

# Updated constraints from radiative $\Upsilon$ decays on a light CP-odd Higgs

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## Abstract

The possible existence of a light CP-odd Higgs state in many new-physics models could lead to observable effects in the bottomonium sector. Experimental bounds on radiative  $\Upsilon$  decays through such a pseudoscalar state and possible mixings with the  $\eta_b^0$  states are reviewed. Combining these two effects, we set constraints on the properties of the CP-odd Higgs in the limit of small photon energy of  $BR(\Upsilon \rightarrow \gamma \tau^+ \tau^-)$ , that is on the pseudoscalar mass-range  $\sim 8 - 10$  GeV.

## 1 Introduction

The bottomonium sector is well-known as a possibly sensitive probe of light CP-odd Higgs states  $A^0$  [1–7]. Such pseudoscalar states have indeed vanishing  $V - V - A^0$  ( $V = Z, W$ ) couplings, which allows them to circumvent most of the direct collider constraints (contrary to CP-even states). Several new-physics models provide a natural framework to embed these particles: MSSM with CP-violating Higgs sector [8, 9],  $U(1)$ -extensions of the MSSM [10], Little-Higgs models, (non-supersymmetric) Two-Higgs Doublet models [11], etc.

As an illustrative case, we consider the example of the NMSSM (see [12] for a review), where a light CP-odd Higgs can emerge (*e.g.*) naturally from an approximate and spontaneously broken R or Peccei-Quinn symmetry. A phenomenological possibility in this model is the so-called “ideal Higgs scenario”, where the lightest CP-even Higgs  $h_1$  decays unconventionally into a pair of light CP-odd Higgs states with masses below the  $B - \bar{B}$  threshold [13–20] so as to avoid LEP bounds on  $e^+e^- \rightarrow Z + b\bar{b}$  (since  $A^0 \rightarrow B\bar{B}$  is kinematically forbidden). This mechanism allows for comparatively light CP-even Higgs states  $\gtrsim 86$  GeV and can also lead to a successful interpretation of the  $2.3\sigma$  excess observed in  $e^+e^- \rightarrow Z + b\bar{b}$  for  $M_{b\bar{b}} \sim 98$  GeV [16]. The recent analysis of  $e^+e^- \rightarrow Z + 4\tau$  in ALEPH data [21] constrains this scenario further but, according to [22], these bounds might also be circumvented by allowing for increased  $A^0 \rightarrow c\bar{c}$  decays; it is otherwise possible to simply increase the singlet component of the lightest CP-even state, which has reduced couplings to the SM-sector, *e.g.* to the  $Z$  boson. Note also that the presence of a light CP-odd state in the NMSSM spectrum is not necessarily associated with this “ideal Higgs scenario” and may as well accompany a heavier CP-even ( $h_1$ ) state (up to  $m_{h_1} \sim 140$  GeV in the NMSSM). Investigating the existence of such an  $A^0$  in low-energy signals is thus a necessary probe, complementary to Higgs searches at high energy. In this context, the bottomonium sector is of particular interest since light CP-odd Higgs states could be produced in  $\Upsilon$  decays or mix with  $\eta_b$ -states, provided their coupling to  $b$ -quarks is sufficiently large [23, 24].

Radiative  $\Upsilon$  decays through a CP-odd Higgs state are constrained by several experimental limits, the most recent ones originating from  $\Upsilon(1S) \rightarrow \gamma A^0$ ,  $A^0 \rightarrow l^+l^-$  searches at CLEO III [25] and  $\Upsilon(3S) \rightarrow \gamma A^0$ ,  $A^0 \rightarrow l^+l^-$  searches at BABAR [26] ( $l = \mu, \tau$ ). These experimental limits on a light NMSSM CP-odd Higgs have already been studied, in [5, 23] for the CLEO III bounds and [22] for the BABAR bounds, and were

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shown to constrain most of the significantly coupled region  $X_d \gtrsim 0.5 - 1$  ( $X_d$  being the reduced coupling of the  $A^0$  to down-type quarks).

Such constraints should however be considered with caution in the region  $m_A \gtrsim 8$  GeV, where  $BR(\Upsilon \rightarrow \gamma A^0)$  is not well controlled theoretically and could suffer from large corrections. Moreover, possibly relevant mixing effects with  $\eta_b$  (or  $\chi_0$ ) states could complicate the situation further [1, 6, 24]. Such limitations leave all the mass region  $m_A \sim 8 - 10.5$  GeV essentially unconstrained (or unreliably constrained) by these previous analyses and, in particular, the higher reach in mass of the BABAR bounds ( $m_{\tau\tau} \leq 10.10$  GeV) would seem impossible to exploit.

Alternative searches for a light CP-odd Higgs signal through a breakdown of lepton universality in inclusive leptonic  $\Upsilon$  decays have likewise failed so far [27]. Yet, for the same reasons as above, few conclusions can be drawn on the mass range  $m_A \gtrsim 8$  GeV.

The mixing between the  $A^0$  and  $\eta_b$  states was also studied and, in particular, it was suggested in [24] that this effect be responsible for a displacement of the observed  $\eta_b(1S)$  mass, which (depending on the QCD-based model) is generically lower than what was expected for the hyperfine splitting  $m_{\Upsilon(1S)} - m_{\eta_b(1S)}$  [28, 29].

The purpose of this paper consists in investigating the impact of the mixing effect on radiative  $\Upsilon$  decays with small photon energy, that is in the CP-odd mass-range  $m_A \sim 8 - 10$  GeV. We show that even when the direct decay  $\Upsilon(1S/3S) \rightarrow \gamma A^0$ , which had been considered in [5, 22, 23], is neglected, the additional and indirect contributions  $\Upsilon(1S/3S) \rightarrow \gamma(\eta_b^0 \rightsquigarrow A^0)$  resulting from the mixing effect, lead to severe limits on  $X_d$  when compared to the experimental bounds from CLEO and BABAR. Given the large uncertainty on hadronic parameters and the limited validity of our approximations, such limits should however be considered cautiously and from a qualitative, rather than quantitative, viewpoint. In the following section, we summarize the status of  $BR(\Upsilon \rightarrow \gamma A^0 (\rightarrow l^+ l^-))$  in view of the CLEO and BABAR limits, in the approximation of a pure  $A^0$  Higgs state. We will also present briefly the mixing effect between the  $A^0$  and the  $\eta_b$  states. In the third section, the consequences for  $BR(\Upsilon \rightarrow \gamma \tau^+ \tau^-)$  will be analysed in the limit where  $m_A \sim m_\Upsilon$ , under the assumption of negligible direct decay  $\Upsilon \rightarrow \gamma A^0$ . The numerical bounds on  $X_d$  will be discussed in the last section before a short conclusion.

In the following, we will assume the existence of a NMSSM-like CP-odd Higgs state with mass  $m_A \lesssim 10.5$  GeV and coupling to down-type quarks and leptons of the form  $\frac{m_f X_d}{\sqrt{2}v}$  (where  $m_f$  is the fermion mass and  $v = (2\sqrt{2}G_F)^{-1/2}$  is the electroweak vacuum expectation value), while the coupling to up-type quarks is given by  $\frac{m_f X_d}{\sqrt{2}v \tan^2 \beta}$ . In the NMSSM,  $X_d = \cos \theta_A \tan \beta$ , where  $\cos \theta_A$  quantifies the amount of doublet-component in  $A^0$  while  $\tan \beta$  is the usual ratio of the doublet vacuum expectation values. Significant corrections to the Yukawa couplings are known to develop at large  $\tan \beta$ , due to loops of supersymmetric particles (see *e.g.* [30]). They would alter the relation  $X_d = \cos \theta_A \tan \beta$  somewhat. Such effects will be neglected here, but we will see that constraints from the bottomonium sector can become relevant already at moderate  $\tan \beta$ .

Moreover  $\tan \beta > 1$ , so that the decay  $A^0 \rightarrow \tau^+ \tau^-$  is dominant (if kinematically allowed). In fact, the dominant branching ratios are essentially independent from  $\tan \beta$ , once  $\tan \beta \gtrsim 2 - 3$  [22]. The results that we present in the following will assume the branching ratios at  $\tan \beta = 5$  (about 0.9 to 0.75 for  $BR(A^0 \rightarrow \tau^+ \tau^-)$  in the range  $m_A = 8 - 10$  GeV; however,  $BR(A^0 \rightarrow \tau^+ \tau^-) \sim 1$  would have been a qualitatively acceptable approximation). Note that for  $\tan \beta \lesssim 2$ , the coupling of the  $A^0$  to  $b$ -quarks is reduced and would generate little effect on the bottomonium sector (except for  $\cos \theta_A \sim 1$ ). In the general case, the full  $(m_A, \tan \beta)$  dependence can be retained. The branching ratio  $BR(A^0 \rightarrow \tau^+ \tau^-)$  (also  $BR(A^0 \rightarrow \mu^+ \mu^-)$ ) that we use is obtained with the public code NMSSMTools [31]. It was noted that the corresponding value can be slightly different from the output of Hdecay [22]. However, such minor effects will have little impact on the discussion that follows. Note also that the very-light mass region  $m_A \lesssim 5$  GeV is already essentially excluded by low-energy constraints [32], such as  $B_s \rightarrow \mu^+ \mu^-$  or  $B \rightarrow X_s \mu^+ \mu^-$  [33].

Beyond the NMSSM, this analysis should be essentially valid for any model with a light CP-odd state and based on a Two-Higgs Doublet model of type II (or even of type I), provided  $X_d$  is suitably chosen.

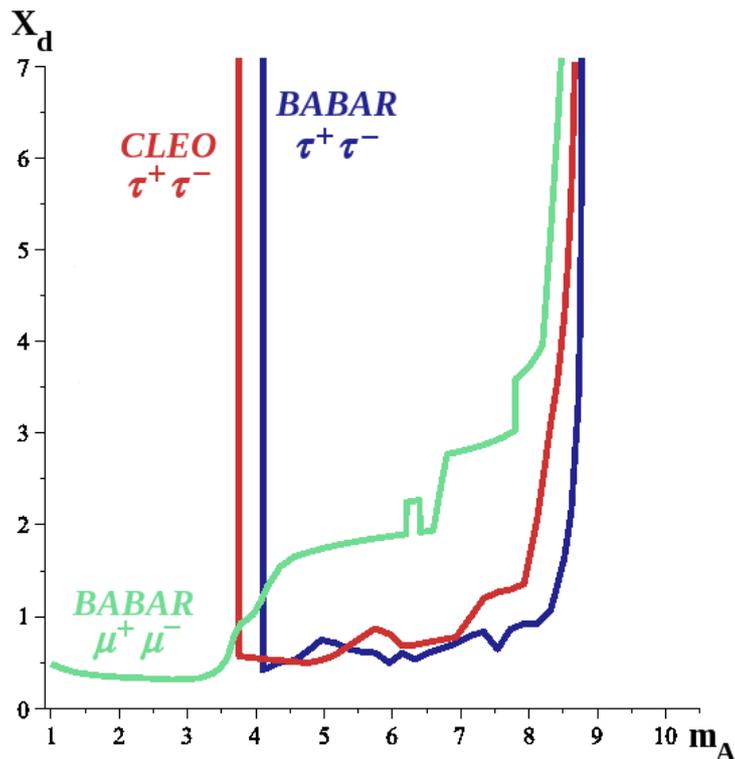


Figure 1: Constraints from  $BR(\Upsilon(nS) \rightarrow \gamma(A^0 \rightarrow \tau^+\tau^-/\mu^+\mu^-))$  on the mass / coupling plane of the  $A^0$  state. The curves correspond to upper bounds on  $X_d$  resulting from the experimental limits. The red, dark- and light-blue curves correspond, respectively, to CLEO limits on  $\Upsilon(1S) \rightarrow \gamma(A^0 \rightarrow \tau^+\tau^-)$ , BABAR limits on  $\Upsilon(3S) \rightarrow \gamma(A^0 \rightarrow \tau^+\tau^-)$  and BABAR limits on  $\Upsilon(3S) \rightarrow \gamma(A^0 \rightarrow \mu^+\mu^-)$ . The correction factor  $F$  of eq. (1) was assumed to be that shown in [23] and vanishes for  $m_A \gtrsim 8.8$  GeV.

## 2 Direct bounds on $BR(\Upsilon \rightarrow \gamma A^0)$ and Mixing $A^0 - \eta_b$

In first approximation, Upsilon decays through a CP-odd Higgs are described by the so-called Wilczek formula [34]:

$$\frac{BR(\Upsilon(nS) \rightarrow \gamma A^0)}{BR(\Upsilon(nS) \rightarrow \mu^+\mu^-)} = \frac{G_F m_b^2 X_d^2}{\sqrt{2}\pi\alpha} \left(1 - \frac{m_A^2}{m_{\Upsilon(nS)}^2}\right) \times F \quad (1)$$

This formula was first established in the non-relativistic approximation and the hard-photon limit, *i.e.*  $m_A \ll m_\Upsilon$ . Several corrections, due to relativistic, QCD and bound-state effects, were then included within the factor  $F$  (see [35] for a summary). Yet our control over this factor fails for  $m_A \gtrsim 8$  GeV, in the limit of low-energy photons, where large corrections from *e.g.* bound-state or relativistic effects are expected. Within the known approximate computations, the correction factor  $F$  vanishes for  $m_A \gtrsim 8.8$  GeV (depending on the  $b$ -mass), hence leaving this mass range apparently unconstrained [23]. One can indeed expect the  $A^0$  contribution to radiative  $\Upsilon$  decays to vanish in this limit, since, for large wavelengths, the photon simply probes an overall neutral state. Yet, the cancellation for  $m_A \gtrsim 8.8$  GeV overestimates probably this feature. In fact, we will show in the following sections that the mixing effect, for  $m_A \sim m_{\eta_b^0}$ , leads to sizable contributions, which are not captured by eq. (1).

Comparing eq. (1) with the experimental limits from CLEO [25] and BABAR [26] on  $BR(\Upsilon(1S/3S) \rightarrow \gamma(A^0 \rightarrow \mu^+\mu^-/\tau^+\tau^-))$ , one obtains upper bounds on  $X_d$  as a function of  $m_A$  [5, 22, 23], as shown in fig. 1. For the mass region beneath  $m_A \sim 8.8$  GeV, only moderate to small values of  $X_d \lesssim 0.5$  are allowed. Note

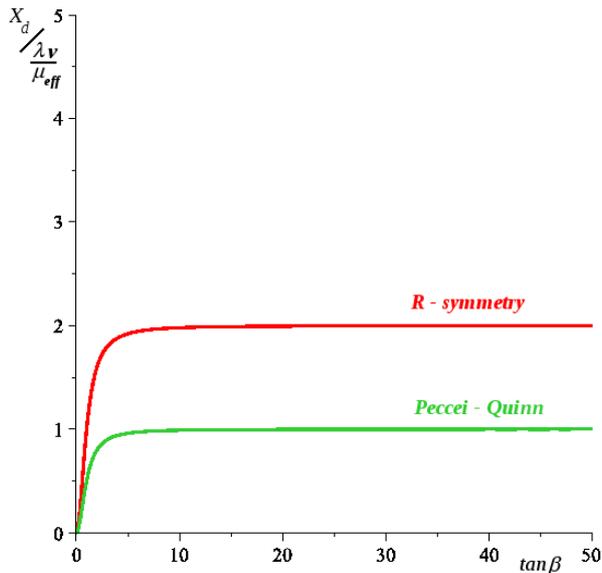


Figure 2: Reduced coupling of the light CP-odd Higgs,  $X_d \equiv \cos \theta_A \tan \beta$ , as a function of  $\tan \beta$ , in units of  $\frac{\lambda v}{\mu_{\text{eff}}}$ , for the R (red curve) and PQ (green) limits.  $\lambda$  and  $\mu_{\text{eff}}$  are NMSSM parameters [12], with  $v \sim 175$  GeV,  $\lambda \leq 0.8$  and  $\mu_{\text{eff}} \geq 100$  GeV. We thus obtain that  $X_d$  is generically small in these limits.

that this region of weak coupling is however that which is usually favoured from the theoretical point of view: the R and Peccei-Quinn symmetry limits of the NMSSM would naturally predict small  $X_d$  (see fig. 2). On the other hand, such weakly-coupled  $A^0$  will prove difficult to observe experimentally.

As we discussed in the introduction, the mass range  $m_A \sim 9 - 10.5$  GeV is (due to the smallness of the factor  $F$  in eq. (1), within the known computations) free of the latter constraints, so that large values of  $X_d$  seem *a priori* allowed. In this range however, one expects to find the  $\eta_b(nS)$ ,  $n = 1, 2, 3$ , states. Such states carry the same quantum numbers as the CP-odd Higgs, so that a mixing  $A^0 - \eta_b$  develops. This scenario was studied in [1, 6, 24]. We shall use the notations of [24]. The mixing can be described by an effective mass-matrix for the  $\eta_b - A^0$  states:

$$\mathcal{M}^2 = \begin{pmatrix} m_{\eta_b^0(1S)}^2 & 0 & 0 & \delta m_1^2 \\ 0 & m_{\eta_b^0(2S)}^2 & 0 & \delta m_2^2 \\ 0 & 0 & m_{\eta_b^0(3S)}^2 & \delta m_3^2 \\ \delta m_1^2 & \delta m_2^2 & \delta m_3^2 & m_A^2 \end{pmatrix} \quad (2)$$

The diagonal elements  $m_{\eta_b^0(nS)}^2$ ,  $n = 1, 2, 3$ , correspond to the masses of the pure QCD  $b\bar{b}$  states which can be estimated through the hyperfine-splitting  $m_{\Upsilon(nS)}^2 - m_{\eta_b^0(nS)}^2$  [28]:

$$m_{\eta_b^0(1S)} \simeq 9418 \pm 13 \text{ MeV} \quad , \quad m_{\eta_b^0(2S)} \simeq 10002 \text{ MeV} \quad , \quad m_{\eta_b^0(3S)} \simeq 10343 \text{ MeV} \quad (3)$$

The quoted value for  $m_{\eta_b^0(1S)}$  is the perturbative QCD (pQCD) prediction. Slightly different predictions can be obtained in other approaches [29]. The uncertainty on the heavier  $\eta_b^0$  masses will be neglected. The off-diagonal terms  $\delta m_n^2$  can be computed in a non-relativistic quark-potential model [1, 6, 23]:

$$\delta m_n^2 = \left( \frac{3m_{\eta_b(nS)}^3}{8\pi v^2} \right)^{1/2} |R_{\eta_b(nS)}(0)| \times X_d \quad \Rightarrow \quad \begin{cases} \delta m_1^2 \simeq (0.14 \pm 10\%) \text{ GeV}^2 \times X_d, \\ \delta m_2^2 \simeq (0.11 \pm 10\%) \text{ GeV}^2 \times X_d, \\ \delta m_3^2 \simeq (0.10 \pm 10\%) \text{ GeV}^2 \times X_d \end{cases} \quad (4)$$

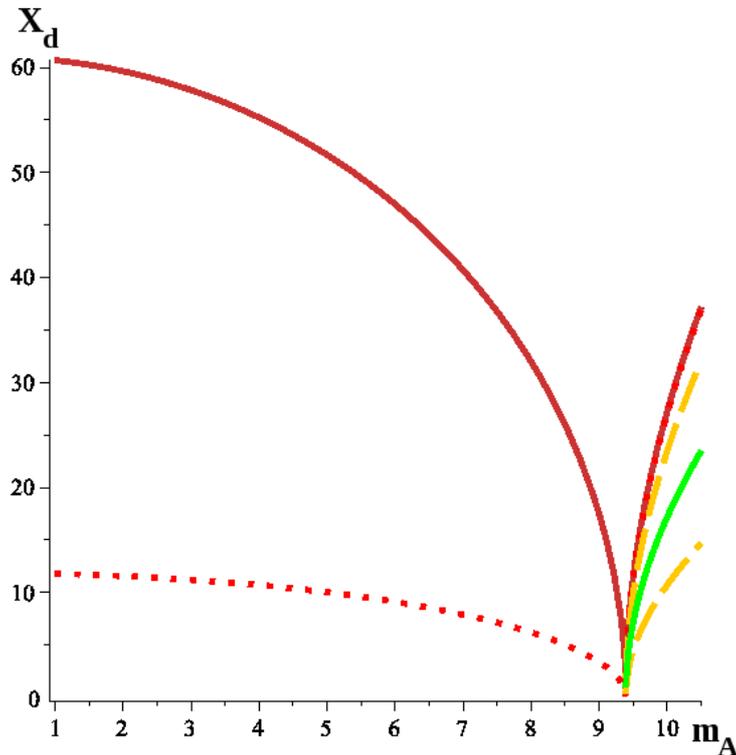


Figure 3: Relations and constraints in the plane  $(m_A, X_d)$  originating from the splitting effect in the pseudoscalar mass matrix and the observed  $\eta_b$  mass. The red curves represent upper bounds on  $X_d$ : the full line corresponds to the conservative limits  $-30 \text{ MeV} \leq m_{\eta_b^0(1S)} - m_{\text{obs}} \leq 54 \text{ MeV}$ ; the dotted one corresponds to the pQCD  $2\sigma$  range. In the domain between the orange dashed curves, the splitting between the observed  $\eta_b$  mass and the pQCD prediction is generated by the mixing effect within  $1\sigma$ . The green curve is the “favoured” line obtained with the central value of the pQCD prediction [24].

where the  $\eta_b^0(nS)$  wave-functions  $R_{\eta_b(nS)}$  were estimated through those of the  $\Upsilon(nS)$  states, which can be extracted, in turn, from the leptonic decays  $\Upsilon(nS) \rightarrow l^+l^-$  [36]. The other off-diagonal entries vanish as a result of the orthogonality between  $\eta_b^0$  states. The physical mass-states are denoted as:

$$\eta_i = P_{i1} \eta_b^0(1S) + P_{i2} \eta_b^0(2S) + P_{i3} \eta_b^0(3S) + P_{i4} A^0 \quad (5)$$

The only experimental information at our disposal concerning the mass matrix of eq. (2) originates from the observation at BABAR, in  $\Upsilon(3S)$  [37] and  $\Upsilon(2S)$  [38] decays, of a state with mass  $m_{\text{obs}} = 9390.9 \pm 3.1 \text{ MeV}$  [38] which was interpreted as the  $\eta_b^0(1S)$ . It is remarkable that this value for the mass is in slight tension with the pQCD prediction (eq. (3)). Within most of the QCD models, one expects a slightly smaller hyperfine splitting [29] (although possibly within  $1\sigma$ ). As a consequence, the mixing effect with the  $A^0$  was suggested as a possible interpretation of a displaced  $\eta_b^0(1S)$  mass [24]. The requirement that  $m_{\text{obs}}^2$  be an eigenvalue of the mass matrix (2) leads to the relation:

$$m_A^2 = m_{\text{obs}}^2 + \frac{\delta m_1^4}{m_{\eta_b^0(1S)}^2 - m_{\text{obs}}^2} + \frac{\delta m_2^4}{m_{\eta_b^0(2S)}^2 - m_{\text{obs}}^2} + \frac{\delta m_3^4}{m_{\eta_b^0(3S)}^2 - m_{\text{obs}}^2} \quad (6)$$

For given QCD-predicted  $\eta_b$  masses (see eq. (3)) and the mixing elements of eq. (4), eq. (6) determines  $m_A$  in terms of  $X_d$ . Varying the input from eq. (3) and (4) within the error bars, this relation results in bounds on the plane  $(m_A, X_d)$  [23]. These limits originate simply from the observation that, in the presence of a mixing,



Figure 4: Contributions to the radiative Upsilon decay to a Higgs pseudoscalar.

a) Direct decay, corresponding to the description of eq. (1).

b) Additional mixing contribution due to the  $A^0 - \eta_b$  mixing.

a mass-splitting is generated so that the observed mass cannot coincide with a diagonal entry of the mass matrix (to which it couples). Assuming the pQCD result to be valid, one can define a favoured region in the plane  $(m_A, X_d)$  where the observed mass can be reproduced within  $1\sigma$ . This is illustrated in fig. 3. The  $2\sigma$  limits are also shown. However, given the larger number of predictions in alternative QCD-models, we retain, in what follows, the bounds resulting from the more conservative range  $-30 \text{ MeV} \leq m_{\eta_b^0(1S)} - m_{\text{obs}} \leq 54 \text{ MeV}$ .

### 3 Radiative $\Upsilon$ decays in the mixing scenario

$BR(\Upsilon(nS) \rightarrow \gamma A^0)$ , as we presented it in the previous section, corresponds to the diagram of fig. 4a. In the presence of a  $A^0 - \eta_b$  mixing, however, a second contribution, as in fig. 4b, arises. This contribution was already described in [6] under the denomination “resonant decay”. Assuming that the contribution of fig. 4a vanishes for small  $A^0 - \Upsilon$  mass differences, as we discussed in section 2, the diagram of fig. 4b is then dominant and could lead to a significant effect. In this section, we will explicitly neglect this diagram of fig. 4a when  $m_A \gtrsim 8 \text{ GeV}$  and focus exclusively on the contribution b due to mixing.

Let us first consider the transition between pure bottomonium states. In the non-relativistic approach to quarkonium bound states,  $BR(\Upsilon(nS) \rightarrow \gamma \eta_b^0(jS))$  can be written as [39]:

$$BR(\Upsilon(nS) \rightarrow \gamma \eta_b^0(jS)) = \frac{4 \alpha e_b^2}{3 m_b^2 \Gamma_{\Upsilon(nS)}} |I_{nj}|^2 k^3 \cdot \Theta(k) \quad (7)$$

where  $\alpha = (137.036)^{-1}$  is the fine-structure constant,  $e_b = -1/3$ , the  $b$ -quark charge,  $k$ , the photon energy and  $\Gamma_{\Upsilon(nS)}$ , the  $\Upsilon(nS)$  total width ( $\Theta$  is the Heaviside distribution). We shall use  $\Gamma_{\Upsilon(1S)} = (54.02 \pm 1.25) \text{ keV}$  and  $\Gamma_{\Upsilon(3S)} = (20.32 \pm 1.85) \text{ keV}$  [36]. The transition factor  $I_{nj}$  is defined as:

$$I_{nj} = \langle \eta_b^0(jS) | j_0(kr/2) | \Upsilon(nS) \rangle \quad (8)$$

where  $j_0(x) = \frac{\sin x}{x}$ . Taking  $r_0 \sim 1.4 \text{ GeV}^{-1}$  as a typical scale of confinement, we see that this function can be safely expanded in the limit of low-energy photons as  $j_0(kr/2) \simeq 1 - (kr/2)^2/6 + \dots$ . Using that the wave functions for the  $\eta_b^0$  and  $\Upsilon$  states are almost identical, one obtains that in first approximation  $I_{nj} \simeq \delta_{nj}$ :  $nS \rightarrow nS$  transitions are *a priori* favoured. The resulting  $k^3$  dependence in eq. (7) is then characteristic of point-like interactions of the bottomonium states, which could be anticipated for large wavelengths.

Note however that for QCD  $b\bar{b}$  states, the photon energies are much smaller for a  $nS \rightarrow nS$  transition than for a  $3S \rightarrow 1S/2S$  transition, resulting in comparable branching ratios. The coefficients  $I_{nj}$ ,  $n \neq j$ , were estimated in QCD (see *e.g.* [39]), taking into account relativistic corrections, but such predictions range over one to two orders of magnitude. Fortunately, this situation will be slightly changed in the case of mixed states (see eq. (10) below) and it will be possible to focus on the dominant  $nS \rightarrow nS$  channel. For the time being, let us use that  $I_{nj} \simeq \delta_{nj}$  and that photon energies are small, and parametrize:

$$I_{nj} \simeq \delta_{nj} - k^2 \cdot R_{nj}^2 \quad (9)$$

So far the  $R_{nj}^2$  coefficients are simply unknowns, which might be extracted from the QCD computations of the  $I_{nj}$ . Under the assumption that the  $nS \rightarrow nS$  transition dominates, it will be possible to neglect the  $R_{nj}^2$ ,  $n \neq j$ .

In fact, for large mixing (that is, for  $m_A$  in the vicinity of the  $\eta_b^0$  masses), processes involving the CP-odd Higgs or the  $\eta_b^0$  states are more adequately described in the context of the mixing formalism which we have presented in section 2. In particular, the production of any mass state  $\eta_i$ ,  $i = 1 - 4$ , in radiative  $\Upsilon$  decays may be obtained (under the assumption that the contribution of fig. 4a is negligible) by a generalization of eq. (7) as:

$$BR(\Upsilon(nS) \rightarrow \gamma\eta_i) = \frac{4 \alpha e_b^2}{3 m_b^2 \Gamma_{\Upsilon(nS)}} k_i^3 \cdot \Theta(k_i) \sum_{j,l=1}^3 I_{nj}^* I_{nl} P_{ij} P_{il} \quad (10)$$

Note that the photon energy is  $k_i \equiv \frac{m_{\Upsilon(nS)}^2 - m_{\eta_i}^2}{2m_{\Upsilon(nS)}}$  for all the terms on the right-hand side of eq. (10), including pure  $nS \rightarrow nS$  transitions as well as  $nS \rightarrow \tilde{n}S$ ,  $n \neq \tilde{n}$ , transitions or interference terms. Under such conditions, the  $nS \rightarrow nS$  transition dominates eq. (10) due to its larger transition factor. In the following, we will neglect the interference terms in eq. (10): such terms cannot be large unless the  $\eta_i$  has simultaneously large  $\eta_b^0(jS)$  and  $\eta_b^0(lS)$ ,  $j \neq l$ , components, which (almost) never happens since the  $\eta_b^0$  states are orthogonal; moreover, both transition factors  $I_{nj}$ ,  $I_{nl}$ ,  $j \neq l$ , would have to be large, which contradicts eq. (9) for small photon energy. Eq. (10) thus reduces to:

$$BR(\Upsilon(nS) \rightarrow \gamma\eta_i) \simeq \sum_{j=1}^3 P_{ij}^2 BR_i(\Upsilon(nS) \rightarrow \gamma\eta_b^0(jS)) \quad (11)$$

where  $BR_i(\Upsilon(nS) \rightarrow \gamma\eta_b^0(jS))$  is given by eq. (7), while the index  $i$  reminds us however that the relevant mass for the  $\eta^0$  state is that of the mass state  $\eta_i$ . Note that the simplification from eq. (10) to eq. (11) will work well due to the dominant  $nS \rightarrow nS$  channel. This also implies, however, that we restrict ourselves to small photon energies ( $k_i$ ), so that  $I_{nj} \simeq \delta_{nj}$  remains a good approximation: therefore, we will require in the following that  $k_i \lesssim 1$  GeV.

Then, the decay of the mass state  $\eta_i$  to a tauonic pair can be assumed to proceed through the  $A^0$  component [24]:

$$BR(\eta_i \rightarrow \tau^+ \tau^-) = \frac{P_{i4}^2 \Gamma_i(A^0 \rightarrow \tau^+ \tau^-)}{\sum_{j=1}^3 P_{ij}^2 \Gamma_{\eta_b^0(jS)} + P_{i4}^2 \Gamma_{A^0}} \quad (12)$$

where the widths  $\Gamma_{\eta_b^0(1,2,3S)}$  can be estimated to 10, 5 and 5 MeV respectively<sup>1</sup>.  $\Gamma_{A^0}$  and  $\Gamma_i(A^0 \rightarrow \tau^+ \tau^-)$  can be computed. Again, the index  $i$  indicates that the partial width should be calculated using the mass of the state  $\eta_i$  instead of that of the diagonal  $A^0$ . The  $BR(\eta_i \rightarrow \tau^+ \tau^-)$  has already been proposed as a probe for light pseudoscalars in [41].

We can eventually write the radiative  $\Upsilon$  decay to a tauonic pair as:

$$BR(\Upsilon(nS) \rightarrow \gamma \tau^+ \tau^-) \simeq \sum_{i=1}^4 BR(\Upsilon(nS) \rightarrow \gamma\eta_i) \cdot BR(\eta_i \rightarrow \tau^+ \tau^-) \quad (13)$$

where  $BR(\Upsilon(nS) \rightarrow \gamma\eta_i)$  and  $BR(\eta_i \rightarrow \tau^+ \tau^-)$  are given respectively by eq. (11) and (12). We stress that, in this analysis, the main contribution to eq. (13) is associated with the subprocess:

$$\Upsilon(nS) \rightarrow \gamma ( \eta_b^0(nS) \rightsquigarrow A^0 \rightarrow \tau^+ \tau^- ) \quad (14)$$

the  $nS \rightarrow nS$  transition being favoured by its large transition factor while the mixing effect allows it to access photon energies comparable to those of the other transitions.

At this point, neglecting the direct contribution  $\Upsilon \rightarrow \gamma A^0$  of fig. 4a was the most important assumption. If the factor  $F$  of eq. (1) is indeed small in the considered mass-range ( $m_A \sim 8 - 10$  GeV), as its cancellation in known computations seems to indicate, we may infer that the amplitude of fig. 4a remains small, so that

<sup>1</sup>After  $\Gamma_{\eta_b^0(nS)}/\Gamma_{\eta_c^0(nS)} \simeq (m_b/m_c)[\alpha_S(m_b)/\alpha_S(m_c)]^5 \simeq 0.25 - 0.75$  [40].

our assumption in neglecting the direct contribution is justified. In practice, the limits on  $X_d$  that we obtain (in section 4) using the indirect decay (fig. 4b) only, are comparable in the mass range  $m_A = 8 - 10$  GeV to those that eq. (1) alone would have given with  $F \sim 0.5 - 1$  (for the BABAR bounds) or  $F \sim 1$  (for the CLEO bounds): therefore, we conclude that a factor  $F \lesssim 0.1$  should be a sufficient condition to ensure that the contribution of fig. 4b captures the dominant effect, hence that our approach is valid. Note that, among the neglected terms, the non-interferent contribution given by eq. (1) could be directly included in eq. (11) (if  $F$  were satisfactorily known) and, being positive, would only strengthen the bounds that we obtain in section 4. However, if the amplitude of fig. 4a were non-negligible, the interference between the diagrams of fig. 4a and 4b could be significant: since its sign is not controlled, this would spoil the validity of our analysis.

A remark concerning the choice of the diagonal  $\eta_b^0(1S)$  mass is necessary. In the presence of the mixing structure of eq. (2) and knowing one of the eigenvalues  $m_{\text{obs}}$ , one cannot choose  $m_{\eta_b^0(1S)}$ ,  $m_A$  and  $X_d$  independently (or  $m_{\text{obs}}$  will not be recovered in the general case). Therefore, we will determine  $m_{\eta_b^0(1S)}$  in terms of  $m_A$ ,  $X_d$  and  $m_{\text{obs}}$  through eq. (6). Should  $m_{\eta_b^0(1S)}$  reach unrealistic values (that is, if the condition  $-30 \text{ MeV} \leq m_{\eta_b^0(1S)} - m_{\text{obs}} \leq 54 \text{ MeV}$  is violated), the corresponding points will be excluded through the mixing constraint (see end of section 2 and fig. 3).

Finally, the advantage of considering the three  $\eta_b^0$  states simultaneously lies in a more accurate and conservative description of the  $\Upsilon(nS) \rightarrow \gamma\eta_i$  transition: whereas the  $\eta_b^0(j = nS)$  component is the most relevant one, the presence of the other  $\eta_b^0(j \neq nS)$  components reduces the size of the corresponding mixing element, hence alleviates slightly the impact of the experimental bounds.

We thus conclude that, if only through the contribution of fig. 4b, the experimental bounds on  $BR(\Upsilon(nS) \rightarrow \gamma\tau^+\tau^-)$  should translate into constraints on the plane  $(m_A, X_d)$ , even in the limit  $m_A \sim 9 - 10$  GeV.

## 4 Bounds on the light CP-odd Higgs

In this section, we present the consequences of the analysis of section 3 for the light CP-odd Higgs in the limit of small photon energies. Given the large uncertainties on the transition factors  $I_{nj}$ , one cannot expect to obtain strict bounds. Instead, we evaluate eq. (13) under several approximations and compare the results to the experimental limits.

The BABAR limits on  $BR(\Upsilon(3S) \rightarrow \gamma\tau^+\tau^-)$  have the highest reach in mass ( $m_{\tau\tau} \leq 10.10$  GeV, limited kinematically by the  $\Upsilon(3S)$  mass and the requirement of observable, not-too-soft photons), hence are of particular interest. As a first approach, we will consider only the  $3S \rightarrow 3S$  transition with the approximation that  $|I_{33}|^2 \simeq 1$ , neglecting the coefficients  $R_{nj}^2$ . Then we will continue to neglect the transition factors  $I_{nj}$ ,  $n \neq j$ , but assume  $|I_{33}|^2$  is reduced down to  $1/2$ . Finally, we will use eq. (9) with the values  $R_{31}^2 \sim 0.02 \text{ GeV}^{-2}$ ,  $R_{32}^2 \sim 0.25 \text{ GeV}^{-2}$  and  $R_{33}^2 \sim 0.02 \text{ GeV}^{-2}$ . This choice of  $R_{31}^2$  allows to reproduce approximately the measured  $BR(\Upsilon(3S) \rightarrow \gamma\eta_{\text{obs}}) = (4.8 \pm 1.3) \cdot 10^{-4}$  [37] in the limit where no mixing is assumed.  $R_{32}^2$  was chosen so as to have the same approximate ratio with  $R_{31}^2$  as in [39].  $R_{33}^2$  is simply the order 1 term in the expansion of  $j_0(kr_0/2)$ ,  $r_0$  being the typical scale of confinement. The choice of these values is of course arguable: the main purpose here consists in checking that the leading effect is captured by the  $3S \rightarrow 3S$  transition. The corresponding limits on the  $(m_A, X_d)$  plane are shown in fig. 5a. We varied all the experimental input within  $2\sigma$  before extracting the bounds. The  $\eta_b^0$  widths were however kept at 10, 5 and 5 MeV respectively: their effect is studied separately, in fig. 5b. We observe that values of  $X_d \gtrsim 1 - 2$  are constrained severely and more or less equivalently in the three cases we considered and we regard this fact as a sign of robustness of the corresponding limits: despite significant variations of the  $I_{nj}$  factors in the three cases under consideration, the excluded region in the  $(m_A, X_d)$  plane remains essentially unchanged. The underlying reason for this is related to the dominant  $3S \rightarrow 3S$  transition. Fig. 5b leads to a similar observation: increased  $\eta_b^0$  widths alleviate slightly the bounds as the total widths of the  $\eta_i$  state is increased (leading to a smaller branching ratio into  $\tau^+\tau^-$ ). Yet despite a doubling of the (conjectured)  $\eta_b^0$  widths, the bounds on the properties of the  $A^0$  remain little affected. The dip for  $m_A \sim m_{\text{obs}}$  corresponds to unacceptable values of  $m_{\eta_b^0(1S)}$ . For  $m_A \lesssim 9.3$  GeV, the lightest state (dominantly  $A^0$ ) leads to larger photon energies so that the plotted bounds become unreliable.

Although our approach to the BABAR limits should fail for  $m_A \lesssim 9.3$  GeV, the CLEO limits can then be

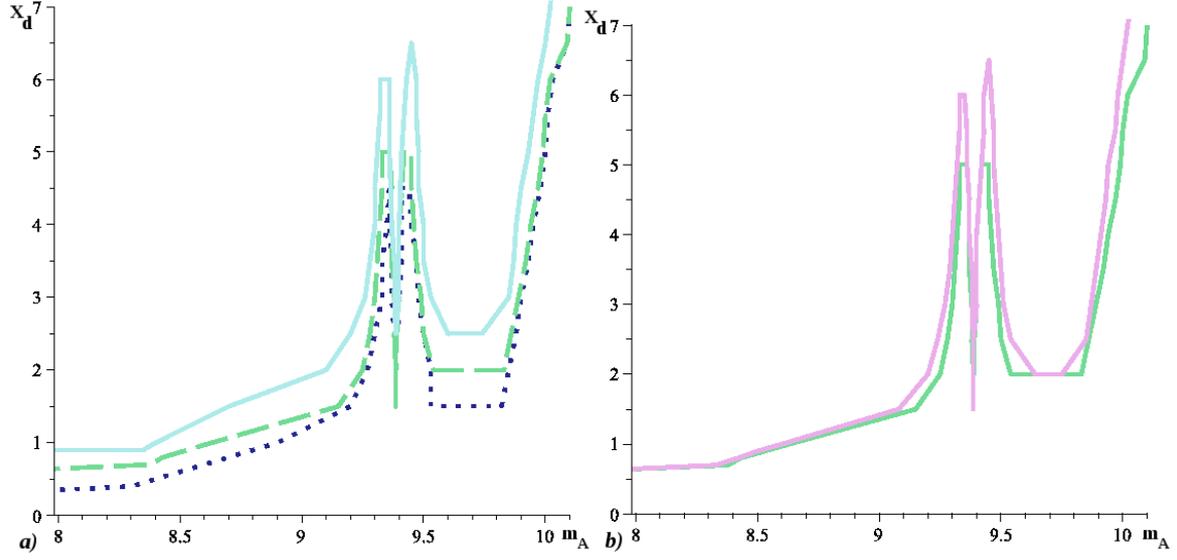


Figure 5: Upper bounds on  $X_d$  as a function of  $m_A$  due to the mixing effect in  $BR(\Upsilon(3S) \rightarrow \gamma \tau^+ \tau^-)$  (using BABAR data).

a) Influence of the  $I_{nj}$  coefficients on the bounds.  $\Gamma_{\eta_b^0(1,2,3S)} = 10, 5, 5$  MeV respectively. The full light-blue curve assumes  $R_{3j}^2 = 0$  and  $|I_{33}|^2 = 1/2$ ; the dashed middle-blue curve corresponds to  $R_{3j}^2 = 0$  and  $|I_{33}|^2 = 1$ ; the dotted dark-blue curve assumes the  $R_{3j}^2$  as described in the main text.

b) Influence of the  $\eta_b^0$  widths on the bounds.  $R_{3j}^2 = 0$  and  $|I_{33}|^2 = 1$ , with  $\Gamma_{\eta_b^0(1,2,3S)} = 10, 5, 5$  MeV (middle-blue curve) and  $\Gamma_{\eta_b^0(1,2,3S)} = 20, 10, 10$  MeV (pink curve).

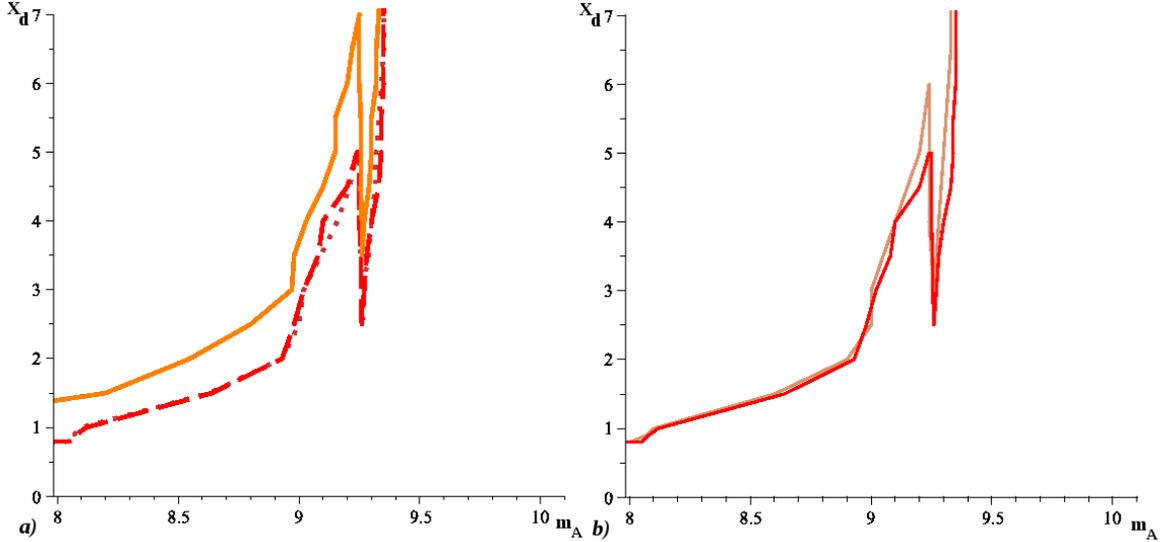


Figure 6: Upper bounds on  $X_d$  as a function of  $m_A$  due to the mixing effect in  $BR(\Upsilon(1S) \rightarrow \gamma \tau^+ \tau^-)$  (using CLEO data).

a) Influence of the  $I_{nj}$  coefficients on the bounds.  $\Gamma_{\eta_b^0(1,2,3S)} = 10, 5, 5$  MeV respectively. The full orange curve assumes  $R_{1j}^2 = 0$  and  $|I_{11}|^2 = 1/2$ ; the dashed red curve corresponds to  $R_{1j}^2 = 0$  and  $|I_{11}|^2 = 1$ ; the dotted dark-red curve assumes the  $R_{1j}^2$  as described in the main text.

b) Influence of the  $\eta_b^0$  widths on the bounds.  $R_{1j}^2 = 0$  and  $|I_{11}|^2 = 1$ , with  $\Gamma_{\eta_b^0(1,2,3S)} = 10, 5, 5$  MeV (red curve) and  $\Gamma_{\eta_b^0(1,2,3S)} = 20, 10, 10$  MeV (brown curve).

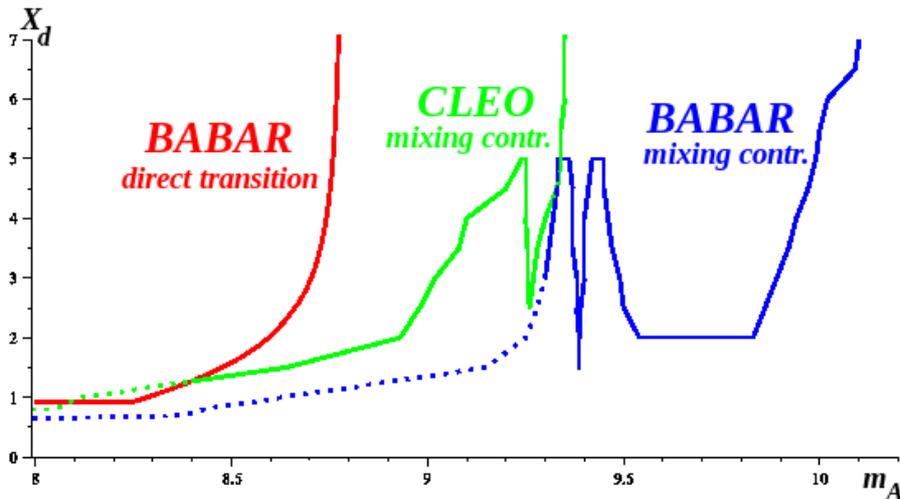


Figure 7: Conclusion: upper bounds on  $X_d$  for a pseudoscalar  $A^0$  in the mass-range 8 – 10.1 GeV.

- in red, from BABAR, resulting from the direct  $\Upsilon$  decay to a  $A^0$  state (fig. 4a): similar to fig. 1;
- in green, from CLEO, resulting from the mixed contribution (fig. 4b); the full line corresponds to the approximate validity range, while the dotted curve extends to regions where the condition on the photon energy  $k \leq 1$  GeV is not satisfied (*i.e.*, where the assumption  $I_{nn} \sim 1$ , hence the corresponding bounds, are unreliable);
- in blue, from BABAR, resulting from the mixed contribution (fig. 4b); the full line corresponds to the approximate validity range, while the dotted curve extends to regions where the condition on the photon energy  $k \leq 1$  GeV is not satisfied.

applied. The favoured transition is now  $1S \rightarrow 1S$ . As before, we consider the cases  $|I_{11}|^2 \simeq 1$ ,  $|I_{11}|^2 \simeq 1/2$  and finally  $R_{11}^2 \sim R_{33}^2$ ,  $R_{12}^2 \sim R_{32}^2$ ,  $R_{13}^2 \sim R_{31}^2$ . The result is shown in fig. 6 and constrains  $X_d \lesssim 2 - 4$  in the range  $m_A \sim 8.4 - 9.4$  GeV. Again, we observe little variations of the general aspect of the bounds in the several considered cases.

We conclude this analysis with fig. 7 where we collect all the previously discussed limits on the  $(m_A, X_d)$  plane in their approximate range of validity. (We chose the case  $I_{nn} = 1$ ,  $I_{n \neq j} = 0$ .) We stress that, due to the limited control on the coefficients  $I_{nj}$  (and on the  $A^0 - \eta_b$  interference), the precise limits on this  $(m_A, X_d)$  plane should be considered with caution, that is, as a qualitative trend, rather than a strict exclusion boundary. In view of the results of fig. 5 and 6 however, we conclude that values of  $X_d$  beyond  $\sim 2 - 3$  should lead to unacceptably large  $BR(\Upsilon(nS) \rightarrow \gamma \tau^+ \tau^-)$ , at least within the range of the experimental constraints.

Concerning the possibility of explaining the tension between the pQCD prediction and the observed  $\eta_b(1S)$  mass, most of the “favoured” region, where this discrepancy can be interpreted (at least within  $1\sigma$ ) by the presence of the  $A^0$ , is now excluded. The variations of  $BR(\Upsilon(3S) \rightarrow \gamma \tau^+ \tau^-)$  in the approximation of eq. (13), along this “favoured region”, are shown in fig. 8 (with  $X_d$  now determined in terms of  $m_A$  through eq. (6)). Only two mass ranges remain, where the tension between the observed and the pQCD-predicted  $\eta_b^0$  masses may be interpreted as an  $A^0$  effect: in the first case,  $m_A \sim 9.39$  GeV, the required  $X_d$  is sufficiently small to escape the BABAR bounds. This coincides with the region of natural NMSSM couplings. Note however that the acceptable mass range is now only of a few 10 MeV around  $m_{\text{obs}}$ , which supposes a very fine accidental relation between the  $A^0$  and  $\eta_b^0(1S)$  masses. The second region  $m_A \gtrsim 10.1$  GeV is beyond the reach of the BABAR searches and requires large  $X_d \sim 20$ . We must therefore regard the possibility of generating the  $\eta_b$  mass shift in the context of a light CP-odd Higgs only as a marginal possibility.

To summarize, we reviewed experimental constraints from CLEO and BABAR on radiative  $\Upsilon$  decays

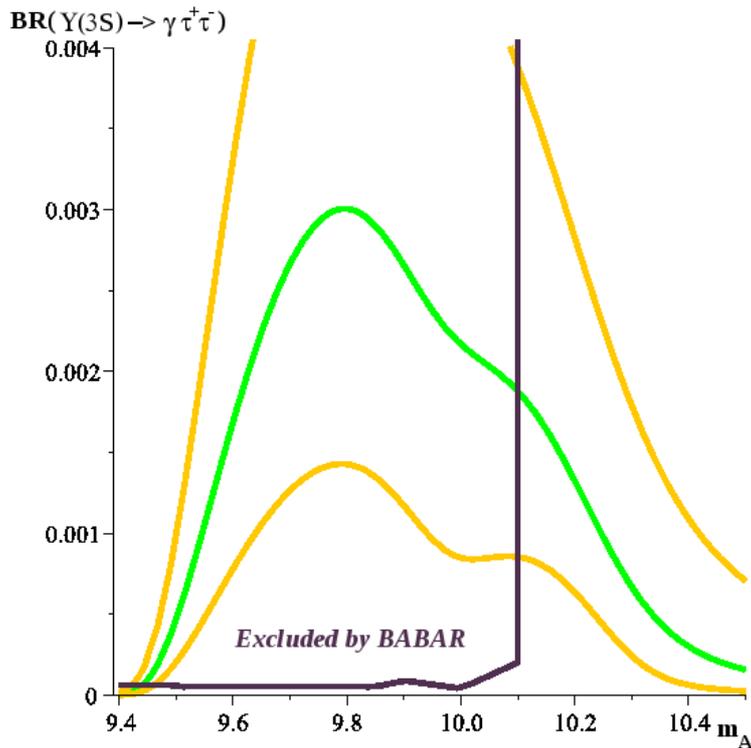


Figure 8: Mixed  $\eta_b^0 - A^0$  contribution to  $BR(\Upsilon(3S) \rightarrow \gamma\tau^+\tau^-)$  compared to the BABAR limits (dark curve) in the “favoured” scenario of [24]. The result is shown along the “favoured” line (green curve), and the two  $1\sigma$  boundaries (orange curves). Except for a few 10 MeV around  $m_{\text{obs}} \sim 9.39$  GeV and the large mass region ( $m_A \geq 10.1$  GeV), unconstrained by BABAR, the essential part of the mass range in this scenario would lead to excessive contributions.

through a pseudoscalar. Even though the pure Higgs contribution is little controlled in the limit of low-energy photons, we showed that the mixing effect should lead to a sizable contribution, which can be used as a theoretical estimate for  $BR(\Upsilon(3S) \rightarrow \gamma\tau^+\tau^-)$ . Taking this estimate seriously, we obtain that the reduced coupling of the  $A^0$  to  $b$  quarks must be small,  $X_d \lesssim 2$ , even in the range  $m_A \sim 9 - 10.1$  GeV. Note however that this region with small couplings is that which is the most naturally achieved in the NMSSM (in *e.g.* the R or Peccei-Quinn symmetry limits). The possibility of interpreting the small tension between the observed and pQCD-predicted  $\eta_b^0(1S)$  masses through a mixing with the  $A^0$  is nevertheless largely narrowed and confined to the ranges  $m_A \sim m_{\text{obs}}$  or  $m_A \gtrsim 10.1$  GeV.

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