),\\
P_6 &= (\bar{s}_L\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} T^a b_L)\textstyle\sum_q(\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} T^a q),\\
P_7 &= \tfrac{e}{16\pi^2} m_b(\bar{s}_L\sigma^{\mu\nu} b_R)F_{\mu\nu},\\
P_8 &= \tfrac{g}{16\pi^2} m_b(\bar{s}_L\sigma^{\mu\nu} T^a b_R)G^a_{\mu\nu}.
\end{aligned} \tag{1.3}$$

Sums over q in $P_{3,\ldots,6}$ include all the active flavors $q = u, d, s, c, b$.

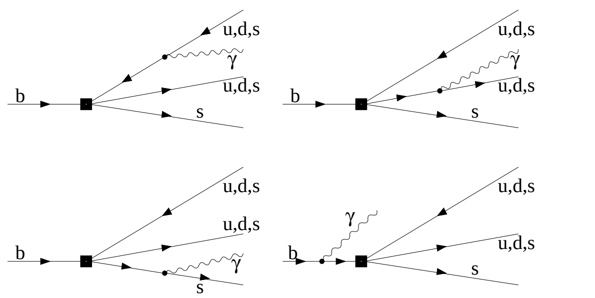

FIG. 1: Tree-level Feynman diagrams for $b \to s\bar{q}q\gamma$ at the LO. Black squares denote four-quark operator insertions.

Our goal in the present work is to evaluate $\mathcal{O}\left(\alpha_s^0\right)$ leading order (LO) contributions to the r.h.s. of Eq. (1.1) that originate from $b \to q\bar{q}s\gamma$ with $q = u, d, s$. They can be generated either by the current-current operators $P_{1,2}^u$ or by the QCD penguin ones $P_{3,\ldots,6}$. The corresponding Feynman diagrams are shown in Fig. 1.

At first glance, it may seem surprising that the considered LO effects have not been evaluated so far while the analysis of other contributions has already reached the $\mathcal{O}\left(\alpha_s^2\right)$ next-to-next-to-leading order (NNLO) level [2]. Let us recall that the Wilson coefficients of all the operators in Eq. (1.3) acquire non-zero values already at the LO once the QCD logarithms have been resummed using renormalization group evolution from the electroweak scale $\mu_0 \sim M_W, m_t$ down to the low-energy scale $\mu_b \sim m_b/2$. However, despite being non-vanishing at the LO, Wilson coefficients of the QCD penguin operators remain rather small $|C_{3,\ldots,6}(\mu_b)/C_7(\mu_b)|^2 < |C_4(\mu_b)/C_7(\mu_b)|^2 \sim 0.1$, while the tiny CKM matrix element ratio $|V_{us}^* V_{ub}/V_{ts}^* V_{tb}| \simeq 0.02$ makes the current-current operators $P_{1,2}^u$ even more suppressed. Moreover, when the lower photon energy cutoff E_0 is at $1.6\,\mathrm{GeV}$ [2] or higher [1], the considered tree-level contributions to the branching ratio undergo severe phase-space suppression, which justifies neglecting them at the leading and next-to-leading orders in α_s (see App. E of Ref. [6]).

Given the current and expected future progress in the NNLO calculations [7], reliable uncertainty estimates in the SM prediction for $\mathcal{B}(\bar{B} \to X_s\gamma)$ can no longer be made without evaluating the diagrams in Fig. 1 and checking what the actual size of their contribution to the r.h.s of Eq. (1.1) is. This fact serves as the main motivation for our present work. Since the corrections are expected (and found) to be quite small for $E_0 = 1.6\,\mathrm{GeV}$, rough estimates of their size are sufficient. Actually, nothing more is available within perturbation theory alone because collinear logarithms $\ln(m_b/m_q)$ involving light quark masses m_q $(q = u, d, s)$ remain in the final expressions. While using the so-called current masses for the light quarks is not adequate in such a case, the above-mentioned rough estimates can be obtained by assuming that m_q are of the same order as masses of pions and kaons or, equivalently, as the constituent quark masses. We shall do it by varying m_b/m_q from 10 to 50, which covers the necessary range. A refined approach would require taking non-perturbative fragmentation into account, as it has been done in Refs. [8, 9] for contributions that are proportional to $|C_8|^2$ or in Ref. [10] for the $b \to u\bar{u}d\gamma$ background. Such an analysis is beyond the scope of the present paper.

The article is organized as follows. Our final perturbative results together with a discussion of their numerical impact on the total decay rate are presented in Sec. II. The next two sections contain brief descriptions of two alternative methods that we have used for integration over the four-body phase space. In Sec. III, partly massive phase-space integration in $D = 4$ dimensions is outlined. In Sec. IV, a calculation involving dimensional regularization and splitting functions is described. We conclude in Sec. V. App. A contains a collection of several intermediate results that may be useful for other calculations of electromagnetic bremsstrahlung corrections to decays mediated by four-fermion operators. App. B is devoted to summarizing the necessary properties of the splitting functions.

II. RESULTS

Our final result can be expressed in terms of three functions $T_k(\delta)$ $(k = 1, 2, 3)$ that depend on the photon energy cut $E_0 = \frac{m_b}{2}(1-\delta)$ and on logarithms of the quark masses. Each of the functions gets multiplied by a quadratic polynomial in the Wilson coefficients C_i that are evaluated at the scale μ_b. We assume that all the C_i are real, as it is the case in the SM. The LO tree-level contribution to $\Gamma[b \to X_s^p\gamma]$ arising from the four-quark operators $P_{1,2}^u$ and $P_{3,\ldots,6}$ reads

$$\begin{aligned}\Delta\Gamma[b \to X_s^p\gamma]^{\rm LO}_{\text{4-quark}} &= \frac{G_F^2 \alpha_{\rm em} m_b^5}{32\pi^4}\, |V_{ts}^* V_{tb}|^2 \Bigg[T_1(\delta)\left(C_3^2 + 20 C_3 C_5 + \frac{2}{9}C_4^2 + \frac{40}{9}C_4C_6 + 136 C_5^2 + \frac{272}{9}C_6^2 \right) \\ &+ T_2(\delta)\left(\frac{2}{9}|A_1|^2 + |A_2|^2 + \left(\frac{8}{9}C_3 - \frac{4}{27}C_4 + \frac{128}{9}C_5 - \frac{64}{27}C_6\right)\mathrm{Re}\,A_1 + \left(\frac{2}{3}C_3 + \frac{8}{9}C_4 + \frac{32}{3}C_5 + \frac{128}{9}C_6\right)\mathrm{Re}\,A_2 \right) \\ &+ T_3(\delta)\left(C_3^2 + \frac{4}{3}C_3C_4 + 32C_3C_5 + \frac{128}{3}C_3C_6 - \frac{2}{9}C_4^2 + \frac{128}{3}C_4C_5 - \frac{64}{9}C_4C_6 + 256C_5^2 + \frac{1024}{3}C_5C_6 - \frac{512}{9}C_6^2 \right)\Bigg], \end{aligned} \tag{2.1}$$

where $A_i = -C_i \frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}}$, $i = 1, 2$, and

$$\begin{aligned} T_1(\delta) &= \left(-\frac{5}{3}\rho(\delta) - \frac{2}{9}\omega(\delta)\right)\ln\frac{m_b^2\,\delta}{\sqrt[3]{m_u^4 m_d m_s}} + \frac{109}{18}\delta \\ &+ \frac{17}{18}\delta^2 - \frac{191}{108}\delta^3 + \frac{23}{16}\delta^4 + \frac{79}{18}\ln(1-\delta) \\ &- \frac{5}{3}\mathrm{Li}_2(\delta) + \frac{1}{9}\rho(\delta)\ln\frac{m_s^5}{m_u^4 m_d}, \end{aligned} \tag{2.2}$$

$$\begin{aligned} T_2(\delta) &= \left(-\frac{1}{2}\rho(\delta) - \frac{2}{27}\omega(\delta)\right)\ln\frac{m_b^2\,\delta}{m_u^2} + \frac{187}{108}\delta \\ &+ \frac{7}{18}\delta^2 - \frac{395}{648}\delta^3 + \frac{1181}{2592}\delta^4 + \frac{133}{108}\ln(1-\delta) \\ &- \frac{1}{2}\mathrm{Li}_2(\delta) + \frac{1}{9}\rho(\delta)\ln\frac{m_s}{m_u}, \end{aligned} \tag{2.3}$$

$C_1 = -0.8144$	$C_3 = -0.0125$	$C_5 = 0.0012$	$C_7 = -0.3688$
$C_2 = \ \ 1.0611$	$C_4 = -0.1224$	$C_6 = 0.0026$	$C_8 = -0.1710$

TABLE I: The LO Wilson coefficients C_i at $\mu_b = 2.5$ GeV. The matching scale μ_0 has been set to 160 GeV in their evaluation.

$$T_3(\delta) = \left(-\frac{1}{9}\rho(\delta) - \frac{1}{81}\omega(\delta)\right)\ln\frac{m_b^2\,\delta}{m_s^2} + \frac{35}{81}\delta + \frac{1}{36}\delta^2 - \frac{89}{972}\delta^3 + \frac{341}{3888}\delta^4 + \frac{26}{81}\ln(1-\delta) - \frac{1}{9}\mathrm{Li}_2(\delta), \quad (2.4)$$

with

$$\rho(\delta) = \delta + \frac{1}{6}\delta^4 + \ln(1-\delta),$$
$$\omega(\delta) = \frac{3}{2}\delta^2 - 2\delta^3 + \delta^4. \quad (2.5)$$

The function $T_3(\delta)$ originates from cross-terms in $b \to ss\bar{s}\gamma$ where the s-quark lines are interchanged in one of the interfered diagrams (see Fig. 2). All the other contributions from the penguin operators $P_{3,\ldots,6}$ alone are described by $T_1(\delta)$. Finally, $T_2(\delta)$ comes from $P^u_{1,2}$ and their interference with $P_{3,\ldots,6}$.

We have retained the light quark masses m_q in the collinear logarithms *only*, i.e., all the power-like corrections proportional to m_q^2/m_b^2 have been neglected in the above expressions. Such an approximation breaks down at some point, which manifests itself in non-physical negative values of $T_{1,2}(\delta)$ when $\ln(m_b^2\,\delta/m_q^2)$ is not big enough.

Determining the size of the calculated correction is now straightforward. Numerical values of the LO Wilson coefficients $C_i \equiv C_i^{(0)}(\mu_b)$ are summarized in Tab. I. For the CKM element ratio we use $\frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}} = -0.0079 + 0.018\,i$ [11]. As far as the light quark masses $m_{u,d,s}$ are concerned, we set all of them equal in the numerical examples to be discussed below.

In Tab. II, we present the calculated correction (2.1) as a fraction of the leading contribution to the decay rate $\Gamma^{(0)} = G_F^2 \alpha_{\rm em} m_b^5\, C_7^2\, |V_{ts}^* V_{tb}|^2/(32\pi^4)$ for two values of E_0 and two values of $\frac{m_b}{m_q}$. Strong dependence on the collinear logarithms is clearly visible. On the other hand, the non-logarithmic terms turn out to be relevant

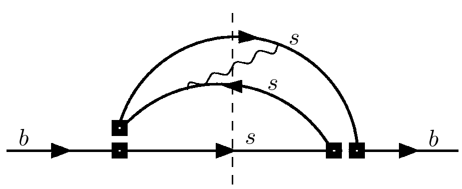

FIG. 2: Sample contribution to the $b \to ss\bar{s}\gamma$ cross-terms that give rise to the function $T_3(\delta)$. It is represented as a cut propagator diagram, with the cut denoted by the vertical dashed line.

E_0	δ	$\Delta\Gamma^{\rm LO}_{\text{4-quark}}/\Gamma^{(0)}$		
			$\frac{m_b}{m_q} = 50$	$\frac{m_b}{m_q} = 10$
1.6 GeV	0.316	$-0.45\% + 0.21\% \ln\frac{m_b}{m_q}$	$+0.36\%$	$+0.02\%$
$\frac{m_b}{20}$	0.9	$-8.1\% + 5.2\% \ln\frac{m_b}{m_q}$	$+12.2\%$	$+3.8\%$

TABLE II: The correction from Eq. (2.1) as a fraction of the leading contribution $\Gamma^{(0)}$ for several values of E_0 and m_b/m_q.

in the considered range of $\frac{m_b}{m_q}$. For a low value of the photon energy cutoff $E_0 \simeq m_b/20$ that has often been used in the literature, the correction can enhance the inclusive rate by more than 10%. On the other hand, for $E_0 = 1.6\,\text{GeV} \simeq \frac{m_b}{3}$ or higher, the effect does not exceed 0.4%, which is obviously due to phase-space suppression that becomes efficient when we approach the high-energy endpoint $E_0 \simeq \frac{m_b}{2}$, i.e. when δ becomes small. In this limit, our correction in Eq. (2.1) behaves like $\mathcal{O}\left(\delta^2 \ln\delta\right)$. In all the four cases shown in Tab. II, the contribution from $T_1(\delta)$ is the dominant one, while $T_3(\delta)$ ($T_2(\delta)$) give about 5 (50) times smaller effects. Thus, the CKM-suppressed contributions that come with $T_2(\delta)$ are minuscule indeed, and no precise knowledge of $\frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}}$ is necessary here. On the other hand, our LO results exhibit significant dependence on the renormalization scale μ_b that comes from $C_{3,\ldots,6}(\mu_b)$. It could be stabilized only after including $\mathcal{O}(\alpha_s)$ contributions to $b \to q\bar{q}s\gamma$, in particular the ones generated by $P_{1,2}$.

III. CALCULATION INVOLVING PARTLY MASSIVE PHASE-SPACE INTEGRALS

Let us now briefly describe the calculation. First, we consider the diagrams in Fig. 1 with an operator

$$\widetilde{P}_3 = (\bar{s}_L \gamma_\mu b_L)(\bar{q}_1 \gamma^\mu q_2), \quad (3.1)$$

where no sum over flavors is present (contrary to P_3 in Eq. (1.3)), and the electric charges $\{Q_s, Q_b, Q_1, Q_2\}$ are retained arbitrary. The invariant matrix element $\mathcal{M}$ is calculated in the Feynman gauge, so collinear divergences are allowed to occur in interferences between different diagrams (rather than in self-interference terms alone) [12]. The Dirac algebra is performed in $D = 4 - 2\epsilon$ dimensions without neglecting the light quark masses. At this point, we proceed in two different ways. One of them is to integrate the spin-averaged $|\overline{\mathcal{M}}|^2$ over the partly massive four-body phase space in $D = 4$ dimensions. The other way is to initially neglect the light quark masses, integrate $|\overline{\mathcal{M}}|^2$ in $D = 4 - 2\epsilon$ dimensions over the massless phase space, and convert the collinear $1/\epsilon$ poles to logarithms of masses only afterwards (see Sec. IV).

In the $D = 4$ case, we impose the photon energy cut (in the b-quark rest frame $\mathcal{F}$) after expressing E_γ in terms of invariants, namely $2m_b E_\gamma = m_b^2 - s_{123}$, where s_{123} is the invariant mass squared of the $q_1 \bar{q}_2 s$ system. A boost to the rest frame $\hat{\mathcal{F}}$ of this system is performed along the $-\vec{k}$ direction, where $\vec{k}$ is the photon three-momentum in $\mathcal{F}$. The $q_1\bar{q}_2 s$ system has energy equal to $\sqrt{s_{123}}$ in $\hat{\mathcal{F}}$, while the three-momenta of its constituents define a plane. The direction of $\vec{k}$ with respect to this plane is parametrized by two polar angles $\hat{\theta}$ and $\hat{\varphi}$. The remaining three phase-space variables are the quark energy fractions $\hat{x}_i = \frac{2\hat{E}_i}{\sqrt{s_{123}}}$ $(i = 1, 2)$ and s_{123}.

We integrate first over $\hat{\theta}$ and $\hat{\varphi}$, and obtain results containing large logarithms $\ln \frac{m_b^2}{m_q^2}$. Integration of the interference terms $\frac{p_i p_j}{(p_i q)(p_j k)}$ involves angular ordering (see Ref. [13]), which applies also to the case $p_i^2 \equiv m_q^2 \ll m_b^2$. At this stage, all the collinear logarithms have been already identified, which allows us to neglect masses of the outgoing particles in the remaining terms. Next, integrations over the quark energy fractions and s_{123} are performed. Finally, charge conservation $Q_b = Q_1 - Q_2 + Q_s$ is imposed, but the charges on the r.h.s. are still retained arbitrary. Several intermediate results with such arbitrary charges are collected in App. A.

Once the final result for $\widetilde{P}_3$ with arbitrary charges is at hand, obtaining the corresponding one for P_3 $(T_1(\delta))$ is just a matter of substituting the actual values of the charges and summing over flavors. Extending the calculation to $P_{4,5,6}$ and taking into account the $b \to s s \bar{s} \gamma$ cross-terms $(T_3(\delta))$ requires modifying the Dirac algebra and color factors but no essential difference in the phase-space integration is encountered. As far as $T_2(\delta)$ is concerned, it originates from the operators $P^u_{1,2}$, including their interference with $P_{3,...,6}$. In this case, it is sufficient to express $P^u_{1,2}$ in $D = 4$ dimensions as linear combinations of the $(\bar{s}b)(\bar{u}u)$-parts of $P_{3,...,6}$, namely

$$
\begin{aligned}
P_1^u &= -\tfrac{4}{27} P_3^u + \tfrac{1}{9} P_4^u + \tfrac{1}{27} P_5^u - \tfrac{1}{36} P_6^u \\
P_2^u &= -\tfrac{1}{9} P_3^u - \tfrac{2}{3} P_4^u + \tfrac{1}{36} P_5^u + \tfrac{1}{6} P_6^u \,,
\end{aligned}
\tag{3.2}
$$

The above expressions are easily derived using Fierz identities for the Dirac and Gell-Mann matrices.

IV. CALCULATION WITH THE HELP OF DIMENSIONAL REGULARIZATION

A calculation with the help of dimensional regularization is technically simpler but involves a few subtleties. To start with, all the particles in the final state are assumed to be massless, and the phase-space integration is performed in $D = 4 - 2\epsilon$ dimensions [14]. The results are collected in App. A. Given that the collinear divergences appear as $1/\epsilon$ poles, one should make sure that no ambiguities arise from Dirac traces with odd numbers of γ_5's [15]. Fortunately, such traces being purely imaginary give no contribution to the decay rate in the case of $P_{3,...,6}$ alone. As far as $P^u_{1,2}$ are concerned, we do not need to consider them at this level. Once the collinear divergences are re-expressed in terms of logarithms of masses (see below), we can pass to $D = 4$ and use the identities (3.2).

For definiteness, let us consider the operator $\widetilde{P}_3$ from Eq. (3.1) again, but this time with the s-quark denoted by q_3 to keep the notation symmetric. Before integrating over the photon energy (but after integration over all the other phase-space variables), the differential decay width for $b \to q_1 \bar{q}_2 q_3 \gamma$ reads

$$
\frac{d\Gamma}{dx} = \frac{d\Gamma_\epsilon}{dx} + \frac{d\Gamma_{\rm shift}}{dx} \,, \tag{4.1}
$$

where $x = 2E_\gamma/m_b$. The first term on the r.h.s. above is the dimensionally regulated expression, while the second one converts the dimensional regulators to logarithms of masses. Its explicit form is given below.

We shall need a D-dimensional expression for the total width of the three-body decay $b \to q_1 \bar{q}_2 q_3$. Denoting momenta of the final-state massless particles by p_i, $i = 1, 2, 3$, and parametrizing the phase space by $s_{ij} = 2p_i p_j / m_b^2$, one can write

$$
\Gamma_{3\text{-body}} = \frac{\widetilde{\mu}^{4\epsilon}}{2m_b} \int_0^1 ds_{12}\, ds_{13}\, ds_{23}\, |\overline{\mathcal{M}'}|^2\, M_3, \tag{4.2}
$$

where $\mathcal{M}'$ is the corresponding invariant matrix element, $\widetilde{\mu}^2 = \mu^2 e^{\gamma_E}/4\pi$, and

$$
M_3 = \frac{\delta(1 - s_{12} - s_{13} - s_{23})\, \pi^{-\frac{5}{2}+2\epsilon}\, (m_b^2)^{1-2\epsilon}}{2^{8-6\epsilon}\, \Gamma\left(\frac{3}{2} - \epsilon\right)\, \Gamma(1-\epsilon)\, (s_{12}s_{13}s_{23})^{\epsilon}} \tag{4.3}
$$

describes the phase-space measure [16]. The second term in Eq. (4.1) can now be written as

$$
\begin{aligned}
\frac{d\Gamma_{\rm shift}}{dx} \;&=\; \frac{\widetilde{\mu}^{4\epsilon}}{2m_b} \int_0^1 ds_{12}\, ds_{13}\, ds_{23}\, |\overline{\mathcal{M}'}|^2\, M_3\, \frac{\alpha_{\rm em}}{2\pi x} \\
&\times\; \left\{ Q_1^2 \left[1 + \left(1 - \frac{x}{1 - s_{23}} \right)^2 \right] \Theta\left(1 - s_{23} - x\right) \right. \\
&\times\; \left. \left[\frac{1}{\epsilon} - 1 + 2 \ln \frac{(1 - s_{23})\mu}{x m_{q_1}} \right] + (\text{cyclic}) \right\}.
\end{aligned}
\tag{4.4}
$$

Properties of the splitting functions that have been necessary to derive the above formula are summarized in App. B. One should remember that all the collinear $1/\epsilon$ poles cancel in Eq. (4.1) only after the charge conservation $Q_b = Q_1 - Q_2 + Q_s$ has been imposed.

The structure of Eq. (4.4) remains the same irrespective of what interaction generates the b-quark decay. Thus, it is applicable as it stands to the operators $P_{3,...,6}$. Next, as already mentioned, Eq. (3.2) is used in $D = 4$ dimensions to take $P^u_{1,2}$ into account. This way the final expression in Eq. (2.1) has been obtained once again, in perfect agreement with the results of Sec. III.

V. CONCLUSIONS

In the present paper, we have evaluated the LO contributions to the partonic decay width $\Gamma(b \to X_s^p \gamma)$ that originate from the four-quark operators $P^u_{1,2}$ and $P_{3,\ldots,6}$. They can be sizeable (above 10%) for low photon energy cutoffs E_0 but become very small (below 0.4%) in the phenomenologically interesting domain $E_0 \geq 1.6\,\mathrm{GeV}$, i.e., for $\delta \equiv 1 - 2E_0/m_b \lesssim 0.32$. For small δ, they behave like $\mathcal{O}(\delta^2 \ln \delta)$, which determines their phase-space suppression near the endpoint.

The presence of collinear logarithms $\ln(m_b^2 \delta / m_q^2)$ involving light quark masses m_q ($q = u, d, s$) implies that our perturbative results (with m_b/m_q varied from 10 to 50) may serve only as rough estimates of the corresponding contributions to the inclusive hadronic $\bar{B} \to X_s \gamma$ decay rate. Having such rough estimates at hand is both advantageous and sufficient at present, given that the overall non-perturbative uncertainty remains at the $\pm 5\%$ level [3]. However, once our control over non-perturbative corrections improves in the future, the current contributions will need to be supplemented with hadronic fragmentation effects, along the lines of Refs. [8, 9].

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APPENDIX A: INTERMEDIATE RESULTS

Here, we present several intermediate results that might be useful for studying hard photon emission in other processes mediated by four-fermion operators, like muon decays or semileptonic heavy quark decays. Although radiative corrections to such processes have been calculated long ago, none of the published results that we are aware of leaves the photon energy as the only phase-space variable that has not been integrated over (or, equivalently, integrated with an arbitrary cutoff). To make our results applicable to such cases, it is enough to present them for arbitrary electric charges of the final-state fermions.

As in Sec. III, we replace P_3 (1.3) by $\widetilde{P}_3$ (3.1) and assume that $q_1 \neq s$, which means that no cross-terms ($T_3(\delta)$) arise. The contribution of $\widetilde{P}_3$ to the decay rate is as in the $C_3^2\, T_1(\delta)$ term in Eq. (2.1) but with $T_1(\delta)$ replaced by

$$\begin{aligned} \widetilde{T}_1(\delta) &= (Q_1^2 + Q_2^2)\, F_{11}(\delta) + Q_s^2\, F_{ss}(\delta) \\ &+ Q_1 Q_2 \left(-\frac{5}{6}\rho(\delta) - \frac{1}{9}\omega(\delta) - \frac{1}{4}\delta^2 \right) \\ &+ (Q_1 - Q_2)\, Q_s \left(\frac{7}{6}\rho(\delta) + \frac{1}{18}\omega(\delta) \right), \end{aligned} \tag{A.1}$$

where

$$\begin{aligned} F_{11}(\delta) &= \left(-\rho(\delta) - \frac{1}{6}\omega(\delta) \right) \ln \frac{m_b^2\, \delta}{m_q^2} + 4\delta + \frac{11}{12}\delta^2 \\ &- \frac{17}{12}\delta^3 + \frac{79}{72}\delta^4 + 3\ln(1-\delta) - \mathrm{Li}_2(\delta)\,, \\ F_{ss}(\delta) &= -\rho(\delta)\, \ln \frac{m_b^2\, \delta}{m_q^2} + \frac{23}{6}\delta - \frac{1}{12}\delta^3 + \frac{61}{144}\delta^4 \\ &+ \frac{17}{6}\ln(1-\delta) - \mathrm{Li}_2(\delta)\,. \end{aligned} \tag{A.2}$$

In Eq. (A.1), charge conservation $Q_b = Q_1 - Q_2 + Q_s$ has been already imposed. In effect, collinear logarithms remain only in the terms that come with Q_i^2, $i = 1, 2, s$. For simplicity, all the light quark masses have been set equal and denoted by m_q. However, it is easy to relax this assumption and identify them by the corresponding charges despite the fact that charge conservation has already been used (see App. B).

Now, let us consider the case when $q_1 = s$ in Eq. (3.1). Then, apart from $\widetilde{T}_1(\delta)$ (A.1), we get an additional contribution from the cross-terms

$$\begin{aligned} \widetilde{T}_3(\delta) &= Q_2^2\, S_{22}(\delta) + Q_s^2\, S_{ss}(\delta) \\ &+ Q_2 Q_s \left(-\frac{5}{9}\rho(\delta) - \frac{2}{27}\omega(\delta) - \frac{1}{6}\delta^2 \right), \end{aligned} \tag{A.3}$$

where

$$\begin{aligned} S_{22}(\delta) &= \left(-\frac{1}{3}\rho(\delta) - \frac{1}{9}\omega(\delta) \right) \ln \frac{m_b^2\, \delta}{m_q^2} + \frac{25}{18}\delta + \frac{11}{18}\delta^2 \\ &- \frac{11}{12}\delta^3 + \frac{85}{144}\delta^4 + \frac{19}{18}\ln(1-\delta) - \frac{1}{3}\mathrm{Li}_2(\delta)\,, \\ S_{ss}(\delta) &= -\frac{2}{3}\rho(\delta)\, \ln \frac{m_b^2\, \delta}{m_q^2} + \frac{55}{18}\delta - \frac{1}{12}\delta^2 - \frac{1}{18}\delta^3 \\ &+ \frac{79}{216}\delta^4 + \frac{43}{18}\ln(1-\delta) - \frac{2}{3}\mathrm{Li}_2(\delta)\,. \end{aligned} \tag{A.4}$$

The above results are valid only for the particular Dirac structure of the operator $\widetilde{P}_3$. To generalize them to all the four-fermion operators with chirality-conserving currents, it is sufficient to consider

$$\widetilde{P}_5 = (\bar{s}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} b_L)(\bar{q}_1 \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} q_2). \tag{A.5}$$

Its interference with $\widetilde{P}_3$ gives

$$20\,\widetilde{T}_1(\delta) + (Q_1^2 - Q_2^2)\, r_1(\delta) + (Q_1 + Q_2)\, Q_s\, r_2(\delta), \tag{A.6}$$

and $32\,\widetilde{T}_3(\delta)$ for the cross-terms. Its self-interference gives

$$136\,\widetilde{T}_1(\delta) + 10\left[(Q_1^2 - Q_2^2)\,r_1(\delta) + (Q_1 + Q_2)\,Q_s\,r_2(\delta)\right], \tag{A.7}$$

and $256\,\widetilde{T}_3(\delta)$ for the cross-terms. The functions $r_{1,2}(\delta)$ are given by

$$\begin{aligned} r_1(\delta) &= 2\omega(\delta)\ln\frac{m_b^2\,\delta}{m_q^2} - 2\rho(\delta) - 11\delta^2 + 16\delta^3 - \frac{31}{4}\delta^4\,, \\ r_2(\delta) &= 4\rho(\delta) - \frac{2}{3}\omega(\delta) - 3\delta^2\,. \end{aligned} \tag{A.8}$$

The corresponding results for massless final-state fermions in the dimensional regularization have been obtained with the help of Eq. (A.5) of Ref. [16] where a compact expression for the D-dimensional four-body phase-space measure is given. Only the functions that come proportional to squared charges differ from their $D = 4$ counterparts. For $\widetilde{P}_3$ alone, they read

$$\begin{aligned} F_{11}^{(\epsilon)}(\delta) &= \left(\rho(\delta) + \frac{1}{6}\omega(\delta)\right)C_\epsilon(\delta) + \frac{45}{4}\delta + \frac{5}{2}\delta^2 - \frac{23}{9}\delta^3 \\ &+ \frac{175}{72}\delta^4 + \frac{37}{4}\ln(1-\delta) + \ln^2(1-\delta) - 2\mathrm{Li}_2(\delta)\,, \\ F_{ss}^{(\epsilon)}(\delta) &= \rho(\delta)\,C_\epsilon(\delta) + \frac{133}{12}\delta + \frac{7}{24}\delta^2 + \frac{4}{9}\delta^3 + \frac{65}{72}\delta^4 \\ &+ \frac{109}{12}\ln(1-\delta) + \ln^2(1-\delta) - 2\mathrm{Li}_2(\delta)\,, \\ S_{22}^{(\epsilon)}(\delta) &= \left(\frac{1}{3}\rho(\delta) + \frac{1}{9}\omega(\delta)\right)C_\epsilon(\delta) + \frac{125}{36}\delta + \frac{101}{72}\delta^2 \\ &- \frac{44}{27}\delta^3 + \frac{83}{72}\delta^4 + \frac{101}{36}\ln(1-\delta) + \frac{1}{3}\ln^2(1-\delta) \\ &- \frac{2}{3}\mathrm{Li}_2(\delta)\,, \\ S_{ss}^{(\epsilon)}(\delta) &= \frac{2}{3}\,\rho(\delta)\,C_\epsilon(\delta) + \frac{65}{9}\delta + \frac{1}{9}\delta^2 + \frac{8}{27}\delta^3 + \frac{31}{54}\delta^4 \\ &+ \frac{53}{9}\ln(1-\delta) + \frac{2}{3}\ln^2(1-\delta) - \frac{4}{3}\mathrm{Li}_2(\delta)\,, \end{aligned} \tag{A.9}$$

where $C_\epsilon(\delta) = 1/\epsilon - 2\ln[\delta(1-\delta)]$.

In the case of $\widetilde{P}_3$-$\widetilde{P}_5$ interference (Eq. (A.6) and below), the following replacements need to be made:

$$\begin{aligned} (Q_1^2 - Q_2^2)\,r_1 \;\to\; &-2(Q_1^2 - Q_2^2)\left((C_\epsilon + 8)\,\omega + 7\,\rho - \frac{\delta^2}{4}\right) \\ &- 12\,Q_s^2\,\rho - 4\,Q_2^2(6\rho + \omega)\,, \\ 32\,\widetilde{T}_3 \;\to\; &32\,\widetilde{T}_3^{(\epsilon)} - \frac{16}{9}Q_s^2(3\rho - \omega) - \frac{8}{3}Q_2^2(\rho + \omega)\,, \end{aligned} \tag{A.10}$$

where $\widetilde{T}_3^{(\epsilon)}(\delta)$ is given in terms of $S_{22}^{(\epsilon)}(\delta)$ and $S_{ss}^{(\epsilon)}(\delta)$, in analogy to Eq. (A.3). The corresponding replacements for the $\widetilde{P}_5$ self-interference (Eq. (A.7) and below) read

$$\begin{aligned} &(Q_1^2 - Q_2^2)\,r_1(\delta) \;\to \\ &\qquad -2(Q_1^2 - Q_2^2)\left(\left(C_\epsilon + \frac{83}{10}\right)\omega + 10\,\rho - \frac{\delta^2}{4}\right) \\ &\qquad + Q_s^2\left(-18\rho + \frac{8}{5}\omega\right) - 4\,Q_2^2\left(9\rho + \frac{19}{10}\omega\right), \\ &256\,\widetilde{T}_3 \;\to\; 256\,\widetilde{T}_3^{(\epsilon)} - \frac{64}{9}Q_s^2(21\rho - 4\,\omega) - \frac{32}{3}Q_2^2(7\rho + 5\omega)\,. \end{aligned} \tag{A.11}$$

Conversion of the collinear regulators in Eq. (4.1) is most conveniently performed before substitution of charges, at the level of the functions from Eqs. (A.9)–(A.11).

Our final result in Eq. (2.1) has been obtained by forming appropriate linear combinations of the functions appearing in Eqs. (A.1), (A.3) and (A.8), according to the values of charges, color factors and sums over flavors.

APPENDIX B: SPLITTING FUNCTIONS

Our conversion formula (4.4) involves a difference between splitting functions derived in the dimensional regularization and in the regularization with masses. Their derivation along the lines of Refs. [17, 18] is briefly described in the following.

Let us consider an amplitude $\mathcal{M}(q, k; \ldots)$ of a process where an external massive fermion radiates a photon with momentum k ($k^2 = 0$). After the radiation, the fermion is on shell ($q^2 = m^2$). We shall assume that the calculation is performed in the light-cone axial gauge $n \cdot A = 0$, where n is a lightlike vector ($n^2 = 0$), and the sum over photon polarizations gives

$$\sum_\lambda \epsilon_\mu^{\lambda *}\epsilon_\nu^\lambda = -g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{kn}\,. \tag{B.1}$$

In such a gauge, interference terms between different diagrams are free of collinear singularities [12].

In the course of the splitting function derivation, it is convenient to introduce the Sudakov parametrization in terms of

$$\begin{aligned} p &\equiv q + k - \frac{qk}{(q+k)n}\,n, \\ k_\perp &\equiv z\,p - q + z(1-z)\,\frac{(q+k)q}{qn}\,n, \end{aligned} \tag{B.2}$$

where $z \equiv (qn)/[(q+k)n] \in \left[m^2/(m^2 + 2qk)\,, 1\right]$. It is easy to verify that $p^2 = m^2$, and that the spacelike vector $k_\perp$ is orthogonal to both n and p. Inverting the relations (B.2), one finds

$$\begin{aligned} q &= z\,p - k_\perp + \frac{\mathbf{k}_\perp^2 + (1-z^2)m^2}{z\,(2pn)}\,n, \\ k &= (1-z)\,p + k_\perp + \frac{\mathbf{k}_\perp^2 - (1-z)^2 m^2}{(1-z)\,(2pn)}\,n, \end{aligned} \tag{B.3}$$

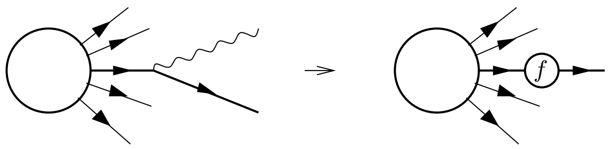

FIG. 3: A schematic picture of the factorization formula (B.5)

where $\mathbf{k}_\perp^2$ stands for $-k_\perp^\mu k_{\perp\mu}$ when expressed in a frame-independent way. In such a parametrization, propagator denominators that are responsible for collinear singularities appear as

$$\frac{1}{(q+k)^2-m^2} = \frac{1}{2qk} = \frac{z(1-z)}{\mathbf{k}_\perp^2+(1-z)^2m^2}. \qquad \text{(B.4)}$$

The transverse momentum $k_\perp$ parametrizes how far off-shell the radiating fermion is. In the massless case ($m=0$ and $\epsilon \neq 0$), the collinear limit is defined by $k_\perp \to 0$, which determines the phase-space region where the $1/\epsilon$ singularity arises. In the case of a massive fermion ($m \neq 0$ and $\epsilon = 0$), the quasi-collinear limit has to be considered [19]. In this limit, the collinear region is defined by taking simultaneously $\mathbf{k}_\perp^2 \to 0$ and $m \to 0$, but keeping the ratio $m^2/\mathbf{k}_\perp^2$ fixed. Both limits lead to the factorization formula [17, 20] illustrated in Fig. 3

$$|\overline{\mathcal{M}}(q,k;\ldots)|^2 \;\simeq\; \frac{Q_j^2}{2qk}\,\hat{P}(z)\,|\overline{\mathcal{M}}(p;\ldots)|^2\,, \qquad \text{(B.5)}$$

where $\mathcal{M}(p;\ldots)$ is the amplitude of the process without radiation where $p \to q/z$ in the collinear or quasi-collinear limits, and Q_j is the fermion charge. The splitting function $\hat{P}(z)$ in the collinear limit and in $D=4-2\epsilon$ dimensions reads (see Eqs. (53) and (54) of Ref. [17])

$$\hat{P}_\epsilon(z) = 8\pi\,\alpha_e\left[\frac{1+z^2}{1-z} - \epsilon\,(1-z)\right]. \qquad \text{(B.6)}$$

It becomes the Altarelli-Parisi splitting function [21] for the gluon emission off quark when $8\pi\alpha_e$ is replaced by C_F. Its extension to the massive quark case in $D=4$ dimensions is

$$\hat{P}_m(z) = 8\pi\,\alpha_e\left[\frac{1+z^2}{1-z} - \frac{m^2}{qk}\right]. \qquad \text{(B.7)}$$

For definiteness, let us assume that the number of final-state particles is as in Fig 1. In the collinear region, it is possible to disentangle the four-body phase space $d\Phi_4$ into a convolution of the three-body phase space of the non-radiative process, and the phase space corresponding to the radiation process alone [20, 22],

$$d\Phi_4 = d\Phi_3 \otimes d\Phi\,. \qquad \text{(B.8)}$$

One proceeds with integration over the low-$k_\perp$ region using the following phase-space measure [17]

$$\begin{aligned} d\Phi \;&=\; (\tilde{\mu})^{2\epsilon}\frac{d^dk}{(2\pi)^{d-1}}\delta_+(k^2) = \\ &=\; \frac{1}{16\pi^2}\frac{1}{\Gamma(1-\epsilon)}d\mathbf{k}_\perp^2\left(\frac{4\pi\tilde{\mu}^2}{\mathbf{k}_\perp^2}\right)^\epsilon\frac{dz}{z(1-z)}\Theta(z(1-z))\,. \end{aligned} \qquad \text{(B.9)}$$

The integration is performed from $\mathbf{k}_\perp^2 = 0$ up to $\mathbf{k}_\perp^2 = E^2$, where E is chosen to remain in the low-$k_\perp$ region, to preserve the factorization formula.

The splitting functions integrated over $\mathbf{k}_\perp^2$ read

$$\begin{aligned} f_\epsilon(z) \;&=\; \frac{\alpha_e}{\pi}\left[\frac{1+z^2}{1-z}\left(-\frac{1}{2\epsilon}+\ln\frac{E}{\mu}\right)+\frac{1-z}{2}\right], \\ f_m(z) \;&=\; \frac{\alpha_e}{\pi}\left[\frac{1+z^2}{1-z}\ln\frac{E}{(1-z)m} - \frac{z}{1-z}\right], \end{aligned} \qquad \text{(B.10)}$$

while their difference is

$$\begin{aligned} \Delta f(z) \;&\equiv\; f_m(z) - f_\epsilon(z) \\ &=\; \frac{\alpha_e}{2\pi}\,\frac{1+z^2}{1-z}\left[\frac{1}{\epsilon} - 1 - 2\ln\frac{(1-z)m}{\mu}\right]. \end{aligned} \qquad \text{(B.11)}$$

The dependence on E cancels in Eq. (B.11) because both splitting functions in Eq. (B.10) have been consistently derived in the corresponding regularizations, and they contain the same high-$k_\perp$ finite terms. The formula (B.11) could have also been obtained with the splitting functions from Ref. [23].

The mass-regulated and dimensionally regulated radiative decay widths with $E_\gamma > E_0$ satisfy the following relation

$$\begin{aligned} \Gamma_m \;&=\; \Gamma_\epsilon + \sum_j Q_j^2 \int d\widetilde{\Phi}_3 \int_0^1 dz\,\Delta f(z)\,|\overline{\mathcal{M}}(p;\ldots)|^2 \\ &\times\; \Theta\left[(1-z)\,p_j^0 - E_0\right]\,, \end{aligned} \qquad \text{(B.12)}$$

where the sum goes over all the radiating fermions. The three-body final-state phase-space measure $d\widetilde{\Phi}_3$ integrated over the "blind" angular variables reads [16]

$$d\widetilde{\Phi}_3 \equiv \tilde{\mu}^{4\epsilon}\int_\Omega d\Phi_3 = \tilde{\mu}^{4\epsilon}\,M_3\,ds_{12}\,ds_{13}\,ds_{23}\,, \qquad \text{(B.13)}$$

where M_3 has been given in Eq. (4.3).

Eq. (B.12) has been derived in the light-cone axial gauge but it is actually gauge-independent once charge conservation has been imposed. Thanks to this fact, we could have used it in our Feynman-gauge calculation. Actually, the conversion formula (4.4) is obtained from Eq. (B.12) just by differentiation with respect to E_0. Even after imposing charge conservation, the splitting functions derived in the same regularization may differ by finite terms which depend on the chosen gauge and also on the high-$k_\perp$ integration limit. Only the difference (B.11) of the two splitting functions is gauge- and convention-independent.

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