Heavy-quark masses from low-energy moments of heavy-quark currents

Peter Marquard

Institut für Theoretische Teilchenphysik
Karlsruher Institut für Technologie (KIT)
76131 Karlsruhe

Abstract

We present the newest results for the determination of heavy-quark masses from low-energy moments of the vacuum polarization function. The results presented here update an earlier determination of the quark masses and incorporate new theoretical calculations of the low-energy moments of the vacuum polarization and new experimental data at the bottom threshold.

Keywords: heavy-quark masses, sum rules

1. Introduction

The precise determination of charm and bottom quark masses has always been an important task both for theory and experiment. The most precise values have been obtained [1] from an analysis of the ITEP sum rules [2] (for reviews see Refs. [3–5]), combining data for the heavy-quark production cross section in electron-positron collision with dispersion relations and a four-loop evaluation of the vacuum polarization induced by the heavy quark current. In this contribution, we summarize the most recent progress, which includes data recently published by the BABAR collaboration [6] and new perturbative results.

2. Analytic Results

The determination of the heavy quark masses in [1] follows closely Refs. [7–9]. It is based on the direct comparison of the theoretical and experimental evaluations of the contributions to the derivatives of the polarization function $\Pi_Q(q^2)$, the former evaluated in perturbative QCD, the latter through moments of the measured cross section for heavy-quark production in electron-positron annihilation. Using dispersion relations, the moments of R_Q [1],

$$\mathcal{M}_n \equiv \int \frac{\mathrm{d}s}{s^{n+1}} R_Q(s)\,, \tag{1}$$

can be related to the derivatives of the vacuum polarization function at $q^2 = 0$,

$$\mathcal{M}_n = \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_Q(q^2)\bigg|_{q^2=0}\,. \tag{2}$$

In its domain of analyticity $\Pi_Q(q^2)$ can be cast into the form

$$\Pi_Q(q^2) = Q_Q^2 \frac{3}{16\pi^2} \sum_{n\geq 0} \bar{C}_n z^n\,, \tag{3}$$

with $z = q^2/(4m_Q^2)$. Here $m_Q = m_Q(\mu)$ is the heavy quark mass with charge Q_Q in the $\overline{\mathrm{MS}}$ scheme at the scale μ. The coefficients $\bar{C}_n$ depend on α_s and on the heavy quark mass through logarithms of the form $l_{m_Q} = \ln(m_Q^2(\mu)/\mu^2)$. Equating theoretically calculated and experimentally measured moments, the heavy quark mass is given by

$$m_Q(\mu) = \frac{1}{2}\left(\frac{9Q_Q^2 \bar{C}_n}{4\mathcal{M}_n^{\mathrm{exp}}}\right)^{1/(2n)}\,. \tag{4}$$

[1] For the precise definition of R_Q, in particular the treatment of gluon splitting into $Q\bar{Q}$, the subtraction of singlet contributions, and the role of nonperturbative terms in the case of charm quarks we refer to Ref. [7].

Email address: peter.marquard@kit.edu (Peter Marquard)

	$\bar{C}_1^{(30)}$	$\bar{C}_2^{(30)}$	$\bar{C}_3^{(30)}$	$\bar{C}_4^{(30)}$
charm	-5.6404	-3.4937	-2.8395	-3.349(11)
bottom	-7.7624	-2.6438	-1.1745	-1.386(10)

Table 1: Lowest four expansion coefficients $\bar{C}_n^{(30)}$ for charm and bottom quarks. The first three coefficients are known analytically, the next is known with good accuracy from a Padé approximation [19].

	$\bar{C}_4^{(3)}$	$\bar{C}_5^{(3)}$	$\bar{C}_6^{(3)}$
Hoang et al	-4.2 ± 1.2	-5.0 ± 1.7	-5.3 ± 2.0
Greynat, Peris	-3.6 ± 0.5	-4.4 ± 1.2	-4.7 ± 1.8
Kiyo et al	$-3.349(11)$	$-3.737(32)$	$-3.735(61)$

Table 2: Comparison of different estimates for the low-energy expansion coefficients $\bar{C}_n^{(30)}$.

As a perturbative series the coefficients $\bar{C}_n$ can be written as

$$\begin{aligned}\bar{C}_n = &\bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{m_Q}\right)\\ &+ \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{m_Q} + \bar{C}_n^{(22)} l_{m_Q}^2\right)\\ &+ \left(\frac{\alpha_s(\mu)}{\pi}\right)^3 \left(\bar{C}_n^{(30)} + \bar{C}_n^{(31)} l_{m_Q} + \bar{C}_n^{(32)} l_{m_Q}^2\right.\\ &\left.+ \bar{C}_n^{(33)} l_{m_Q}^3\right) + \ldots .\end{aligned} \tag{5}$$

The terms of order α_s^2 were evaluated up to $n = 8$ in Refs. [10–12] and even up to n=30 in Refs. [13, 14]. The four-loop contributions to $\bar{C}_0$ and $\bar{C}_1$ were calculated in Refs. [15, 16]. For the higher moments the analysis of [7] was based on estimates for $\bar{C}_n^{(30)}$ with $n = 2, 3, 4$, which lead to an additional uncertainty in the mass determination. Recently, the exact results for the second [17] and third [18] moments were obtained. Combining these coefficients with additional information on the threshold and the high-energy behaviour and using the analyticity of $\Pi_Q(q^2)$ and Padé approximations, fairly precise numerical results were obtained [19] for the higher coefficients up to $n = 10$. (For an earlier analysis along similar lines see Ref. [20].) For the lowest four moments the four-loop coefficients $\bar{C}_n^{(30)}$ are listed in Tab. 1 both for the charm and the bottom quark. Using a different approach based on threshold resummation the low-energy expansion coefficients have also been estimated in [21]. All determinations agree within the error, with [19] giving the smallest errors. A comparison between the works is shown in Tab. 2. It should be emphasized that these results are well within the estimates used in the analysis of [7]. The impact of these new results on the quark mass determination will be shown below.

3. Bottom Production Close to Threshold

The determination of the bottom quark mass, as performed in [7, 8] relies heavily on the precise measurement of $R = \sigma(e^+e^- \to \text{hadrons})/\sigma_{\text{pt}}$ (with $\sigma_{\text{pt}} = \frac{4\pi\alpha^2}{3s}$), which enters the moments as defined above. Specifically, it is the contribution from the heavy quark current denoted as R_b with the light-quark contribution subtracted. It is convenient to split the integration region into three pieces: The lowest region covering the narrow resonances, an intermediate "threshold" region between 10.62 GeV and 11.24 GeV, and the perturbative region above 11.24 GeV, where the measurement is replaced by the perturbative QCD prediction. The choice of 11.24 GeV corresponds to the upper end of the energy range covered by a CLEO measurement more than 20 years ago [22]. It also coincides approximately with the energy reach of a recent BABAR measurement [6]. In the analysis of [7], $\Upsilon(4S)$ with its mass $M_{\Upsilon(4S)} = 10.5794(12)$ GeV and width $\Gamma_{\Upsilon(4S)} = 20.5$ MeV has been considered together with the three lower, narrow resonances and thus the continuum part of the bottom cross section was taken from 10.62 GeV upwards. Until recently the only measurement in the threshold region has been the one from the CLEO collaboration, which quotes a systematic error of about 6%. No radiative corrections had been applied. In Ref. [7] it has been argued, that a normalization factor 1/1.28 is necessary to reconcile these data with more recent and more precise CLEO results below the $\Upsilon(4S)$-resonance and with perturbative QCD at the high end. These "rescaled" data were the basis of the subsequent extraction of the bottom quark mass. However, in view of these uncertainties an overall systematic error of 10% was attributed to the contribution of the moments from this region. Thus, although this contribution to the moments is relatively small, its impact on the error was larger or equal than the one from the other two regions combined.

Recently a measurement of R_b in the energy region between 10.54 GeV and 11.20 GeV was performed by the BABAR collaboration with significantly improved statistics and with a correlated systematic error between 2.5% and 3% [6]. This allows an independent determination of the contribution to the moments with significantly reduced systematic error. However, no radiative corrections were applied to the published data and the radiative tails of the four lower Υ resonances were included in the quantity denoted R_b. Therefore, the data has to be corrected for these effects. The details of this procedure can be found in [1].

n	$m_b(10\text{ GeV})$	exp	α_s	μ	total	$m_b(m_b)$
1	3597	14	7	2	16	4151
2	3610	10	12	3	16	4163
3	3619	8	14	6	18	4172
4	3631	6	15	20	26	4183

Table 3: Results for $m_b(10\text{ GeV})$ and $m_b(m_b)$ in MeV obtained from Eq. (4). The errors are from experiment, α_s and the variation of μ.

4. Quark Masses

Using the new moments with their significantly reduced experimental error, one obtains the results for the bottom quark mass at the scale $\mu = 10\,\text{GeV}$ as listed in Tab. 3. In comparison with the previous determination a minute upwards shift of 1 MeV (resulting from an upward shift of +3 MeV from the new data and a downward shift of −2 MeV from the new theory input) and a reduction of both experimental and theory error is observed. The three results based on $n = 1$, 2 and 3 are of comparable precision. The relative size of the contribution from the continuum above 11.24 GeV which is modelled by perturbative QCD decreases for the higher moments $n = 2$ and 3. On the other hand the theory uncertainty, exemplified by the μ dependence is still acceptable. We therefore adopt the result from $n = 2$ (which is roughly between the $n = 1$ and $n = 3$ values and which exhibits the smallest error) as our final result

$$\begin{aligned} m_b(10\text{ GeV}) &= 3610(16)\,\text{MeV}\,, \\ m_b(m_b) &= 4163(16)\,\text{MeV}\,. \end{aligned} \tag{6}$$

These values are well consistent with the previous determination [7] $m_b(10\text{ GeV}) = 3609(25)$ MeV and $m_b(m_b) = 4164(25)$ MeV. Recently, the bottom-quark mass was also obtained from lattice calculations [23] and ratios of moments [24].

In the absence of new data the analysis of m_c will be based on the moments listed in Tab. 6 of Ref. [7]. As emphasized in [7, 8] it is convenient to consider as primary quantity the running quark mass at scale 3 GeV. This is the natural scale for the sum rule (corresponding roughly to the charm threshold) and, as a consequence of the smaller strong coupling constant, the perturbative series exhibits a more stable behaviour.

If not stated otherwise, all input parameters and assumptions are identical to those of Ref. [7]. In particular we adopt $\alpha_s(M_Z) = 0.1189$. The results for all four moments are nicely consistent, and the three lowest moments exhibit comparable errors. Note, that the relative composition of the experimental input varies strongly from low to high moments: For $n = 1$ the contributions from narrow resonances and continuum are roughly comparable, for $n = 3$ the continuum contribution amounts to about 10%. Furthermore, the experimental contribution to the error decreases with increasing n, the μ-dependence, reflecting the theory uncertainly, increases.Despite the significant differences in the composition of the errors, the results are perfectly consistent. Since the result from $n = 1$ has the smallest dependence on the strong coupling and the smallest total error we take as our final value

$$m_c(3\text{ GeV}) = 986(13)\,\text{MeV}\,, \tag{7}$$

and consider its consistency with $n = 2$, 3 and 4 as additional confirmation.

Transforming this to the scale-invariant mass $m_c(m_c)$ [25], including the four-loop coefficients of the renormalization group functions one finds $m_c(m_c) = 1279(13)$ MeV. Let us recall at this point that a recent lattice determination, combining a lattice simulation for the data for the pseudoscalar correlator with the perturbative three- and four-loop result [12, 18, 26] has led to $m_c(3\text{ GeV}) = 986(10)$ MeV [27] in remarkable agreement with most recent analysis [1].

5. Summary

Based on new four-loop results for the higher derivatives of the vacuum polarization function and new BABAR data for bottom quark production in the threshold region, a reanalysis of the charm- and bottom-quark mass determination has been performed. The new data, a posteriori, give additional support to the analysis of CLEO data presented in Ref. [22] and, furthermore, lead to a significant reduction of the experimental error. The new theory results for the higher moments lead to a further reduction of the theory uncertainty and, equally important, demonstrate the consistency between the analysis based on different moments. The final results, $m_c(3\text{GeV}) = 0.986(13)$ GeV and $m_b(10\text{GeV}) = 3.610(16)$ GeV are consistent with the earlier determination in Ref. [7] and, together with Ref. [27], constitute the most precise determination of charm- and bottom-quark masses to date.

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