

# Matching heavy-quark fields in QCD and HQET at three loops

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## Abstract

The relation between the heavy-quark field in QCD and the corresponding field in HQET is derived up to three loops.

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QCD problems with a single heavy quark  $Q$  can be treated in a simpler effective theory — HQET, if there exists a 4-velocity  $v$  such that the heavy-quark momentum is  $p = mv + k$  ( $m$  is the on-shell mass) and the characteristic residual momentum is small:  $k \ll m$ . QCD operators can be written as series in  $1/m$  via HQET operators; the coefficients in these series are determined by matching on-shell matrix elements in both theories. For example, the heavy–light quark currents have been considered at the leading (zeroth) order in  $1/m$  up to three-loop accuracy [1,2].

Here we shall consider the heavy-quark field  $Q$ . At the tree level, it is related to the corresponding HQET field  $Q_v$  (satisfying  $\not{v}Q_v = Q_v$ ) by [3–6]

$$Q(x) = e^{-imv \cdot x} \left( 1 + \frac{i\not{D}_\perp}{2m} + \dots \right) Q_v(x), \quad D_\perp^\mu = D^\mu - v^\mu v \cdot D. \quad (1)$$

The matrix elements of the bare fields between the on-shell quark with momentum  $p = mv + k$  and the vacuum in both theories are given by the on-shell wave-function renormalization constants:

$$\langle 0|Q_0|Q(p)\rangle = (Z_Q^{\text{os}})^{1/2} u(p), \quad \langle 0|Q_{v0}|Q(p)\rangle = (\tilde{Z}_Q^{\text{os}})^{1/2} u_v(k) \quad (2)$$

(HQET renormalization constants are denoted by  $\tilde{Z}$ ). The Dirac spinors are related by the Foldy–Wouthuysen transformation

$$u(mv + k) = \left[ 1 + \frac{\not{k}}{2m} + \mathcal{O}\left(\frac{k^2}{m^2}\right) \right] u_v(k).$$

Therefore, the bare fields are related by

$$Q_0(x) = e^{-imv \cdot x} \left[ z_0^{1/2} \left( 1 + \frac{i\mathcal{D}_\perp}{2m} \right) Q_{v0}(x) + \mathcal{O}\left(\frac{1}{m^2}\right) \right], \quad (3)$$

where the bare matching coefficient is

$$z_0 = \frac{Z_Q^{\text{os}}(g_0^{(n_l+1)}, a_0^{(n_l+1)})}{\tilde{Z}_Q^{\text{os}}(g_0^{(n_l)}, a_0^{(n_l)})} \quad (4)$$

(we use the covariant gauge: the gauge-fixing term in the Lagrangian is  $-(\partial_\mu A_0^{\mu})/(2a_0)$ , and the free gluon propagator is  $(-i/p^2)(g_{\mu\nu} - (1 - a_0)p_\mu p_\nu/p^2)$ ; the number of flavours in QCD is  $n_f = n_l + 1$ ). The  $\mathcal{O}(1/m)$  matching coefficient in (3) is equal to the leading one,  $z_0$ ; this reflexes the reparametrization invariance [7]. The  $\overline{\text{MS}}$  renormalized fields are related by the formula similar to (3), with the renormalized decoupling coefficient

$$z(\mu) = \frac{\tilde{Z}_Q(\alpha_s^{(n_l)}(\mu), a^{(n_l)}(\mu))}{Z_Q(\alpha_s^{(n_l+1)}(\mu), a^{(n_l+1)}(\mu))} z_0. \quad (5)$$

If there are no massive flavours except  $Q$ , then  $\tilde{Z}_Q^{\text{os}} = 1$  because all loop corrections are scale-free. The QCD on-shell renormalization constant  $Z_Q^{\text{os}}$  contains the single scale  $m$  in this case; it has been calculated [8] up to three loops. The three-loop  $\overline{\text{MS}}$  anomalous dimensions of  $Q_v$  [8,9] and  $Q$  [10] are also known. We have to express all three quantities  $Z_Q^{\text{os}}(g_0^{(n_l+1)}, a_0^{(n_l+1)})$ ,  $Z_Q(\alpha_s^{(n_l+1)}(\mu), a^{(n_l+1)}(\mu))$ ,  $\tilde{Z}_Q(\alpha_s^{(n_l)}(\mu), a^{(n_l)}(\mu))$  via the same variables, say,  $\alpha_s^{(n_l)}(\mu)$ ,  $a^{(n_l)}(\mu)$ , see [11]. An explicit formula expressing  $\alpha_s^{(n_l+1)}(\mu)$  via  $\alpha_s^{(n_l)}(\mu)$  and  $L = 2 \log(\mu/m)$  ( $m$  is the on-shell mass) can be found in [12]. The corresponding relation between  $a^{(n_l+1)}(\mu)$  and  $a^{(n_l)}(\mu)$  is

$$\begin{aligned} \frac{a^{(n_l+1)}(\mu)}{a^{(n_l)}(\mu)} &= 1 - \left[ \frac{4}{3}L + \frac{6L^2 + \pi^2}{9}\varepsilon + \frac{2L^3 + \pi^2 L - 4\zeta_3}{9}\varepsilon^2 + \dots \right] T_F \frac{\alpha_s^{(n_l)}(\mu)}{4\pi} \\ &- \left[ C_A L^2 + (4C_F + 5C_A)L + 15C_F - \frac{13}{12}C_A \right. \\ &+ \left( C_A L^3 + (4C_F + 5C_A)L^2 + \left( 30C_F + \frac{\pi^2 - 13}{6}C_A \right)L \right. \\ &\left. \left. + \left( \frac{\pi^2}{3} + \frac{31}{2} \right)C_F + \frac{5\pi^2 + 169}{12}C_A \right) \varepsilon + \dots \right] T_F \left( \frac{\alpha_s^{(n_l)}(\mu)}{4\pi} \right)^2 + \dots \end{aligned} \quad (6)$$

Our main result is the renormalized matching coefficient

$$\begin{aligned}
z(\mu) = & 1 - (3L + 4)C_F \frac{\alpha_s^{(n_l)}(\mu)}{4\pi} + (z_{22}L^2 + z_{21}L + z_{20}) C_F \left( \frac{\alpha_s^{(n_l)}(\mu)}{4\pi} \right)^2 \\
& + (z_{33}L^3 + z_{32}L^2 + z_{31}L + z_{30}) C_F \left( \frac{\alpha_s^{(n_l)}(\mu)}{4\pi} \right)^3 + \dots
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
z_{22} = & \frac{9}{2}C_F - \frac{11}{2}C_A + 2T_F n_l, \\
z_{21} = & \frac{27}{2}C_F - \frac{215}{6}C_A + \frac{38}{3}T_F n_l + 2T_F, \\
z_{20} = & \left( 16\pi^2 \log 2 - 24\zeta_3 - 13\pi^2 + \frac{241}{8} \right) C_F \\
& + \left( -8\pi^2 \log 2 + 12\zeta_3 + 5\pi^2 - \frac{1705}{24} \right) C_A \\
& + \left( \frac{4}{3}\pi^2 + \frac{113}{6} \right) T_F n_l + \left( -\frac{16}{3}\pi^2 + \frac{947}{18} \right) T_F, \\
z_{33} = & -\frac{9}{2}C_F^2 + \frac{33}{2}C_F C_A - \frac{121}{9}C_A^2 - 6C_F T_F n_l \\
& + \frac{88}{9}C_A T_F n_l + \frac{1}{3}a^{(n_l)}(\mu) C_A T_F - \frac{16}{9}T_F^2 n_l^2, \\
z_{32} = & -\frac{45}{2}C_F^2 + 135C_F C_A - \frac{2671}{18}C_A^2 - 42C_F T_F n_l - 4C_F T_F \\
& + \frac{938}{9}C_A T_F n_l - \left( \frac{13}{6}a^{(n_l)}(\mu) + 1 \right) C_A T_F - \frac{152}{9}T_F^2 n_l^2 + \frac{8}{3}T_F^2, \\
z_{31} = & \left[ -48\pi^2 \log 2 + 72\zeta_3 + 39\pi^2 - \frac{783}{8} \right] C_F^2 \\
& + \left[ \frac{424}{3}\pi^2 \log 2 - 224\zeta_3 - \frac{331}{3}\pi^2 + \frac{13307}{24} \right] C_F C_A \\
& + \left[ \left( -\frac{2}{45}\pi^4 + \frac{9}{4}\zeta_3 + \frac{1}{4} \right) a^{(n_l)}(\mu) \right. \\
& \quad \left. - \frac{176}{3}\pi^2 \log 2 + \frac{325}{4}\zeta_3 - \frac{2}{15}\pi^4 + \frac{110}{3}\pi^2 - \frac{73981}{108} \right] C_A^2 \\
& + \left( -\frac{128}{3}\pi^2 \log 2 + 16\zeta_3 + \frac{92}{3}\pi^2 - \frac{613}{6} \right) C_F T_F n_l + \left( 16\pi^2 - \frac{1013}{6} \right) C_F T_F \\
& + \left( \frac{64}{3}\pi^2 \log 2 + 16\zeta_3 - \frac{32}{9}\pi^2 + \frac{10816}{27} \right) C_A T_F n_l \\
& + \left( \frac{121}{18}a^{(n_l)}(\mu) - \frac{352}{9}\pi^2 + \frac{11278}{27} \right) C_A T_F \\
& - \left( \frac{32}{9}\pi^2 + \frac{1336}{27} \right) T_F^2 n_l^2 + \left( \frac{128}{9}\pi^2 - \frac{3908}{27} \right) T_F^2 n_l - \frac{20}{9}T_F^2,
\end{aligned}$$

$$\begin{aligned}
a_{30} = & \left[ -1792a_4 - \frac{224}{3} \log^4 2 + 96\pi^2 \log^2 2 + \frac{3568}{3} \pi^2 \log 2 - 20\zeta_5 \right. \\
& \left. + 8\pi^2 \zeta_3 - 1256\zeta_3 - \frac{76}{15} \pi^4 - \frac{4801}{9} \pi^2 - \frac{3023}{12} \right] C_F^2 \\
& + \left[ -\frac{32}{3} a_4 - \frac{4}{9} \log^4 2 - \frac{1448}{9} \pi^2 \log^2 2 - \frac{2752}{9} \pi^2 \log 2 + 580\zeta_5 \right. \\
& \left. - 180\pi^2 \zeta_3 - \frac{2312}{3} \zeta_3 + \frac{6697}{270} \pi^4 + \frac{2137}{9} \pi^2 + \frac{24131}{72} \right] C_F C_A \\
& + \left[ \left( -\frac{7}{6} \zeta_5 - \frac{4}{9} \pi^2 \zeta_3 + \frac{13}{4} \zeta_3 - \frac{17}{432} \pi^4 + \frac{1}{4} \pi^2 + \frac{13}{12} \right) a^{(n_l)}(\mu) \right. \\
& \left. + \frac{1360}{3} a_4 + \frac{170}{9} \log^4 2 + \frac{508}{9} \pi^2 \log^2 2 - \frac{1300}{9} \pi^2 \log 2 - \frac{787}{2} \zeta_5 \right. \\
& \left. + \frac{340}{3} \pi^2 \zeta_3 + \frac{23311}{36} \zeta_3 - \frac{20429}{2160} \pi^4 - \frac{8705}{108} \pi^2 - \frac{1656817}{1944} \right] C_A^2 \\
& + \left[ \frac{1024}{3} a_4 + \frac{128}{9} \log^4 2 + \frac{256}{9} \pi^2 \log^2 2 - \frac{1504}{9} \pi^2 \log 2 \right. \\
& \left. + \frac{1096}{3} \zeta_3 - \frac{916}{135} \pi^4 + \frac{904}{9} \pi^2 + \frac{1120}{9} \right] C_F T_F n_l \\
& + \left[ 768a_4 + 32 \log^4 2 - 32\pi^2 \log^2 2 + \frac{1088}{9} \pi^2 \log 2 \right. \\
& \left. + \frac{466}{9} \zeta_3 + \frac{124}{45} \pi^4 - \frac{8848}{81} \pi^2 - \frac{16811}{54} \right] C_F T_F \\
& + \left[ -\frac{512}{3} a_4 - \frac{64}{9} \log^4 2 - \frac{128}{9} \pi^2 \log^2 2 + \frac{752}{9} \pi^2 \log 2 \right. \\
& \left. - \frac{280}{9} \zeta_3 + \frac{152}{135} \pi^4 + \frac{52}{3} \pi^2 + \frac{111791}{243} \right] C_A T_F n_l \\
& + \left[ \left( \frac{8}{3} \zeta_3 - \frac{2461}{108} \right) a^{(n_l)}(\mu) - 512a_4 - \frac{64}{3} \log^4 2 + \frac{64}{3} \pi^2 \log^2 2 + \frac{5120}{9} \pi^2 \log 2 \right. \\
& \left. - 60\zeta_5 + \frac{44}{3} \pi^2 \zeta_3 - \frac{2837}{9} \zeta_3 - \frac{136}{45} \pi^4 - \frac{36268}{81} \pi^2 + \frac{100627}{81} \right] C_A T_F \\
& - \left( \frac{224}{9} \zeta_3 + \frac{304}{27} \pi^2 + \frac{11534}{243} \right) T_F^2 n_l^2 + \left( \frac{208}{9} \pi^2 - \frac{18884}{81} \right) T_F^2 n_l \\
& + \left( \frac{448}{3} \zeta_3 + \frac{128}{45} \pi^2 - \frac{16850}{81} \right) T_F^2
\end{aligned}$$

(here  $a_4 = \text{Li}_4(1/2)$ ). Gauge dependence first appears at three loops, as in  $Z_Q^{\text{os}}$  [8]. The requirement of finiteness of the renormalized matching coefficient (5) at  $\varepsilon \rightarrow 0$  has allowed the authors of [8] to extract  $\tilde{Z}_Q$  from their result for  $Z_Q^{\text{os}}$ .

It would not be too difficult to take into account a lighter massive flavour, say,

$m_c \neq 0$  in  $b$ -quark HQET.  $\tilde{Z}_Q^{\text{os}}$  is no longer equal to 1, but is known at three loops [13];  $Z_Q^{\text{os}}$  contains two scales, and is a non-trivial function of  $m_c/m_b$  [14]. Both  $\tilde{Z}_Q^{\text{os}}$  and  $Z_Q^{\text{os}}$  have no smooth limit at  $m_c \rightarrow 0$ , but the discontinuity cancels in the ratio (4).

Now let's consider  $z(\mu)$  in the large- $\beta_0$  limit (see Chapter 8 in [15] for a pedagogical introduction):

$$z(\mu) = 1 + \int_0^\beta \frac{d\beta}{\beta} \left( \frac{\gamma(\beta)}{2\beta} - \frac{\gamma_0}{2\beta_0} \right) + \frac{1}{\beta_0} \int_0^\infty du e^{-u/\beta} S(u) + \mathcal{O}\left(\frac{1}{\beta_0^2}\right), \quad (8)$$

where  $\beta = \beta_0 \alpha_s / (4\pi)$ ,  $\gamma = \gamma_0 \alpha_s / (4\pi) + \dots$  (differences of  $n_l$ -flavour and  $(n_l + 1)$ -flavour quantities can be neglected at the  $1/\beta_0$  order). The difference of the QCD and HQET anomalous dimensions  $\gamma = \gamma_Q - \tilde{\gamma}_Q$  and the Borel image  $S(u)$  can be expressed as

$$\gamma(\beta) = -2 \frac{\beta}{\beta_0} F(-\beta, 0), \quad S(u) = \frac{F(0, u) - F(0, 0)}{u}, \quad (9)$$

where the function  $F(\varepsilon, u)$  has been calculated in [1] (see also [15]):

$$\begin{aligned} F(\varepsilon, u) &= -2C_F \left(\frac{\mu}{m}\right)^{2u} e^{\gamma_E \varepsilon} \frac{\Gamma(1+u)\Gamma(1-2u)}{\Gamma(3-u-\varepsilon)} D(\varepsilon)^{u/\varepsilon-1} \\ &\quad \times (3-2\varepsilon)(1-u)(1+u-\varepsilon), \quad (10) \\ D(\varepsilon) &= 6e^{\gamma_E \varepsilon} \Gamma(1+\varepsilon) B(2-\varepsilon, 2-\varepsilon) = 1 + \frac{5}{3}\varepsilon + \dots \end{aligned}$$

The anomalous dimension difference [1] is gauge invariant at this order:

$$\gamma(\beta) = 2C_F \frac{\beta}{\beta_0} \frac{(1+\beta)(1+\frac{2}{3}\beta)}{B(2+\beta, 2+\beta)\Gamma(3+\beta)\Gamma(1-\beta)}; \quad (11)$$

the Borel image is [16,15]

$$S(u) = -6C_F \left[ e^{(L+5/3)u} \frac{\Gamma(u)\Gamma(1-2u)}{\Gamma(3-u)} (1-u^2) - \frac{1}{2u} \right]. \quad (12)$$

This Borel image has infrared renormalon poles at each positive half-integer  $u$  and at  $u = 2$ . Therefore, the integral in (8) is not well defined. Comparing its residue at the leading pole  $u = 1/2$  with the residue of the static-quark self-energy at its ultraviolet pole  $u = 1/2$  [17], we can express the renormalon ambiguity of  $z(\mu)$  as

$$\Delta z(\mu) = \frac{3}{2} \frac{\Delta \bar{\Lambda}}{m} \quad (13)$$

( $\bar{\Lambda}$  is the ground-state meson residual energy). This ambiguity is compensated in physical matrix elements by ultraviolet renormalon ambiguities in the lead-

ing  $1/m$  correction (matrix elements of both local and bilocal dimension-5/2 operators), see [16].

The matching coefficient is gauge invariant at the order  $1/\beta_0$ . Expanding  $\gamma(\beta)$  and  $S(u)$  and integrating, we obtain

$$\begin{aligned}
z(\mu) = & 1 - C_F \frac{\alpha_s(\mu)}{4\pi} \left\{ 3L + 4 + \left[ \frac{3}{2}L^2 + \frac{19}{2}L + \pi^2 + \frac{113}{8} \right] \frac{\beta_0 \alpha_s(\mu)}{4\pi} \right. \\
& + \left[ L^3 + \frac{19}{2}L^2 + \left( 2\pi^2 + \frac{167}{6} \right) L + 14\zeta_3 + \frac{19}{3}\pi^2 + \frac{5767}{216} \right] \left( \frac{\beta_0 \alpha_s(\mu)}{4\pi} \right)^2 \\
& + \left[ \frac{3}{4}L^4 + \frac{19}{2}L^3 + \left( 3\pi^2 + \frac{167}{4} \right) L^2 + \left( 36\zeta_3 + 19\pi^2 + \frac{2903}{36} \right) L \right. \\
& \left. \left. + \frac{71}{40}\pi^4 + \frac{467}{4}\zeta_3 + \frac{167}{6}\pi^2 + \frac{103933}{1728} \right] \left( \frac{\beta_0 \alpha_s(\mu)}{4\pi} \right)^3 + \dots \right\}. \tag{14}
\end{aligned}$$

Thus we have confirmed the contributions with the highest power of  $n_l$  in each term in (7), and predicted such a contribution at  $\alpha_s^4$ .

Numerically, the matching coefficient (7) in the Landau gauge at  $\mu = m$  and  $n_l = 4$  can be written as

$$\begin{aligned}
z(m) = & 1 - \frac{4}{3} \frac{\alpha_s^{(4)}(m)}{\pi} - (1.9996\beta_0 - 4.5421) \left( \frac{\alpha_s^{(4)}(m)}{\pi} \right)^2 \\
& - (2.2091\beta_0^2 + 5.1153\beta_0 - 61.5397) \left( \frac{\alpha_s^{(4)}(m)}{\pi} \right)^3 \\
& - (3.3755\beta_0^3 + \dots) \left( \frac{\alpha_s^{(4)}(m)}{\pi} \right)^4 + \dots \\
= & 1 - \frac{4}{3} \frac{\alpha_s^{(4)}(m)}{\pi} - (16.6629 - 4.5421) \left( \frac{\alpha_s^{(4)}(m)}{\pi} \right)^2 \\
& - (153.4076 + 42.6271 - 61.5397) \left( \frac{\alpha_s^{(4)}(m)}{\pi} \right)^3 \\
& - (1953.4013 + \dots) \left( \frac{\alpha_s^{(4)}(m)}{\pi} \right)^4 + \dots \\
= & 1 - \frac{4}{3} \frac{\alpha_s^{(4)}(m)}{\pi} - 12.1208 \left( \frac{\alpha_s^{(4)}(m)}{\pi} \right)^2 - 134.4950 \left( \frac{\alpha_s^{(4)}(m)}{\pi} \right)^3 \\
& - (1953.4013 + \dots) \left( \frac{\alpha_s^{(4)}(m)}{\pi} \right)^4 + \dots \tag{15}
\end{aligned}$$

( $\beta_0$  is for  $n_l = 4$  flavours). Naive nonabelianization [1] works rather well at two and three loops (in the latter case the  $\mathcal{O}(\beta_0)$  and  $\mathcal{O}(1)$  terms partially compensate each other, similarly to [2]). Therefore, we can expect that the

estimate of the  $\alpha_s^4$  term is also reasonably good. Numerical convergence of the series is very poor; this is related to the infrared renormalon at  $u = 1/2$ .

In QED, the  $\overline{\text{MS}}$  renormalized electron field is related to the Bloch–Nordsieck electron field by the gauge-invariant matching coefficient

$$\begin{aligned}
z(\mu) = & 1 - (3L + 4) \frac{\alpha}{4\pi} \\
& + \left( \frac{9}{2} L^2 + \frac{31}{2} L + 16\pi^2 \log 2 - 24\zeta_3 - \frac{55}{3} \pi^2 + \frac{5957}{72} \right) \left( \frac{\alpha}{4\pi} \right)^2 \\
& - \left[ \frac{9}{2} L^3 + \frac{143}{6} L^2 + \left( 48\pi^2 \log 2 - 72\zeta_3 - 55\pi^2 + \frac{19363}{72} \right) L \right. \\
& \quad + 1024a_4 + \frac{128}{3} \log^4 2 - 64\pi^2 \log^2 2 - \frac{11792}{9} \pi^2 \log 2 + 20\zeta_5 \\
& \quad \left. - 8\pi^2 \zeta_3 + \frac{9494}{9} \zeta_3 + \frac{104}{45} \pi^4 + \frac{259133}{405} \pi^2 + \frac{249887}{324} \right] \left( \frac{\alpha}{4\pi} \right)^3 + \dots
\end{aligned} \tag{16}$$

where  $\alpha \approx 1/137$  is in the on-shell scheme.

In conclusion: we have derived the QCD/HQET matching coefficient for the heavy-quark field with three-loop accuracy (7), and the corresponding QED coefficient (16).

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