Constraining the MSSM sfermion mass matrices with light fermion masses

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We study the finite supersymmetric loop corrections to fermion masses and mixing matrices in the generic MSSM. In this context the effects of non-decoupling chirally-enhanced self-energies are studied beyond leading order in perturbation theory. These NLO corrections are not only necessary for the renormalization of the CKM matrix to be unitary, they are also numerically important for the light fermion masses. Focusing on the tri-linear A-terms with generic flavor-structure we derive very strong bounds on the chirality-changing mass insertions $\delta^{f\,LR,RL}_{IJ}$ by applying 't Hooft's naturalness criterion. In particular, the NLO corrections to the up quark mass allow us to constrain the unbounded element $\delta^{u\,RL}_{13}$ if at the same time $\delta^{u\,LR}_{13}$ is unequal to zero. Our result is important for single-top production at the LHC.

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I. INTRODUCTION

A major challenge in particle physics is to understand the pattern of fermion masses and mixing angles. With the discovery of neutrino oscillations flavor has become even more mysterious since the nearly tri-bimaximal mixing strongly differ from the quark sector. The minimal supersymmetric standard model (MSSM) does not provide insight into the flavor problem by contrast the generic MSSM contains even new sources of flavor and chirality violation, stemming from the supersymmetry-breaking sector which are the sources of the so-called supersymmetric flavor problem. The origin of these flavor-violating terms is obvious: In the standard model (SM) the quark and lepton Yukawa matrices are diagonalized by unitary rotations in flavor space and the resulting basis defines the mass eigenstates. If the same rotations are carried out on the squark fields of the MSSM, one obtains the super–CKM/PMNS basis in which no tree–level FCNC couplings are present. However, neither the 3×3 mass terms $m^2_{\tilde{Q}}$, $m^2_{\tilde{u}}$, $m^2_{\tilde{d}}$, $m^2_{\tilde{L}}$ and $m^2_{\tilde{e}}$ of the left–handed and right–handed sfermions nor the tri-linear Higgs–sfermion–sfermion couplings are necessarily diagonal in this basis. The tri-linear $\overline{Q}H_dA^dd_R$, $\overline{Q}H_uA^uu_R$ and $\overline{L}H_dA^le_R$ terms induce mixing between left–handed and right–handed sfermions after the Higgs doublets H_d and H_u acquire their vacuum expectation values (vevs) v_d and v_u, respectively. In the current era of precision flavor physics stringent bounds on these parameters have been derived from FCNC processes in the quark and in the lepton sector, by requiring that the gluino–squark loops and chargino–sneutrinos/neutralino–slepton loops do not exceed the measured values of the considered observables [1–13].

However, in [14, 15] it is shown that all flavor violation in the quark sector can solely originate from trilinear SUSY breaking terms because all FCNC bounds are satisfied for $M_{SUSY} \geq 500$GeV. Dimensionless quantities are commonly defined in the mass insertion parametrization as:

$$\delta^{f\,XY}_{IJ} = \frac{\left(\Delta m^2_F\right)^{IJ}_{XY}}{\sqrt{m^2_{\tilde{f}_{IX}} m^2_{\tilde{f}_{JY}}}}. \qquad (1)$$

In Eq. (1) I and J are flavour indices running from 1 to 3, X, Y denote the chiralities L and R, $\left(\Delta m^2_F\right)^{IJ}_{XY}$ with $F = U, D, L$ is the off-diagonal element of the sfermion mass matrix (see Appendix A 2) and $m_{\tilde{f}^2_{IX}}$, $m^2_{\tilde{f}_{JY}}$ are the corresponding diagonal ones. In this article we are going to complement the analysis of [14] with respect to three important points:

- Electroweak correction are taken into account. Therefore, we are able to constrain also the flavor-violating and chirality-changing terms in the lepton sector.
- The constraints on the flavor-diagonal mass insertions $\delta^{u,d,l\,LR}_{11,22}$ are obtained from the requirement that the corrections should not exceed the measured masses. This has already been done in the seminal paper of Gabbiani et al. [2]. We improve this calculation by taking into account QCD corrections and by using the up-to-date values of the fermion masses.
- The leading chirally-enhanced two-loop corrections are calculated. As we will see, this allows us to constrain the elements $\delta^{f\,RL}_{13}$ (and $\delta^{d\,RL}_{23}$), if at the same time, also $\delta^{f\,LR}_{13}$ ($\delta^{d\,LR}_{23}$) is different from zero.

Our paper is organized as follows: In Sec. II we study the impact of chirally enhanced parts of the self-energies for quarks and leptons on the fermion masses and mixing matrices (CKM matrix and PMNS matrix). First, we introduce the general formalism in Sec. II A and then specify to the MSSM with non-minimal sources of flavor violation in Sec. II B where we compute the chirally enhanced parts of the self-energies for quarks and leptons

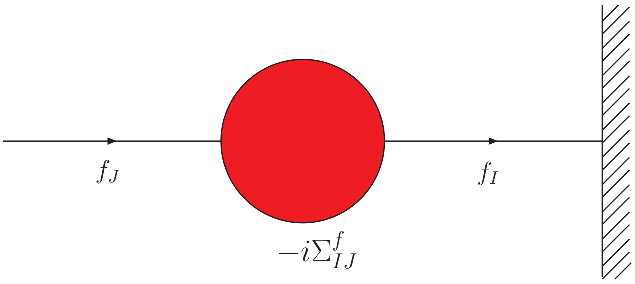

FIG. 1: Flavor-valued wave-function renormalization.

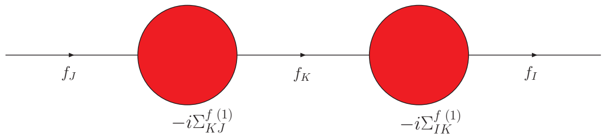

FIG. 2: One-particle irreducible two-loop self-energy constructed out of two one-loop self energies with $I \neq J \neq K$.

taking into account also the leading two-loop corrections. Sec. III is devoted to the numerical analysis. Finally we conclude in Sec. IV.

II. FINITE RENORMALIZATION OF FERMION MASSES AND MIXING MATRICES

We have computed the finite renormalization of the CKM matrix by SQCD effects in Ref. [14, 16] and of the PMNS matrix in Ref. [17]. In this section we compute the finite renormalization of fermion masses and mixing angles induced through one-particle irreducible flavor-valued self-energies beyond leading-order. We first consider the general case and then specify to the MSSM.

A. General formalism

In this section we consider the general effect of one-particle irreducible self-energies. It is possible to decompose any self-energy in its chirality-changing and its chirality-flipping parts in the following way:

$$\begin{aligned}\Sigma^f_{IJ}(p) = &\left(\Sigma^{f\,LR}_{IJ}(p^2) + \not{p}\,\Sigma^{f\,RR}_{IJ}(p^2)\right) P_R \\ &+ \left(\Sigma^{f\,RL}_{IJ}(p^2) + \not{p}\,\Sigma^{f\,LL}_{IJ}(p^2)\right) P_L\,.\end{aligned} \tag{2}$$

Note that chirality-changing parts $\Sigma^{f\,LR}_{IJ}$ and $\Sigma^{f\,RL}_{IJ}$ have mass dimension 1, while $\Sigma^{f\,LL}_{IJ}$ and $\Sigma^{f\,RR}_{IJ}$ are dimensionless. With this convention the renormalization of the fermion masses is given by:

$$m^{(0)}_{f_I} \to m^{(0)}_{f_I} + \Sigma^{f\,LR}_{II}(m^2_{f_I}) + \frac{1}{2} m_{f_I}\left(\Sigma^{f\,LL}_{II}(m^2_{f_I}) + \Sigma^{f\,RR}_{II}(m^2_{f_I})\right) + \delta m_{f_I} = m^{\mathrm{phys}}_{f_I}\,. \tag{3}$$

If the self-energies are finite, the counter-term δm_{f_I} in Eq. (3) is zero in a minimal renormalization scheme like $\overline{\mathrm{MS}}$. In the following we choose this minimal scheme for two reasons: First, A-terms are theoretical quantities which are not directly related to physical observables. For such quantities it is always easier to use a minimal scheme which allows for a direct relation between theoretical quantities and observables. Second, we consider the limit in which the light fermion masses and CKM elements are generated radiatively. In this limit it would be unnatural to have tree-level Yukawa couplings and CKM elements in the Lagrangian which are canceled by counter-terms as in the on-shell scheme.

The self-energies in Eq. (2) do not only renormalize the fermion masses. Also a rotation $1+\Delta U^{f\,L}_{IJ}$ in flavor-space which has to be applied to all external fields is induced through the diagram in Fig. 1:

$$\begin{aligned}\Delta U^{f\,L}_{IJ} &= \frac{1}{m^2_{f_J} - m^2_{f_I}}\left(m^2_{f_J}\Sigma^{f\,LL}_{IJ}\left(m^2_{f_I}\right) + m_{f_J}m_{f_I}\Sigma^{f\,RR}_{IJ}\left(m^2_{f_I}\right) + m_{f_J}\Sigma^{f\,LR}_{IJ}\left(m^2_{f_I}\right) + m_{f_I}\Sigma^{f\,RL}_{IJ}\left(m^2_{f_I}\right)\right) \quad \text{for } I \neq J,\\ \Delta U^{f\,L}_{II} &= \frac{1}{2}\mathrm{Re}\left[\Sigma^{f\,LL}_{II}\left(m^2_{f_I}\right) + 2m_{f_I}\Sigma^{f\,LR\prime}_{II}\left(m^2_{f_I}\right) + m^2_{f_I}\left(\Sigma^{f\,LL\prime}_{II}\left(m^2_{f_I}\right) + \Sigma^{f\,RR\prime}_{II}\left(m^2_{f_I}\right)\right)\right].\end{aligned} \tag{4}$$

The prime denotes differentiation with respect to the argument. The flavor-diagonal part arises from the truncation of flavor-conserving self-energies. Eq. (3) and

Eq. (4) are valid for arbitrary one-particle irreducible self-energies.

B. Self-energies in the MSSM

Self-energies with supersymmetric virtual particles are of special importance because of a possible chiral enhancement which can lead to order-one corrections. In this section we calculate the chirally enhanced (by a factor $\frac{A_f^{IJ}}{M_{SUSY} Y_f^{IJ}}$ or $\tan\beta$) parts of the fermion self-energies in the MSSM. Therefore it is only necessary to evaluate the diagrams at vanishing external momentum.

We choose the sign of the self-energies Σ to be equal to the sign of the mass, e.g. calculating a self-energy diagram yields $-i\Sigma$. Then, with the conventions given in the Appendix A, the gluino contribution to the quark self-energies is given by:

$$\Sigma^{\tilde{g}}_{q_{IL}-q_{JR}} = -\sum_{i=1}^{6} \frac{m_{\tilde{g}}}{16\pi^2} \left(\Gamma^{\tilde{g}\tilde{q}_i}_{q_{JR}}\right)^* \Gamma^{\tilde{g}\tilde{q}_i}_{q_{IL}} B_0(m^2_{\tilde{g}}, m^2_{\tilde{q}_i}) \tag{5}$$

$$= \frac{\alpha_s}{2\pi} C_F \sum_{i=1}^{6} m_{\tilde{g}} W_Q^{(J+3)i*} W_Q^{Ii} B_0(m^2_{\tilde{g}}, m^2_{\tilde{q}_i}) \tag{6}$$

and for the neutralino and chargino contribution to the quark self-energy we receive:

$$\Sigma^{\tilde{\chi}^0}_{d_{IL}-d_{JR}} = -\sum_{i=1}^{6}\sum_{j=1}^{4} \frac{m_{\tilde{\chi}^0_j}}{16\pi^2} \Gamma^{\tilde{\chi}^0_j \tilde{d}_i *}_{d_{JR}} \Gamma^{\tilde{\chi}^0_j \tilde{d}_i}_{d_{IL}} B_0(m^2_{\tilde{\chi}^0_j}, m^2_{\tilde{d}_i}) \tag{7}$$

$$\Sigma^{\tilde{\chi}^\pm}_{d_{IL}-d_{JR}} = -\sum_{i=1}^{6}\sum_{j=1}^{2} \frac{m_{\tilde{\chi}^\pm_j}}{16\pi^2} \Gamma^{\tilde{\chi}^\pm_j \tilde{u}_i *}_{d_{JR}} \Gamma^{\tilde{\chi}^\pm_j \tilde{u}_i}_{d_{IL}} B_0(m^2_{\tilde{\chi}^\pm_j}, m^2_{\tilde{u}_i}) \tag{8}$$

The self-energies in the up-sector are easily obtained by interchanging u and d. We denote the sum of all contribution as:

$$\Sigma^{q\,LR}_{IJ} = \Sigma^{\tilde{g}}_{q_{IL}-q_{JR}} + \Sigma^{\tilde{\chi}^0}_{q_{IL}-q_{JR}} + \Sigma^{\tilde{\chi}^\pm}_{q_{IL}-q_{JR}} \tag{9}$$

Note that the gluino contribution are dominant in the case of non-vanishing A-terms, since they involve the strong coupling constant. In the lepton case, neutralino–slepton and chargino–sneutrino loops contribute the non-decoupling self-energy $\Sigma^{\ell\,LR}_{IJ}$. With the convention in the Appendix A the self-energies are given by:

$$\Sigma^{\tilde{\chi}^\pm}_{\ell_{IL}-\ell_{JR}} = -\sum_{j=1}^{2}\sum_{k=1}^{3} \frac{m_{\tilde{\chi}^\pm_j}}{16\pi^2} \Gamma^{\tilde{\chi}^\pm_j \tilde{\nu}_k *}_{\ell_{JR}} \Gamma^{\tilde{\chi}^\pm_j \tilde{\nu}_k}_{\ell_{IL}} B_0(m^2_{\tilde{\chi}^\pm_j}, m^2_{\tilde{\nu}_k}), \tag{10}$$

$$\Sigma^{\tilde{\chi}^0}_{\ell_{IL}-\ell_{JR}} = -\sum_{i=1}^{6}\sum_{j=1}^{4} \frac{m_{\tilde{\chi}^0_j}}{16\pi^2} \Gamma^{\tilde{\chi}^0_j \tilde{\ell}_i *}_{\ell_{JR}} \Gamma^{\tilde{\chi}^0_j \tilde{\ell}_i}_{\ell_{IL}} B_0(m^2_{\tilde{\chi}^0_j}, m^2_{\tilde{\ell}_i}). \tag{11}$$

Again, we denote the sum of all contribution as:

$$\Sigma^{\ell\,LR}_{IJ} = \Sigma^{\tilde{\chi}^0}_{\ell_{IL}-\ell_{JR}} + \Sigma^{\tilde{\chi}^\pm}_{\ell_{IL}-\ell_{JR}}. \tag{12}$$

With $I = J$ we arrive at the flavor-conserving case. This can lead to significant quantum corrections to fermion masses, but except for the gluino, the pure bino ($\propto g_1^2$) and the negligible small bino-wino mixing ($\propto g_1 g_2$) contribution, they are proportional to tree-level Yukawa couplings. However, if the light fermion masses are generated radiatively from chiral flavor-violation in the soft SUSY-breaking terms, then the Yukawa couplings of the first and second generation even vanish and the latter effect is absent at all. Radiatively generated fermion mass terms via soft tri-linear A-terms corresponds to the upper bound found from the fine-tuning argument where the correction to the mass is as large as the physical mass itself. This fine-tuning argument is based on 't Hooft's naturalness principle: A theory with small parameters is natural if the symmetry is enlarged when these parameters vanish. The smallness of the parameters is then protected against large radiative corrections by the concerned symmetry. If such a small parameter, e.g. a fermion mass, is composed of several different terms there should be no accidental large cancellation between them. We will derive our upper bounds from the condition that the SUSY corrections should not exceed the measured value.

If we restrict ourself to the case with vanishing first and second generation tree-level Yukawa couplings, the off-diagonal entries in the sfermion mass matrices stem from the soft tri-linear terms. Thus we are left with $\delta^{f\,LR}_{IJ}$ only. In the mass insertion approximation with only LR insertion the flavor violating self-energies simplifies. For the gluino (neutralino) self-energies which are relevant for our following discussion for the quark (lepton) case we get:

$$\Sigma^{\tilde{g}}_{q_{IX}-q_{JY}} = \frac{2\alpha_s}{3\pi} M_{\tilde{g}} m_{\tilde{q}_{JY}} m_{\tilde{q}_{IX}} \delta^{q\,XY}_{IJ}\, C_0\left(M_1^2, m^2_{\tilde{q}_{JY}}, m^2_{\tilde{q}_{IX}}\right), \tag{13}$$

$$\Sigma^{\tilde{B}}_{\ell_{IX}-\ell_{JY}} = \frac{\alpha_1}{4\pi} M_1 m_{\tilde{\ell}_{JY}} m_{\tilde{\ell}_{IX}} \delta^{\ell\,XY}_{IJ}\, C_0\left(M_1^2, m^2_{\tilde{\ell}_{JY}}, m^2_{\tilde{\ell}_{IX}}\right). \tag{14}$$

Since the sneutrino mass matrix consists only of a LL block, there are no chargino diagrams in the lepton case with LR insertions at all.

Since the SUSY particles are known to be much heavier than the five lightest quarks it is possible to evaluate the one-loop self-energies at vanishing external momentum and to neglect higher terms which are suppressed by powers of $m^2_{f_I}/M^2_{SUSY}$. The only possibly sizable decoupling effect concerning the W vertex renormalization is a loop-induced right-handed W coupling (see [16]). Therefore Eq. (2) simplifies to

$$\Sigma^{f\,(1)}_{IJ} = \Sigma^{f\,LR\,(1)}_{IJ} P_R + \Sigma^{f\,RL\,(1)}_{IJ} P_L \tag{15}$$

at the one-loop level (indicated by the superscript (1)). In this approximation the self-energies are always chirality changing and contribute to the finite renormalization of the quark masses in Eq. (3) and to the flavor-valued wave-function renormalization in Eq. (4). At the one-loop level we receive the well known result

$$m_{f_I}^{(0)} \to m_{f_I}^{(1)} = m_{f_I}^{(0)} + \Sigma_{II}^{f\;LR\;(1)} \tag{16}$$

for the mass renormalization in the $\overline{\text{MS}}$ scheme. According to Eq. (4) the flavor-valued rotation which has to be applied to all external fermion fields is given by:

$$\Delta U^{f\;L\;(1)} = \begin{pmatrix} 0 & \dfrac{m_{f_2}\Sigma_{12}^{f\;LR\;(1)} + m_{f_1}\Sigma_{12}^{f\;RL\;(1)}}{m_{f_2}^2 - m_{f_1}^2} & \dfrac{m_{f_3}\Sigma_{13}^{f\;LR\;(1)} + m_{f_1}\Sigma_{13}^{f\;RL\;(1)}}{m_{f_3}^2 - m_{f_1}^2} \\ \dfrac{m_{f_1}\Sigma_{21}^{f\;LR\;(1)} + m_{f_2}\Sigma_{21}^{f\;RL\;(1)}}{m_{f_1}^2 - m_{f_2}^2} & 0 & \dfrac{m_{f_3}\Sigma_{23}^{f\;LR\;(1)} + m_{f_2}\Sigma_{23}^{f\;RL\;(1)}}{m_{f_3}^2 - m_{f_2}^2} \\ \dfrac{m_{f_1}\Sigma_{31}^{f\;LR\;(1)} + m_{f_3}\Sigma_{31}^{f\;RL\;(1)}}{m_{f_1}^2 - m_{f_3}^2} & \dfrac{m_{f_2}\Sigma_{32}^{f\;LR\;(1)} + m_{f_3}\Sigma_{32}^{f\;RL\;(1)}}{m_{f_2}^2 - m_{f_3}^2} & 0 \end{pmatrix}. \tag{17}$$

The corresponding corrections to the right-handed wave-functions are obtained by simply exchanging L with R and vice versa in Eq. (17). Note that the contributions of the self-energies $\Sigma_{IJ}^{f\;RL\;(1)}$ with $J > I$ are suppressed by small mass ratios. Therefore, the corresponding off-diagonal elements of the sfermion mass matrices cannot be constrained from the CKM and PMNS renormalization. However, since we treat, in the spirit of Ref. [18], all diagrams in which no flavor appears twice on quark lines as one-particle irreducible, chirally-enhanced self-energies can also be constructed at the two-loop level (see Fig. (2)):

$$\begin{aligned} \Sigma_{IJ}^{f\;RR\;(2)}\left(p^2\right) &= \sum_{K\neq I,J} \frac{\Sigma_{IK}^{f\;RL\;(1)}\Sigma_{KJ}^{f\;LR\;(1)}}{p^2 - m_{f_K}^2}, & \Sigma_{IJ}^{f\;LL\;(2)}\left(p^2\right) &= \sum_{K\neq I,J} \frac{\Sigma_{IK}^{f\;LR\;(1)}\Sigma_{KJ}^{f\;RL\;(1)}}{p^2 - m_{f_K}^2}, \\ \Sigma_{IJ}^{f\;LR\;(2)}\left(p^2\right) &= \sum_{K\neq I,J} m_{f_K}\frac{\Sigma_{IK}^{f\;LR\;(1)}\Sigma_{KJ}^{f\;LR\;(1)}}{p^2 - m_{f_K}^2}, & \Sigma_{IJ}^{f\;RL\;(2)}\left(p^2\right) &= \sum_{K\neq I,J} m_{f_K}\frac{\Sigma_{IK}^{f\;RL\;(1)}\Sigma_{KJ}^{f\;RL\;(1)}}{p^2 - m_{f_K}^2}. \end{aligned} \tag{18}$$

Therefore, the chiral-enhanced two-loop corrections to the masses and the wave-function renormalization are given by:

$$\begin{pmatrix} m_{f_1}^{(0)} \\ m_{f_2}^{(0)} \\ m_{f_3}^{(0)} \end{pmatrix} \to \begin{pmatrix} m_{f_1}^{(0)} + \Sigma_{11}^{f\;LR\;(1)} - \dfrac{\Sigma_{12}^{f\;LR\;(1)}\Sigma_{21}^{f\;LR\;(1)}}{m_{f_2}} - \dfrac{\Sigma_{13}^{f\;LR\;(1)}\Sigma_{31}^{f\;LR\;(1)}}{m_{f_3}} \\ m_{f_2}^{(0)} + \Sigma_{22}^{f\;LR\;(1)} - \dfrac{\Sigma_{23}^{f\;LR\;(1)}\Sigma_{32}^{f\;LR\;(1)}}{m_{f_3}} \\ m_{f_3}^{(0)} + \Sigma_{33}^{f\;LR\;(1)} \end{pmatrix}, \tag{19}$$

$$\Delta U_L^{f\;(2)} = \begin{pmatrix} -\dfrac{\left|\Sigma_{12}^{f\;LR\;(1)}\right|^2}{2m_{f_2}^2} - \dfrac{\left|\Sigma_{13}^{f\;LR\;(1)}\right|^2}{2m_{f_3}^2} & -\dfrac{\Sigma_{13}^{f\;\mathrm{LR}\;(1)}\Sigma_{32}^{f\;\mathrm{LR}\;(1)}}{m_{f_2}m_{f_3}} & \dfrac{\Sigma_{12}^{f\;\mathrm{LR}\;(1)}\Sigma_{23}^{f\;\mathrm{RL}\;(1)}}{m_{f_3}^2} \\ \dfrac{\Sigma_{23}^{f\;\mathrm{RL}\;(1)}\Sigma_{31}^{f\;\mathrm{RL}\;(1)}}{m_{f_2}m_{f_3}} & -\dfrac{\left|\Sigma_{23}^{f\;LR\;(1)}\right|^2}{2m_{f_3}^2} - \dfrac{\left|\Sigma_{12}^{f\;LR\;(1)}\right|^2}{2m_{f_2}^2} & \dfrac{\Sigma_{21}^{f\;\mathrm{LR}\;(1)}\Sigma_{13}^{f\;\mathrm{RL}\;(1)}}{m_{f_3}^2} \\ \dfrac{\Sigma_{32}^{f\;\mathrm{RL}\;(1)}\Sigma_{21}^{f\;\mathrm{RL}\;(1)}}{m_{f_2}m_{f_3}} & -\dfrac{\Sigma_{31}^{f\;\mathrm{RL}\;(1)}\Sigma_{12}^{f\;\mathrm{LR}\;(1)}}{m_{f_2}m_{f_3}} & -\dfrac{\left|\Sigma_{13}^{f\;LR\;(1)}\right|^2}{2m_{f_3}^2} - \dfrac{\left|\Sigma_{23}^{f\;LR\;(1)}\right|^2}{2m_{f_3}^2} \end{pmatrix}, \tag{20}$$

where we have neglected small mass ratios. In the quark case, we already know about the hierarchy of the self-energies from our fine-tuning argument. In this case Eq. (20) is just necessary to account for the unitarity of the CKM matrix [14]. However, the corrections to $m_{f_1}^{(0)}$ in Eq. (19) can be large. For this reason we can also constrain $\Sigma_{31}^{f\,LR\,(1)}$ with 't Hooft's naturalness criterion if at the same time $\Sigma_{13}^{f\,LR\,(1)}$ is different from zero.

III. NUMERICAL ANALYSIS

In this section we are going to give a complete numerical evaluation of the all possible constraints on the SUSY breaking sector from 't Hooft's naturalness argument. This criterion is applicable since we gain a flavor symmetry [14] if the light fermion masses are generated radiatively. Therefore the situation is different from e.g. the little hierarchy problem, where no additional symmetry is involved. First of all, it is important to note that all off-diagonal elements of the fermion mass matrices have to be smaller than the average of their assigned diagonal elements

$$\left(\Delta m_F^2\right)_{XY}^{IJ} < \sqrt{m_{\tilde{f}_{IX}}^2 m_{\tilde{f}_{JY}}^2}, \tag{21}$$

since otherwise one sfermion mass eigenvalue is negative. We note that in Ref. [2] this constraint is not imposed.

All constraints in this section are non-decoupling since we compute corrections to the Higgs-quark-quark coupling which is of dimension 4. Therefore, our constraints on the soft-supersymmetry-breaking parameters do not vanish in the limit of infinitely heavy SUSY masses but rather converge to a constant [14]. However, even though $\delta_{IJ}^{f\,LR}$ is a dimensionless parameter it does not only involve SUSY parameter. It is also proportional to a vacuum expectation and therefore scales like $v/M_{\rm SUSY}$. Thus, our constraints on $\delta_{IJ}^{f\,LR}$ do not approach a constant for $M_{\rm SUSY} \to \infty$ but rather get stronger. Similar effects occur in Higgs-mediated FCNC processes which decouple like $1/M_{\rm Higgs}^2$ rather than $1/M_{\rm SUSY}^2$ [19–21]. However, Higgs-mediated effects can only be induced within supersymmetry in the presence of non-holomorphic terms which are not required for our constraints. An example of a non-decoupling Higgs-mediated FCNC process is the observable $R_K = \Gamma\left(K \to e\nu\right)/\Gamma\left(K \to \mu\nu\right)$ that is currently analyzed by the NA62-experiment. In this case Higgs contributions can induce deviations from lepton flavor universality [10, 22, 23].

A. Constraints on flavor-diagonal mass insertions at one loop

The diagonal elements of the A-terms can be constrained from the fermion masses by demanding that

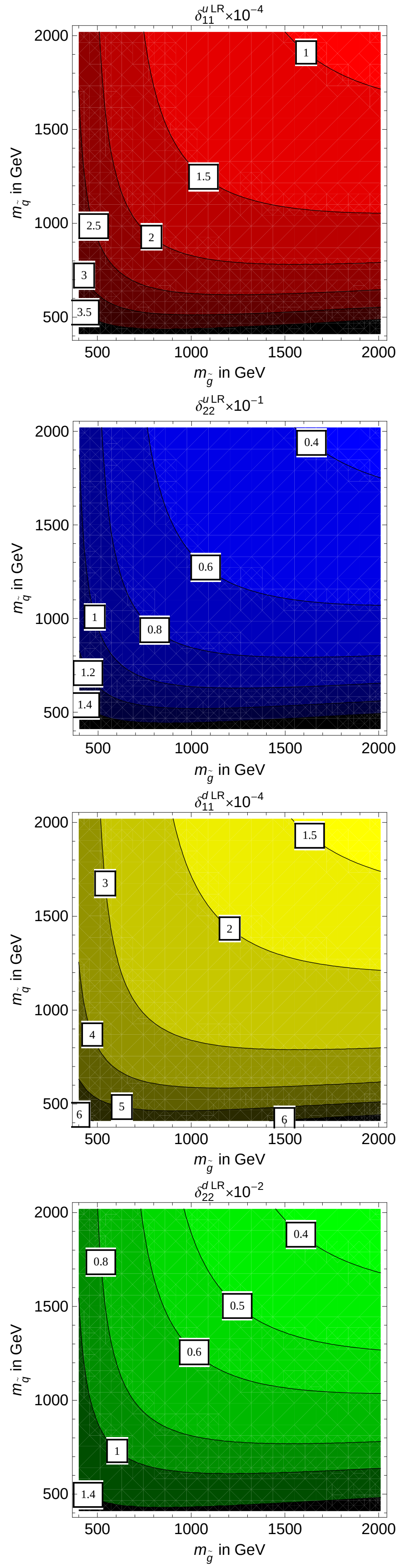

FIG. 3: Constraints on the diagonal mass insertions $\delta_{11,22}^{u,d\,LR}$ obtained by applying 't Hooft's naturalness criterion.

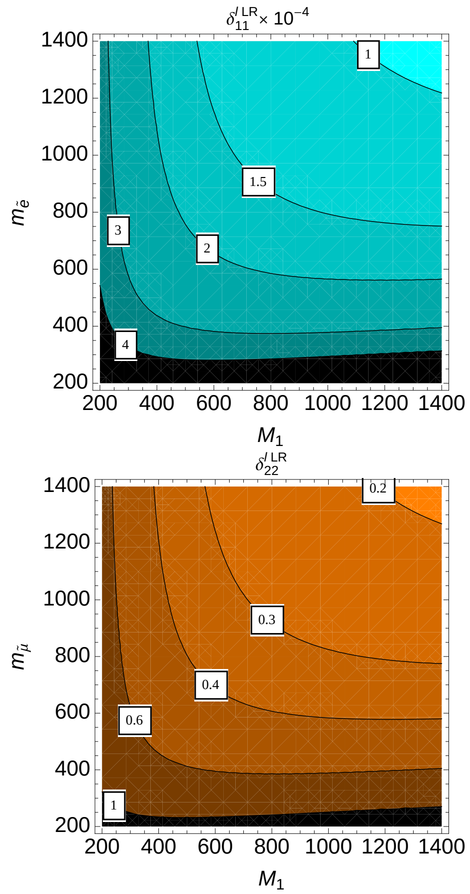

FIG. 4: Contraints on the diagonal mass insertion $\delta^{\ell\,LR}_{11,22}$ as a function of M_1 and $m_{\tilde{e}}$, $m_{\tilde{\mu}}$.

$\Sigma^{f\,LR\,(1)}_{II} \leq m_{f_I}$ [see Eq. (16)]. The bounds on the flavor-conserving A-term for the up, charm, down and strange quarks are shown in Fig. (3) and the constraints from the electron and muon mass are depicted in Fig. (4). The upper bound derived from the fermion mass is roughly given by

$$\left|\delta^{q\,LR}_{II}\right| \lesssim \frac{3\pi\, m_{q_I}(M_{SUSY})}{\alpha_s(M_{SUSY})M_{\rm SUSY}} \tag{22}$$

for quarks and

$$\left|\delta^{\ell\,LR}_{II}\right| \lesssim \frac{8\pi m_{\ell_I}}{\alpha_1 M_{\rm SUSY}} \tag{23}$$

for leptons in the case of equal SUSY masses. In the lepton case Eq. (23) can be further simplified, since we can neglect the running of the masses:

$$\begin{aligned} |\delta^{\ell\,LR}_{11}| &\lesssim 0.0025 \left(\tfrac{500\,\mathrm{GeV}}{M_{\rm SUSY}}\right), \\ |\delta^{\ell\,LR}_{22}| &\lesssim 0.5 \left(\tfrac{500\,\mathrm{GeV}}{M_{\rm SUSY}}\right). \end{aligned} \tag{24}$$

However, as already pointed out in Ref. [24] a muon mass that is solely generated radiatively potentially leads to measurable contributions to the muon anomalous magnetic moment. This arises from the same one-loop diagram as $\Sigma^{\ell\,LR}_{22}$ with an external photon attached. Therefore, the SUSY contribution is not suppressed by a loop factor compared to the case with tree-level Yukawa couplings.

B. Constraints on flavor-off-diagonal mass insertions from CKM and PMNS renormalization

1. CKM matrix

A complete analysis of the constraints for the CKM renormalization was already carried out in Ref. [14]. The numerical effect of the chargino contributions is negligible at low $\tan\beta$ and the neutralino contributions amount only to corrections of about 5% of the gluino contributions. Therefore, we refer to the constraints on the off-diagonal elements $\delta^{q\,LR}_{IJ}$ given in Ref. [14].

2. Threshold corrections to PMNS matrix

Up to now, we have only an upper bound for the matrix element $U_{e3} = \sin\theta_{13} e^{-i\delta}$ and thus for the mixing angle θ_{13}; the best-fit value is at or close to zero: $\theta_{13} = 0.0^{+7.9}_{-0.0}$ [25]. It might well be that it vanishes at tree level due to a particular symmetry and obtains a non-zero value due to corrections. So we can ask the question if threshold corrections to the PMNS matrix could spoil the prediction $\theta_{13} = 0°$ at the weak scale. We demand the absence of fine-tuning for these corrections and therefore require that the SUSY loop contributions do not exceed the value of U_{e3},

$$|\Delta U_{e3}| \leq \left|U^{\rm phys}_{e3}\right|. \tag{25}$$

The renormalization of the PMNS matrix is described in detail in [17], where the on-shell scheme was used. As discussed in Sec. (II) we also use the $\overline{\rm MS}$ scheme in this section. Then the physical PMNS matrix is given by:

$$U^{\rm phys} = U^{(0)} + \Delta U\,, \tag{26}$$

where ΔU should not be confused with the wave function renormalization $\Delta U^{f\,L}$. Then ΔU is given by

$$\Delta U = \left(\Delta U^{\ell\,L}\right)^T U^{(0)}. \tag{27}$$

Note that in contrast to the corrections to the CKM matrix, there is a transpose in $\Delta U^{\ell\,L}$, because the first index of the PMNS matrix corresponds to down-type fermions and not to up-type fermion as in the CKM matrix. Only the corrections to the small element U_{e3} can be sizeable, since all other elements are of order one. If we set all off-diagonal element to zero except for $\delta_{13}^{\ell\,LR} \neq 0$, we get

$$\Delta U_{e3} = \frac{\Delta U_{31}^{\ell\,L} U_{\tau 3}^{\rm phys} - U_{e3}^{\rm phys} \left|\Delta U_{31}^{\ell\,L}\right|^2}{1 + \left|\Delta U_{11}^{\ell\,L}\right|^2} \approx -U_{\tau 3}^{\rm phys} \frac{\Sigma_{31}^{\ell\,RL}}{m_\tau}. \tag{28}$$

Note that here, in contrast to the renormalization of the CKM matrix, the physical PMNS element appears. This is due to the fact that one has to solve the linear system in Eq. (27) as described in [17]. By means of the fine-tuning argument we can in principle derive upper bounds for $\delta_{13}^{\ell\,LR}$. The results depend on the SUSY mass scale $M_{\rm SUSY}$ and the assumed value for θ_{13}.

Here, we consider the corrections stemming from flavor-violating A-terms to the small matrix element U_{e3}. The $\delta_{13}^{\ell\,LL}$-contribution was already studied in [17] with the result that they are negligible small. We also made a comment about the $\delta_{13}^{\ell\,LR}$-contribution which is outlined in more detail. Our results depend on the overall SUSY mass scale, the value of θ_{13} and of $\delta_{13}^{\ell\,LR}$. In Fig. (5) you can see the percentage deviation of U_{e3} through this SUSY loop corrections in dependence of $\delta_{13}^{\ell\,LR}$ (top) and θ_{13} (bottom) for $M_{\rm SUSY} = 1000$ GeV. The constraints on $\delta_{13}^{\ell\,LR}$ get stronger with smaller θ_{13} and with larger $M_{\rm SUSY}$. In Fig. (6) the excluded $\left(\theta_{13}, \delta_{13}^{\ell\,LR}\right)$-region is below the curves for different $M_{\rm SUSY}$ scales. The derived bound can be simplified to

$$\left|\delta_{13}^{\ell\,LR}\right| \lesssim 0.2 \left(\frac{500\,\text{GeV}}{M_{\rm SUSY}}\right) \left|\theta_{13}\,\text{in degrees}\right|. \tag{29}$$

Exemplarily, we get for reasonable SUSY masses of $M_{\rm SUSY} = 1000$ GeV and $\theta_{13} = 3^\circ$ an upper bound of $\left|\delta_{13}^{\ell\,LR}\right| \leq 0.3$. The constraints on $\delta_{13}^{\ell\,LR}$ from $\tau \to e\gamma$ are of the order of 0.02 [17] and in general better than our derived bounds if θ_{13} is non-zero. As an important consequence, we note that $\tau \to e\gamma$ impedes any measurable correction from supersymmetric loops to U_{e3} : E.g. for sparticle masses of 500 GeV we find $|\Delta U_{e3}| \leq 10^{-3}$ corresponding to a correction to the mixing angle θ_{13} of at most 0.06°. That is, if the DOUBLE CHOOZ experiment measures $U_{e3} \neq 0$, one will not be able to ascribe this result to the SUSY breaking sector. Stated positively, $U_{e3} \gtrsim 10^{-3}$ will imply that at low energies the flavor symmetries imposed on the Yukawa sector to motivate tri-bimaximal mixing are violated. This finding confirms the pattern found in [17] where the product $\delta_{13}^{\ell\,LL}\delta_{33}^{\ell\,LR}$ has been studied instead of $\delta_{13}^{\ell\,LR}$.

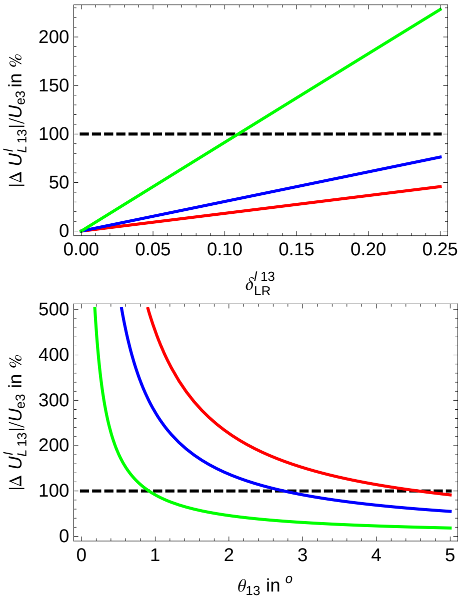

FIG. 5: $|\Delta U_{e3}|/U_{e3}$ in %. Top: as a function of $\delta_{13}^{\ell\,LR}$ for $M_{\rm SUSY} = 1000$ GeV and different values of θ_{13} (green 1°; blue: 3°; red: 5°). Bottom: as a function of θ_{13} for $M_{SUSY} = 1000$ GeV and different values of $\delta_{LR}^{\ell\,13}$ (red: $\delta_{13}^{\ell\,LR} = 0.5$; blue: $\delta_{13}^{\ell\,LR} = 0.3$; green: $\delta_{13}^{\ell\,LR} = 0.1$) (both from top to bottom)

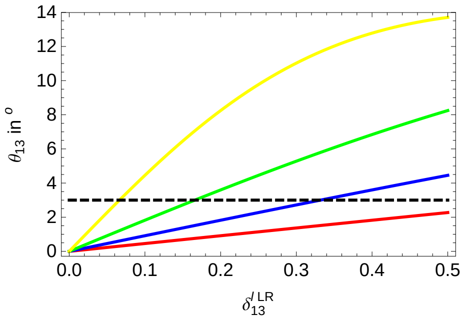

FIG. 6: The excluded $\left(\theta_{13}, \delta_{13}^{\ell\,LR}\right)$-region is below the curves for (from bottom to top) $M_{\rm SUSY} = 500$ GeV (red), 1000 GeV (blue), 2000 GeV (green) and 5000 GeV (yellow). The black dashed line denotes the future experimental sensitivity to $\theta_{13} = 3^\circ$.

C. Constraints from two-loop corrections to fermion masses

Combining two flavor-violating self-energies can have sizable impacts on the light fermion masses according to Eq. (19). Requiring that no large numerical cancellations should occur between the tree-level mass (which is absent in the case of a radiative fermion mass) and the supersymmetric loop corrections we can derive bounds on the products $\delta_{IK}^{f\,LR}\delta_{KI}^{f\,LR}$ which contain the so far less constrained elements $\delta_{KI}^{f\,LR}$, $K > I$.

We apply the fine-tuning argument to the two-loop contribution originating from flavor-violating A-terms, e.g. $\left|\Sigma_{11}^{f\,LR(2)}\right| \leq m_{f_1}$. The bound $\Sigma_{11}^{f\,LR(2)} = m_{f_1}$ corresponds to a 100% change in the fermion mass through supersymmetric loop corrections which is equivalent to the case that the fermion Yukawa coupling vanishes. The upper bound depends on the overall SUSY mass scale and is roughly given as

$$\left|\delta_{I3}^{q\,LR}\delta_{3I}^{q\,LR}\right| \lesssim \frac{9\pi^2\, m_{q_I} m_{q_3}(M_{SUSY})}{(\alpha_s(M_{SUSY})M_{\mathrm{SUSY}})^2},\quad I \neq 3 \qquad (30)$$

for quarks and

$$\left|\delta_{13}^{\ell\,LR}\delta_{31}^{\ell\,LR}\right| \lesssim \frac{64\pi^2 m_{\ell_1} m_{\ell_3}}{(\alpha_1 M_{\mathrm{SUSY}})^2} \qquad (31)$$

for leptons. Again, Eq. (31) can be further simplified

$$\left|\delta_{13}^{\ell\,LR}\delta_{31}^{\ell\,LR}\right| \leq 0.021\left(\frac{500\,\mathrm{GeV}}{M_{\mathrm{SUSY}}}\right)^2 . \qquad (32)$$

The contributions proportional to $\delta_{13}^{f\,LR}\delta_{31}^{f\,LR}$ cannot be important, since these elements are already severely constrained by FCNC processes [26]. As studied in Ref. [27], single-top production involves the same mass insertion $\delta_{31}^{u\,LR}$ which can also induce a right-handed W coupling if at the same time $\delta_{33}^{d\,LR} \neq 0$ [16]. Therefore our bound can be used to place a constraint on this cross section. Also the product $\delta_{23}^{u,\ell\,LR}\delta_{32}^{u,\ell\,LR}$ cannot be constrained, since the muon and the charm are too heavy. However, $\delta_{23}^{d\,LR}\delta_{32}^{d\,LR}$ can be constrained as shown in Fig. (10). Our results for the up, down, and electron mass are depicted in Fig. (8),(9) and (7). In the quark case also the bounds from the CKM renormalization on $\delta_{13,23}^{q\,LR}$ are taken into account.

IV. CONCLUSIONS

According to 't Hooft's naturalness principle, the smallness of a quantity is linked to a symmetry that is restored if the quantity is zero. The smallness of the Yukawa couplings of the first two generations (as well as the small CKM elements involving the third generation) suggest the idea that Yukawa couplings (except for the third generation) are generated through radiative corrections [14, 15, 24, 28–30]. It might well be that the chiral flavor symmetry is broken by soft SUSY-breaking terms rather than by the trilinear tree-level Yukawa couplings.

We use 't Hooft's naturalness criterion to constrain the chirality-changing mass insertion $\delta_{IJ}^{u,d,\ell\,LR}$ from the mass and CKM renormalization. Therefore, we compute the finite renormalization of fermion masses and mixing angles in the MSSM, taking into account the leading two-loop effects. These corrections are not only important, in order to obtain a unitary CKM matrix, they are also numerically important for light fermion masses. This allows us to constrain the product $\delta_{13}^{f\,LR}\delta_{31}^{f\,LR}$ (and $\delta_{23}^{d\,LR}\delta_{32}^{d\,LR}$) which is important, especially with respect to the before unconstrained element $\delta_{13}^{u\,RL}$. All constraints given in this paper are non-decoupling. This means they do not vanish in the limit of infinitely heavy SUSY masses unlike the bounds from FCNC processes. Therefore our constraints are always stronger than the FCNC constraints for sufficiently heavy SUSY (and Higgs) masses.

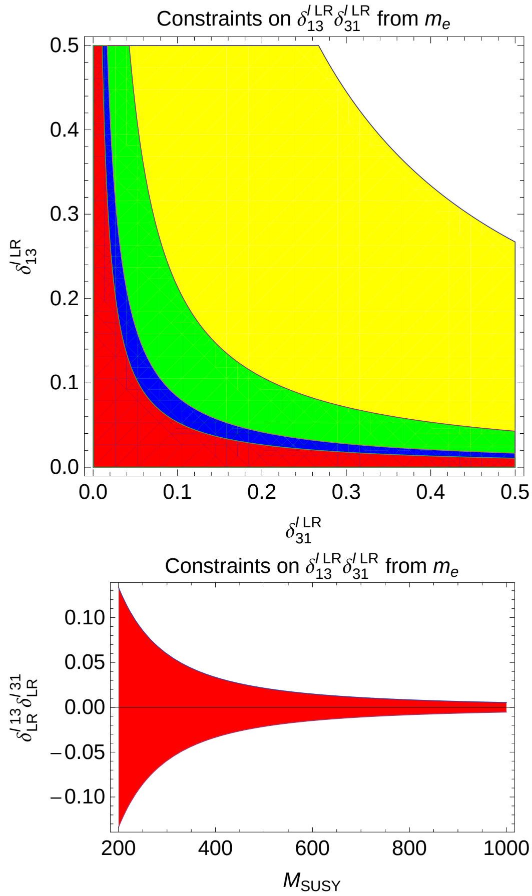

FIG. 7: Results of the two-loop contribution to the electron mass. Above: Region compatible with the naturalness principle for (from top to bottom) $M_{\mathrm{SUSY}} = 200$ GeV (yellow), 500 GeV (green), 800 GeV (blue), 1000 GeV (red). Bottom: Allowed range for $\delta_{13}^{\ell\,LR}\delta_{31}^{\ell\,LR}$ as a function of M_{SUSY}.

The PMNS renormalization is a bit more involved since the matrix is not hierarchical. The radiative decay $\tau \to e\gamma$ severely limits the size of the loop correction ΔU_{e3} to the PMNS element U_{e3}. In a previous paper we have studied this topic for effects triggered by the product $\delta_{13}^{\ell\,LL}\delta_{33}^{\ell\,LR}$ [17]. In this paper we have complemented that analysis by investigating $\delta_{13}^{\ell\,LR}$ instead. Assuming reasonable slepton masses and noting that the Daya Bay neutrino experiment is only sensitive to values of θ_{13} above 3°, we conclude that the threshold corrections to U_{e3} are far below the measurable limit. Consequently, if a symmetry at a high scale imposes tri-bimaximal mixing, SUSY loop corrections cannot spoil this prediction $\theta_{13} = 0$ at the weak scale. This is an important result for the proper interpretation of a measurement of θ_{13}. Thus if DOUBLE CHOOZ or Daya Bay neutrino experiment will measure a non-zero θ_{13} then this is also true at a high energy scale.

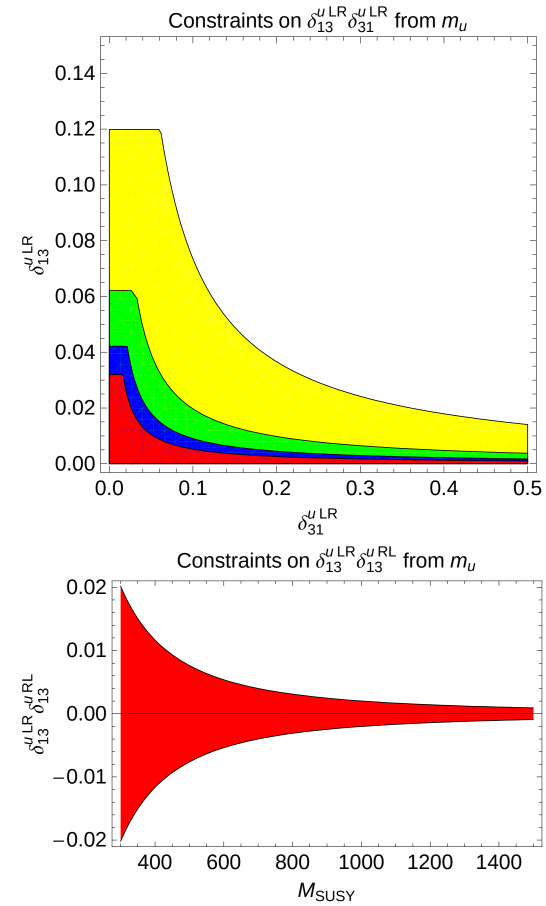

FIG. 8: Results of the two-loop contribution to the up quark mass. Above: Region compatible with the naturalness principle (100% bound) for (from top to bottom) $M_{\mathrm{SUSY}} = 500$ GeV (yellow), 1000GeV (green), 1500 GeV (blue), 2000 GeV (red). Bottom: Allowed range for $\delta_{13}^{u\,LR}\delta_{31}^{u\,LR}$ as a function of M_{SUSY}.

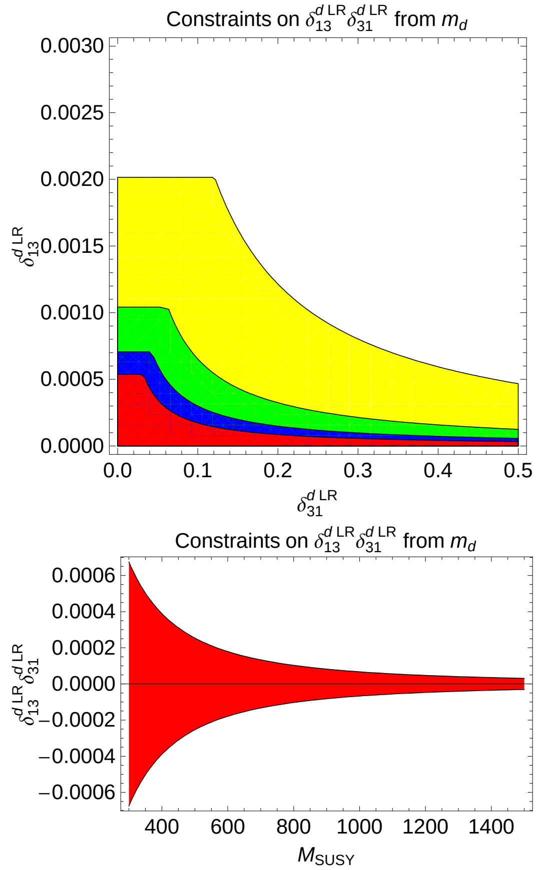

FIG. 9: Results of the two-loop contribution to the down quark mass. Above: Region compatible with the naturalness principle for (from top to bottom) $M_{\mathrm{SUSY}} = 500$ GeV (yellow), 1000GeV (green), 1500 GeV (blue), 2000 GeV (red). Bottom: Allowed range for $\delta_{13}^{d\,LR}\delta_{31}^{d\,LR}$ as a function of M_{SUSY}.

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Appendix A: Conventions

1. Loop integrals

For the self-energies, we need the following loop integrals:

$$B_0(x,y) = -\Delta - \frac{x}{x-y}\ln\frac{x}{\mu^2} - \frac{y}{y-x}\ln\frac{y}{\mu^2}, \quad \text{(A1)}$$

$$\text{with} \quad \Delta = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi.$$

$$C_0(x,y,z) = \frac{xy\ln\frac{x}{y} + yz\ln\frac{y}{z} + xz\ln\frac{z}{x}}{(x-y)(y-z)(z-x)}. \quad \text{(A2)}$$

2. Diagonalization of mass matrices and Feynman rules

For the vacuum expectation value we choose the normalization without the factor $\sqrt{2}$ and define the Yukawa

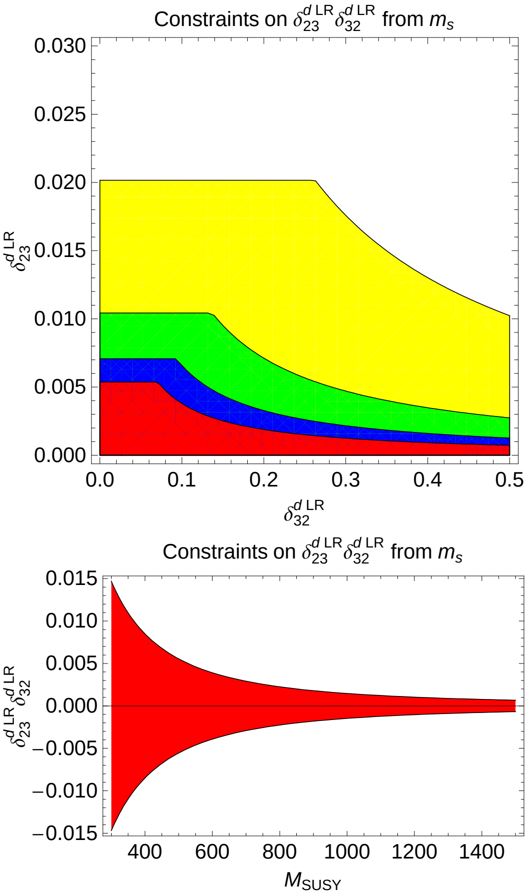

FIG. 10: Results of the two-loop contribution to the strange quark mass. Above: Region compatible with the naturalness principle (100% bound) for (from top to bottom) $M_{\mathrm{SUSY}} =$ 500 GeV (yellow), 1000GeV (green), 1500 GeV (blue), 2000 GeV (red). Bottom: Allowed range for $\delta_{23}^{d\,LR}\delta_{32}^{d\,LR}$ as a function of M_{SUSY}.

couplings in the following way:

$$v = \sqrt{v_u^2 + v_d^2} = 174 \text{ GeV}, \quad \tan\beta = \frac{v_u}{v_d}, \tag{A3}$$

$$m_l = -v_d Y_l, \quad m_d = -v_d Y_d, \quad m_u = v_u Y_u. \tag{A4}$$

Neutralinos $\tilde{\chi}_i^0$

In the following we mainly use the convention of [31].

$$\begin{aligned}
\Psi^0 &= \left(\tilde{B}, \tilde{W}, \tilde{H}_d^0, \tilde{H}_u^0\right), \\
\mathcal{L}_{\tilde{\chi}^0_{mass}} &= -\frac{1}{2}(\Psi^0)^\top M_N \Psi^0 + \text{h.c.} \\
M_N &= \begin{pmatrix} M_1 & 0 & -\frac{g_1 v_d}{\sqrt{2}} & \frac{g_1 v_u}{\sqrt{2}} \\ 0 & M_2 & \frac{g_2 v_d}{\sqrt{2}} & -\frac{g_2 v_u}{\sqrt{2}} \\ -\frac{g_1 v_d}{\sqrt{2}} & \frac{g_2 v_d}{\sqrt{2}} & 0 & -\mu \\ \frac{g_1 v_u}{\sqrt{2}} & -\frac{g_2 v_u}{\sqrt{2}} & -\mu & 0 \end{pmatrix}.
\end{aligned} \tag{A5}$$

M_N can be diagonalised with an unitary transformation such that the eigenvalues are real and positive.

$$Z_N^\top M_N Z_N = M_N^D = \begin{pmatrix} m_{\tilde{\chi}_1^0} & & 0 \\ & \ddots & \\ 0 & & m_{\tilde{\chi}_4^0} \end{pmatrix}. \tag{A6}$$

For that purpose, $Z_N^\dagger M_N^\dagger M_N Z_N = (M_N^D)^2$ can be used. Z_N consists of the eigenvectors of the Hermitian matrix $M_N^\dagger M_N$. Then the columns can be multiplied with phases $e^{i\phi}$, such that $Z_N^T M_N Z_N = M_N^D$ has positive and real diagonal elements.

Charginos $\tilde{\chi}_i^\pm$

$$\begin{aligned}
\Psi^\pm &= \left(\tilde{W}^+, \tilde{H}_u^+, \tilde{W}^-, \tilde{H}_d^-\right), \\
\mathcal{L}_{\tilde{\chi}^\pm_{mass}} &= -\frac{1}{2}\left(\Psi^\pm\right)^\top M_C \Psi^\pm + \text{h.c.} \\
M_C &= \begin{pmatrix} 0 & X^\top \\ X & 0 \end{pmatrix}, \qquad X = \begin{pmatrix} M_2 & g_2 v_u \\ g_2 v_d & \mu \end{pmatrix}.
\end{aligned} \tag{A7}$$

The rotation matrices for the positive and negative charged fermions differ, such that

$$Z_-^T X Z_+ = \begin{pmatrix} m_{\tilde{\chi}_1} & 0 \\ 0 & m_{\tilde{\chi}_2} \end{pmatrix}. \tag{A8}$$

Sleptons

The sleptons $\tilde{L}_2^I = \tilde{e}_{IL}$ and $\tilde{R}^I = \tilde{e}_{IR}^+$ mix to six charged mass eigenstates $\tilde{L}_i$, $i = 1\ldots 6$:

$$\begin{aligned}
&\tilde{L}_2^I = W_L^{Ii*}\tilde{\ell}_i^-, \quad \tilde{R}^I = W_L^{(I+3)i}\tilde{\ell}_i^+, \\
&W_L^\dagger \begin{pmatrix} (m_L^2)_{LL} & (m_L^2)_{LR} \\ (m_L^2)_{RL}^\dagger & (m_L^2)_{RR} \end{pmatrix} W_L = \operatorname{diag}\left(m_{\tilde{\ell}_1}^2, \ldots, m_{\tilde{\ell}_6}^2\right),
\end{aligned}$$

and the slepton mass matrix is composed of

$$\begin{aligned}
(m_L^2)_{LL}^{IJ} &= \frac{e^2\left(v_d^2 - v_u^2\right)\left(1 - 2c_W^2\right)}{4 s_W^2 c_W^2}\delta_{IJ} + v_d^2 Y_{\ell_I}^2 \delta_{IJ} \\
&\quad + (m_{\tilde{L}}^2)_{IJ}^T, \\
(m_L^2)_{RR}^{IJ} &= -\frac{e^2\left(v_d^2 - v_u^2\right)}{2c_W^2}\delta_{IJ} + v_d^2 Y_{\ell_I}^2 \delta_{IJ} + m_{\tilde{e}IJ}^2, \\
(m_L^2)_{LR}^{IJ} &= v_u \mu Y_\ell^{IJ*} + v_d A_\ell^{IJ*}.
\end{aligned}$$

Lepton-slepton-neutralino coupling

Feynman rule for incoming lepton ℓ_I, outgoing neutralino and slepton $\tilde{\ell}_i$:

$$i\Gamma_{\ell_I}^{\tilde{\chi}_j^0\tilde{\ell}_i} = \underbrace{i\left(\frac{W_L^{Ii}}{\sqrt{2}}\left(g_1 Z_N^{1j} + g_2 Z_N^{2j}\right) + Y_{\ell_I} W_L^{(I+3)i} Z_N^{3j}\right)}_{=\Gamma_{\ell_I L}^{\tilde{\chi}_j^0\tilde{\ell}_i}} P_L + \underbrace{i\left(-g_1\sqrt{2}W_L^{(I+3)i} Z_N^{1j*} + Y_{\ell_I} W_L^{Ii} Z_N^{3j*}\right)}_{=\Gamma_{\ell_I R}^{\tilde{\chi}_j^0\tilde{\ell}_i}} P_R. \tag{A9}$$

Lepton-sneutrino-chargino coupling

Feynman rule for incoming lepton ℓ_I, outgoing chargino and sneutrino $\tilde{\nu}_J$:

$$i\Gamma_{\ell_I}^{\tilde{\nu}_J\tilde{\chi}_i^{\pm}} = -i\left(g_2 Z_+^{1i} P_L + Y_{\ell_I} Z_-^{2i*} P_R\right) W_\nu^{IJ*}.$$

Down-squarks

The down-squarks $\tilde{Q}_2^I = \tilde{d}_{IL}$ and $\tilde{D}^I = \tilde{d}_{IR}^*$ mix to six mass eigenstates $\tilde{d}_i$, $i = 1\dots 6$:

$$\tilde{Q}_2^I = W_D^{Ii*}\tilde{d}_i^-, \quad \tilde{D}^I = W_D^{(I+3)i}\tilde{d}_i^+,$$
$$W_D^\dagger \begin{pmatrix} (m_D^2)_{LL} & (m_D^2)_{LR} \\ (m_D^2)_{RL}^\dagger & (m_D^2)_{RR} \end{pmatrix}, W_D = \mathrm{diag}\left(m_{\tilde{d}_1}^2, \dots, m_{\tilde{d}_6}^2\right),$$

and the downs-squark mass matrix is composed of

$$\begin{aligned}
(m_D^2)_{LL}^{IJ} &= -\frac{e^2\left(v_d^2 - v_u^2\right)\left(1 + 2c_W^2\right)}{12 s_W^2 c_W^2}\delta_{IJ} \\
&\quad + v_d^2 Y_{d_I}^2 \delta_{IJ} + (m_{\tilde{Q}}^2)_{IJ}^T, \\
(m_D^2)_{RR}^{IJ} &= -\frac{e^2\left(v_d^2 - v_u^2\right)}{6c_W^2}\delta_{IJ} + v_d^2 Y_{d_I}^2 \delta_{IJ} + m_{\tilde{d}IJ}^2, \\
(m_D^2)_{LR}^{IJ} &= v_u \mu Y_d^{IJ*} + v_d A_d^{IJ*}.
\end{aligned}$$

Up-squarks

Finally, one has six up-squarks $\tilde{u}_i$ composed from fields $\tilde{Q}_1^I = \tilde{u}_{IL}$ and $\tilde{U}^I = \tilde{u}_{IR}^I$

$$\tilde{Q}_1^I = W_U^{Ii}\tilde{u}_i^+, \quad \tilde{D}^I = W_U^{(I+3)i*}\tilde{u}_i^-,$$
$$W_U^T \begin{pmatrix} (m_U^2)_{LL} & (m_U^2)_{LR} \\ (m_U^2)_{RL}^\dagger & (m_U^2)_{RR} \end{pmatrix}, W_U^* = \mathrm{diag}\left(m_{\tilde{u}_1}^2, \dots, m_{\tilde{u}_6}^2\right).$$

$$\begin{aligned}
(m_U^2)_{LL}^{IJ} &= -\frac{e^2\left(v_d^2 - v_u^2\right)\left(1 - 4c_W^2\right)}{12 s_W^2 c_W^2}\delta_{IJ} \\
&\quad + v_u^2 Y_{u_i}^2 \delta_{IJ} + (V m_{\tilde{Q}}^2 V^\dagger)_{IJ}^T, \\
(m_U^2)_{RR}^{IJ} &= \frac{e^2\left(v_d^2 - v_u^2\right)}{3c_W^2}\delta_{IJ} + v_u^2 Y_{u_I}^2 \delta_{IJ} + m_{\tilde{u}IJ}^2, \\
(m_U^2)_{LR}^{IJ} &= -v_d \mu Y_u^{IJ*} - v_u A_u^{IJ*}.
\end{aligned}$$

Quark-squark-gluino coupling

Feynman rule for incoming quark d_I, u_I, outgoing gaugino and squark $\tilde{d}_i$, $\tilde{u}_i$:

$$i\Gamma_{d_I}^{\tilde{g}\tilde{d}_i} = i g_s \sqrt{2} T^a \left(-W_D^{Ii} P_L + W_D^{(I+3)i} P_R\right), \tag{A10}$$

$$i\Gamma_{u_I}^{\tilde{g}\tilde{u}_i} = i g_s \sqrt{2} T^a \left(-W_U^{Ii*} P_L + W_U^{(I+3)i*} P_R\right). \tag{A11}$$

Quark-squark-neutralino coupling

Feynman rule for incoming quark d_I, u_I, outgoing neutralino and squark $\tilde{d}_i$, $\tilde{u}_i$:

$$\begin{aligned}
i\Gamma_{d_I}^{\tilde{\chi}_j^0\tilde{d}_i} =& i\left(\frac{W_D^{Ii}}{\sqrt{2}}\left(-\frac{g_1}{3}Z_N^{1j} + g_2 Z_N^{2j}\right) + Y_{d_I} W_D^{(I+3)i} Z_N^{3j}\right) P_L \\
&+ i\left(-\frac{\sqrt{2}g_1}{3} W_D^{(I+3)i} Z_N^{1j*} + Y_{d_I} W_D^{Ii} Z_N^{3j*}\right) P_R, \\
i\Gamma_{u_I}^{\tilde{\chi}_j^0\tilde{u}_i} =& i\left(\frac{W_U^{Ii*}}{\sqrt{2}}\left(-\frac{g_1}{3}Z_N^{1j} - g_2 Z_N^{2j}\right) - Y_{u_I} W_U^{(I+3)i*} Z_N^{4j}\right) P_L \\
&+ i\left(\frac{2\sqrt{2}g_1}{3} W_U^{(I+3)i*} Z_N^{1j*} - Y_{u_I} W_U^{Ii*} Z_N^{4j*}\right) P_R.
\end{aligned}$$

Quark-squark-chargino coupling

Feynman rule for incoming quark d_I, u_I, outgoing chargino and squark $\tilde{u}_i$, $\tilde{d}_i$:

$$\begin{aligned}
i\Gamma_{d_I}^{\tilde{\chi}_j^{\pm}\tilde{u}_i} =& i\left(-g_2 W_U^{Ji*} Z_+^{1j} + Y_{u_J} W_U^{(J+3)i*} Z_+^{2j}\right) V^{JI} P_L \\
&+ i\left(-Y_{d_I} W_U^{Ji*} Z_-^{2j*}\right) V^{JI} P_R., \\
i\Gamma_{u_I}^{\tilde{\chi}_j^{\pm}\tilde{d}_i} =& i\left(-g_2 W_D^{Ji} Z_-^{1j} - Y_{d_J} W_D^{(J+3)i} Z_-^{2j}\right) V^{JI*} P_L \\
&+ i\left(Y_{u_I} W_D^{Ji} Z_+^{2j*}\right) V^{JI*} P_R.
\end{aligned}$$

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