

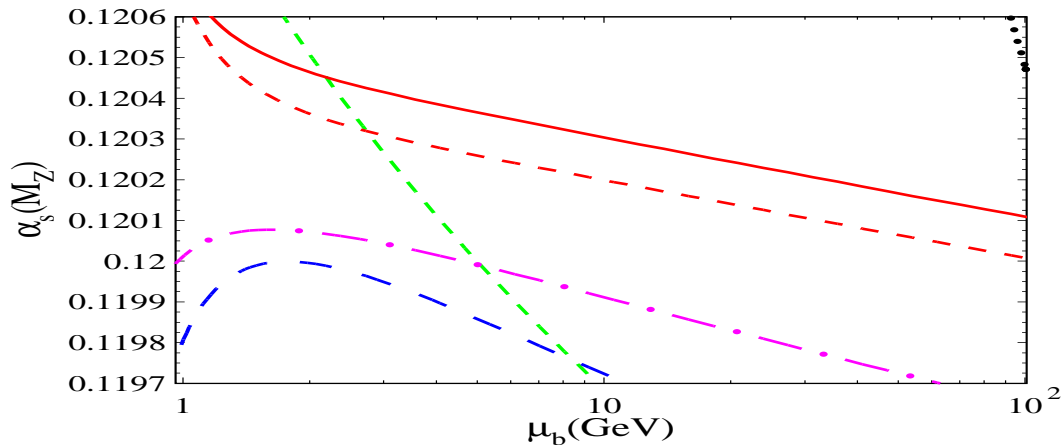
# Running and decoupling of $\alpha_s$

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The  $\beta$  function governs the renormalization scale dependence of the strong coupling and is known to four-loop accuracy since almost 14 years [1][2]. There are different ways to solve the corresponding differential equation. The preferred method is the numerical solution with truncation of  $\beta(\alpha_s)$  at the desired order. There are also several approximate (analytical) expressions, e.g., the one based on the iterative (perturbative) solution where the result for  $\alpha_s(\mu)$  is given as an expansion in  $1/L = 1/\ln(\mu^2/\Lambda^2)$  [3]. This formula should be used with care, in particular for small renormalization scales  $\mu$ . If one considers, e.g.,  $\mu = M_\tau$  one observes a shift of +0.004 after including the four-loop corrections and negative shift of approximately the same order of magnitude at five-loop level. These numbers have to be compared with the current experimental precision which is cited as  $\pm 0.005$  in Ref. [4] (see also the other contributions on  $\alpha_s$  from  $\tau$  decays in these proceedings).

Next to the running itself also the decoupling of heavy quarks from the running of the strong coupling constant is a crucial ingredient of the precision determination of  $\alpha_s$ . Every time a heavy quark threshold is crossed one has to apply the decoupling constants which relate  $\alpha_s$  with  $n_f$  active quark flavours, usually denoted by  $\alpha_s^{(n_f)}$ , to the coupling with only  $n_f - 1$  active quark flavours. The decoupling constants are obtained by matching  $n_f$ -flavour QCD to the effective theory with the number of quarks equal to  $n_f - 1$ . The theoretical framework for the calculation of the decoupling constants has been set up in Ref. [5] where formulae are given relating  $l$ -loop corrections to  $l$ -loop vacuum integrals.



As a consequence of the decoupling relations  $\alpha_s(\mu)$  is not a continuous function of  $\mu$  but has finite steps at the energy scale where the heavy quark is integrated out,  $\mu_{\text{dec}}$ . This energy is not fixed by theory, should, however, be in the vicinity of the heavy quark mass. On general grounds the dependence on  $\mu_{\text{dec}}$  should become weaker if higher order perturbative corrections are included in the analysis. This is demonstrated in the figure above where  $\alpha_s^{(5)}(M_Z)$  is computed using  $\alpha_s^{(3)}(M_\tau)$  as a starting point. The decoupling of the charm

quark is performed at the fixed scale  $\mu_c = 3$  GeV and the decoupling scale of the the bottom quark  $\mu_b$  is varied in the broad range between 1 GeV and 100 GeV.  $N$ -loop running goes along with  $(N - 1)$ -loop decoupling. Results are shown for  $N = 1$  (upper right dotted line),  $N = 2$  (steep dashed line),  $N = 3$  (lower dashed line) and  $N = 4$  (dash-dotted line). One observes a dramatic reduction of the  $\mu_b$  dependence with increasing  $N$  resulting in a quite flat four-loop result (Note that the scale on the ordinate only varies by 0.0009.).

For comparison we show in the figure two more curves which correspond to  $N = 5$ . They incorporate the four-loop decoupling relations [6][7]. For the unknown five-loop coefficient of the  $\beta$  function we have chosen  $\beta_4 = 0$  (solid line) and  $\beta_4 = 150$  (dashed line parallel to the solid one; the normalization corresponding to  $\{\beta_0, \beta_1, \beta_2, \beta_3\} \approx \{1.92, 2.42, 2.83, 18.85\}$  has been chosen).

From the figure above it is possible to estimate an uncertainty on  $\alpha_s^{(5)}(M_Z)$  as obtained from  $\alpha_s^{(3)}(M_\tau)$  due to missing higher order corrections. If we restrict ourselves to a range of  $\mu_b$  between 2 GeV and 10 GeV and take the difference between the three- and four-loop curve as an estimate for the uncertainty we obtain  $\delta\alpha_s^{(5)}(M_Z) \approx 0.0002$ . The difference between the four- and five-loop (dashed) curve would even lead to  $\delta\alpha_s^{(5)}(M_Z) \approx 0.0003$ . The variation of  $\alpha_s^{(5)}(M_Z)$  due to the variation of  $\mu_b$  leads to an additional uncertainty of  $\delta\alpha_s^{(5)}(M_Z) \approx 0.0002$ . A similar uncertainty is obtained from the variation of  $\mu_c$  between 2 GeV and 5 GeV. (This can easily be checked with the program `RunDec` [8].) Thus a total uncertainty of  $\pm 0.0004$  (obtained by adding the three uncertainties in quadrature) should be assigned to  $\alpha_s^{(5)}(M_Z)$ . The uncertainties induced by the errors in the quark masses are much smaller.

## References

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