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Determination of the strong coupling constant from the CLEO measurement of the total hadronic cross section in e^+e^- annihilation below 10.56 GeV

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Abstract

Using recent CLEO III results for the cross section for $e^+e^- \to$ hadrons at seven centre-of-mass energies between 6.964 and 10.538 GeV, we derive a value for the strong coupling constant $\alpha_s(M_Z^2) = 0.110^{-0.010}_{-0.012}{}^{+0.010}_{-0.011}$ where the uncertainties are uncorrelated and correlated, respectively. Our result differs significantly from the one derived by CLEO III, as a consequence of inclusion of quark mass effects and the proper matching between the effective theories with four and five flavours. Combining this new result with an analysis based on earlier cross section measurements in the energy region between 2 and 10.6 GeV, we obtain $\alpha_s(M_Z^2) = 0.119^{+0.009}_{-0.011}$, well consistent with the current world average.

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1 Introduction

A measurement of the total cross section for electron-positron annihilation into hadrons is one of the cleanest methods for the determination of the strong coupling constant α_s [1]. Its determination, based on the hadronic decay rate of the Z boson as measured at the Large Electron-Positron Collider LEP [2], has lead to one of the most precise and theoretically best founded values of this fundamental quantity. Considering the large luminosity of electron-positron colliders at lower energies, similar experimental studies between charm and bottom threshold may lead to an independent measurement of α_s in a completely different energy region, once systematic uncertainties are sufficiently well under control. Although qualitatively similar to the analysis at LEP, the extraction of α_s in this lower energy region differs in many details: i) Radiative corrections lead to relatively large contributions from final states with a hard collinear photon and a hadronic system of correspondingly lower invariant mass. ii) The narrow charmonium and Upsilon resonances contribute through the radiative return and through interferences with the continuum. iii) Since the measurement is performed not very far above threshold for charm production ($2M_D \approx 3.735$ GeV), quark mass effects [3, 4, 5, 6, 7, 8] cannot be neglected. All these points were discussed in detail in Ref. [9], specifically for the energy region close to the Upsilon resonances and accessible to the CLEO experiment at the CESR storage ring. Combining R-measurements between 2 GeV and 10.52 GeV a value of $\alpha_s(M_Z^2) = 0.124^{+0.011}_{-0.014}$ had been derived in Ref. [10] (see also Ref. [11]).

Recently the cross section for $e^+e^- \to$ hadrons has been measured between 6.964 and 10.538 GeV by the CLEO collaboration [12] and expressed in terms of the familiar R ratio, defined by $\sigma(e^+e^- \to \text{hadrons})/\sigma_{\text{point}}$. With a correlated systematic uncertainty of less than 2% this is the most precise measurement in this region. The results for $R(s)$ were used by the CLEO collaboration to extract in a first step $\alpha_s(s)$ for the seven different energies. At this point the approximation of massless quarks was employed. Subsequently, after combining these results and using the renormalization group equation for the running of $\alpha_s(s)$ from the low energy up to M_Z, an average value $\alpha_s(M_Z^2) = 0.126 \pm 0.005^{+0.015}_{-0.011}$ was obtained, where the uncertainties are statistical and systematic, respectively.

In this brief note we will demonstrate that proper inclusion of the aforementioned quark mass effects and performing the renormalization group evolution with the correct matching between the theories with four and five light quark flavours, respectively, leads to a significant shift of the final result for α_s which is equal to the quoted statistical plus systematic uncertainties.

2 Extraction of the strong coupling in the low energy region around $\sqrt{s} = 9$ GeV

As stated in the Introduction, quark mass effects can play a significant role in the analyis of the cross section for $e^+e^- \to$ hadrons, since the centre-of-mass energy is quite comparable to the threshold energy for charm production. From the theory side the complete dependence on the charm quark mass is known up to order α_s^2 [13, 14, 15]. Higher order contributions can be included by taking the massless expansion up to α_s^3 [16, 17, 18, 19, 20, 21] plus the power suppressed terms proportional m_c^2/s [5] and m_c^4/s^2 [8] which are known up to third and (for the quadratic term) even in fourth order [22, 23]. For the analysis discussed below, the α_s^2 approximation is sufficiently precise. However, for completeness all presently known terms up to order α_s^3 are included. Furthermore, mass suppressed terms [24, 25, 26] of order s/m_b^2 from virtual bottom quarks in u, d, s and c production cannot be neglected completely and are included in this analysis. The present analysis is based on the program `rhad` [27], where all these contributions are included.

We start from the results for $R(s)$ as listed in Tab. VII of Ref. [12] and extract

$\sqrt{s}$ (GeV)	$\alpha_s^{(4)}(s)$	$\delta\alpha_s^{\rm stat}$	$\delta\alpha_s^{\rm sys,cor}$	$\delta\alpha_s^{\rm sys,uncor}$	$\alpha_s^{(4)}(s)\vert_{\rm CLEO}$
10.538	0.2113	0.0026	0.0618	0.0444	0.232
10.330	0.1280	0.0048	0.0469	0.0445	0.142
9.996	0.1321	0.0032	0.0516	0.0344	0.147
9.432	0.1408	0.0039	0.0526	0.0291	0.159
8.380	0.1868	0.0187	0.0461	0.0195	0.218
7.380	0.1604	0.0131	0.0404	0.0138	0.195
6.964	0.1881	0.0221	0.0386	0.0134	0.237

Table 1: Results for $\alpha_s^{(4)}(s)$ for the seven different energy values where CLEO performed the measurement of R [12]. Statistical and systematic (common and uncorrelated) uncertainties are displayed separately. The last column shows the result obtained in Ref. [12].

the values for $\alpha_s(s)$. Our results are shown in Tab. 1 with the CLEO values listed for comparison. The systematic and statistical errors, as listed in Tab. 1, are quite similar to those obtained in Ref. [12]. The central values, however, differ significantly.

To combine these results, for each of the seven points a value for the QCD scale parameter $\Lambda \equiv \Lambda_{\rm QCD}$ was derived by the CLEO Collaboration. Subsequently the results were combined into one common value $\Lambda^{(4)}|_{\rm CLEO} = 0.31^{+0.09+0.29}_{-0.08-0.21}$ GeV.

In view of the strong nonlinearity between Λ and α_s, we prefer to use the renormalization group equation to first evolve the seven α_s values to one common energy (taken for convenience 9 GeV) and combine the results (after symmetrizing the errors by adopting the maximum of lower and upper uncertainties, respectively) to

$$\alpha_s^{(4)}(9^2{\rm GeV}^2) \quad = \quad 0.160 \pm 0.024 \pm 0.024\,, \tag{1}$$

where the first error combines statistical and uncorrelated systematic uncertainties and the second one gives the correlated systematic error. The uncertainties have been obtained by minimizing the χ^2 in an analytical way which leads to the proper weights (including correlations) of the individual measurements. The application of standard error propagation[1] leads to the uncertainties given in Eq. (1).

In four-loop accuracy Eq. (1) translates into a QCD scale parameter $\Lambda^{(4)} = 0.18^{+0.14+0.14}_{-0.10-0.10}$ GeV, a result significantly different from the one obtained by the CLEO collaboration ($\Lambda^{(4)}|_{\rm CLEO} = 0.31^{+0.09+0.29}_{-0.08-0.21}$). Adopting the same procedure for the α_s values derived in the massless approximation would lead to $\alpha_s(9^2{\rm GeV}^2) = 0.199 \pm 0.026 \pm 0.039$ and $\Lambda^{(4)}|_{\rm massless} = 0.42^{+0.20+0.31}_{-0.17-0.23}$ GeV. Evidently the results differ again by approximately one standard deviation. The difference between this latter value and $\Lambda^{(4)}|_{\rm CLEO} = 0.31$ GeV is a consequence of the different averaging procedure.

[1] We thank Günter Quast for many discussions in this context.

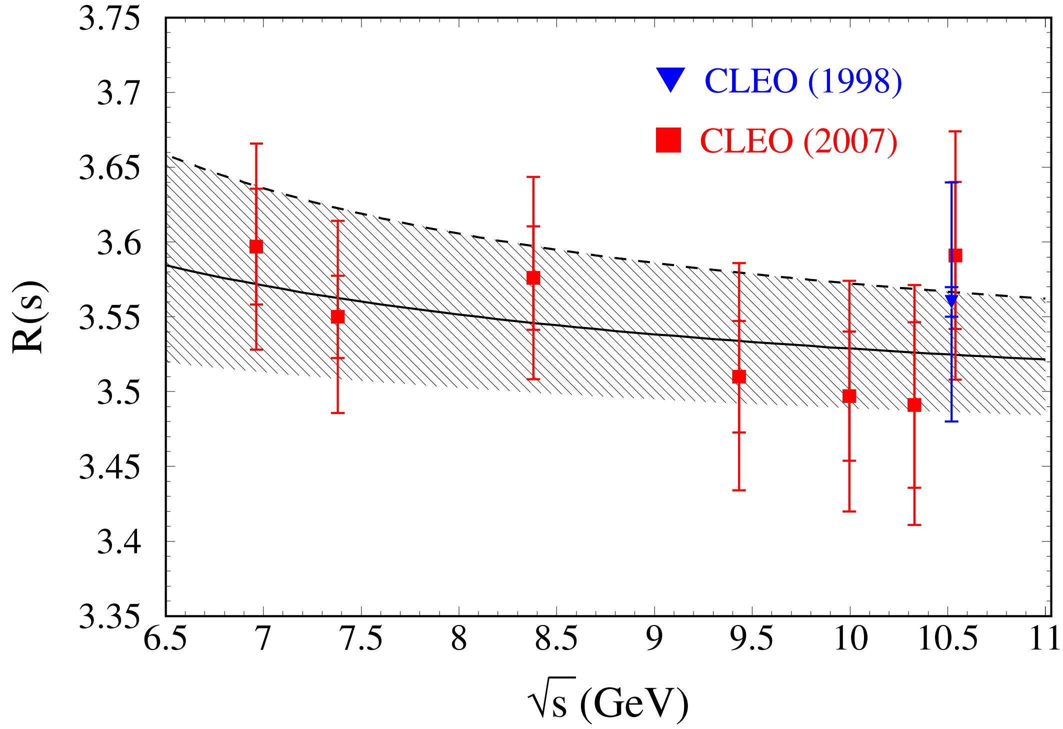

Figure 1: $R(s)$ as measured by CLEO in 1998 [34] and 2007 [12]. The error bars represent the uncorrelated and the total uncertainties. The full solid line and the hatched band correspond to the theory prediction where Eq. (2) has been used as input and the renormalization scale, the charm quark mass and α_s have been varied as described in the text. The dashed line represents the theory prediction where the result from Ref. [12], $\alpha_s^{(5)}(M_Z) = 0.126$, has been used as input.

3 The strong coupling at the scale of M_Z

Using as input the value of Λ as derived before and, furthermore, the three-loop relation between Λ and α_s, evaluated now for five massless flavours, a value for $\alpha_s^{(5)}(M_Z^2)$ is obtained by the CLEO collaboration. However, it is well known [28, 29, 30], that the QCD scale has to be modified ("matching") when crossing flavour thresholds and switching the number of active flavours. Similarly, also the value of α_s has to be adapted when crossing a flavour threshold. (Actually this matching condition is available now up to four-loop order [31, 32].)

Using the `Mathematica` routines provided in the program `RunDec` [33], the $n_f = 4$ result from Eq. (1) can be converted into the strong coupling in the $n_f = 5$ theory, $\alpha_s^{(5)}(9^2\text{GeV}^2) = 0.163 \pm 0.025 \pm 0.025$, which translates into $\Lambda^{(5)} = 0.13^{+0.11+0.11}_{-0.07-0.07}$ GeV.

Using the proper matching and running of the strong coupling from 9 GeV to M_Z we thus obtain from Eq. (1)

$$\alpha_s^{(5)}(M_Z^2) \;=\; 0.110^{+0.010+0.010}_{-0.012-0.011} \;=\; 0.110^{+0.014}_{-0.017}\,, \tag{2}$$

where after the second equality sign the uncertainties have been combined in quadrature. The central value in Eq. (2) differs by one standard deviation[2] from the one of Ref. [12], $\alpha_s^{(5)}(M_Z^2)|_{\rm CLEO} = 0.126 \pm 0.005^{+0.015}_{-0.011}$. In fact, both the inclusion of mass terms in the R ratio and the effect of properly matching[3] at the bottom threshold tend to reduce the result for $\alpha_s^{(5)}(M_Z^2)$. The impact of this difference is evident from Fig. 1 which displays the experimental results for $R(s)$ and the theory predictions based on the α_s value from Eq. (2) (solid line) and the CLEO result ($\alpha_s^{(5)}(M_Z^2) = 0.126$, dashed line). The width of the shaded area represents the uncertainty obtained from the variation of the renormalization scale between $\sqrt{s}/2$ and $2\sqrt{s}$, the charm quark mass between 1.5 GeV and 1.8 GeV, and the error in α_s as given in Eq. (2), where the latter largely dominates. The significant offset of the dashed curve is evident.

It is instructive to combine the result from Eq. (2) with the one obtained in Ref. [10], $\alpha_s^{(4)}(5^2{\rm GeV}^2) = 0.235^{+0.047}_{-0.047}$ and $\alpha_s^{(5)}(M_Z^2) = 0.124^{+0.011}_{-0.014}$, which was based on earlier measurements by BES [35], MD-1 [36] and CLEO [34]. Adding the correlated and uncorrelated errors of the different experiments in quadrature[4], the final result $\alpha_s^{(4)}(9^2{\rm GeV}^2) = 0.182^{+0.022}_{-0.025}$ represents the combined information on the strong coupling from these R measurements in the region below the bottom threshold and corresponds to $\alpha_s^{(5)}(M_Z^2) = 0.119^{+0.009}_{-0.011}$.

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[2] See, e.g., `http://lepewwg.web.cern.ch/LEPEWWG/`.

[2] In case we determine our uncorrelated error under the assumption that the correlated uncertainty is zero we would obtain $\alpha_s(M_Z^2) = 0.110 \pm 0.005^{+0.014}_{-0.016}$, an error decomposition very similar to the one obtained by CLEO.

[3] For completeness let us mention that the CLEO value $\Lambda = 0.31$, if interpreted as $\Lambda^{(4)}$, would translate into $\Lambda^{(5)} = 0.23$ GeV and correspond to $\alpha_s^{(5)}(M_Z^2) = 0.119$.

[4] We treat half of the systematic uncertainty quoted in Ref. [34] as correlated to the new measurements [12] and the other half as uncorrelated.

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