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Flavour $SU(3)$ Symmetry in Charmless B Decays

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Abstract

QCD sum rules are used to estimate the flavour SU(3)-symmetry violation in two-body B decays to pions and kaons. In the factorizable amplitudes the SU(3)-violation manifests itself in the ratio of the decay constants f_K/f_π and in the differences between the $B \to K$, $B_s \to K$ and $B \to \pi$ form factors. These effects are calculated from the QCD two-point and light-cone sum rules, respectively, in terms of the strange quark mass and the ratio of the strange and nonstrange quark-condensate densities. Importantly, QCD sum rules predict that SU(3) breaking in the heavy-to-light form factors can be substantial and does not vanish in the heavy-quark mass limit. Furthermore, we investigate the strange-quark mass dependence of nonfactorizable effects in the $B \to K\pi$ decay amplitudes. Taking into account these effects we estimate the accuracy of several SU(3)-symmetry relations between charmless B-decay amplitudes.

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1 Introduction

The flavour SU(3)-symmetry is frequently used to reduce and control hadronic uncertainties in charmless B decays, while analysing various CP-related observables (for a recent comprehensive review see [1]). The following amplitude relation [2] is a well known example:

$$A(B^- \to \pi^- \bar{K}^0) + \sqrt{2}A(B^- \to \pi^0 K^-) = \sqrt{2}\left(\frac{V_{us}}{V_{ud}}\right) A(B^- \to \pi^- \pi^0)\{1 + \delta_{SU(3)}\}\,, \quad (1)$$

where we neglect electroweak penguin contributions and introduce a parameter $\delta_{SU(3)}$ to quantify the SU(3)-violation, so that $\delta_{SU(3)} = 0$ in the exact symmetry limit.

For a reliable use of Eq. (1) it is desirable to have a QCD-based estimate of $\delta_{SU(3)}$. A usual phenomenological remedy is to relate SU(3) violation to the ratio of the kaon and pion decay constants f_K/f_π and/or to the ratio of $B \to K$ and $B \to \pi$ form factors. Such estimates, however, rely on the factorization approximation with its limited accuracy. Adding nonfactorizable effects, e.g., in the spirit of QCD factorization [3], one has the following schematic expression for a given $B \to P_1P_2$ amplitude ($B = B_{u,d,s}$; $P_{1,2} = \pi, K$):

$$A(B \to P_1P_2) = A_{fact}(B \to P_1P_2)\left\{1 + \frac{\alpha_s C_F}{\pi}\sum_{i=E,P,A,..}\delta_i^{(BP_1P_2)} + \sum_{i=E,P,A,..}\frac{\lambda_i^{(BP_1P_2)}}{m_B}\right\}, \quad (2)$$

where

$$A_{fact}(B \to P_1P_2) = i\frac{G_F}{\sqrt{2}}(m_B^2 - m_{P_1}^2)f_{P_2}f^0_{BP_1}(m_{P_2}^2) \quad (3)$$

is the factorizable amplitude, P_2 being the "emitted" meson with the decay constant f_{P_2}, and $f^0_{BP_1}$ is the $B \to P_1$ transition form factor. For simplicity, all CKM and short-distance factors are not shown. The nonfactorizable corrections are suppressed either by α_s or by inverse powers of the b-quark mass. In Eq. (2) they are parametrized by the process-dependent parameters $\delta_i^{(BP_1P_2)}$ and $\lambda_i^{(BP_1P_2)}$, respectively. The sums indicate that nonfactorizable contributions stem from different effective operators and topologies (emission, penguin, annihilation, etc.). Moreover, certain decay channels receive two factorizable contributions, so that the term $f_{P_1}f_{BP_2}(m_{P_2}^2)$, with its nonfactorizable corrections, has to be added to Eq. (2). There are several sources of SU(3) violation in the $A(B \to P_1P_2)$ amplitudes. The inequalities $f_K \neq f_\pi$ and $f_{BK} \neq f_{B\pi} \neq f_{B_sK}$ reflect flavour-symmetry breaking in the factorizable amplitudes. In addition, differences between the nonfactorizable contributions may also play a role. All separate SU(3)-violating effects have to be accounted and added up in order to obtain the estimate of $\delta_{SU(3)}$ in Eq. (1).

Only the ratio f_K/f_π is known from experiment, revealing quite a noticeable SU(3) violation: $f_K = 160$ MeV and $f_\pi = 131$ MeV. For heavy-to-light form factors and nonfactorizable effects one has to rely on theoretical predictions. Important questions concern therefore the parametrical dependence of various SU(3)-violation effects on the quark-mass difference $m_s - m_{u,d}$. We will take into account all effects of the first order in $m_s - m_{u,d}$ and in several cases also those of $O(m_s^2)$. It is also important to distinguish the SU(3)-violation effects proportional to $(m_s - m_d)/m_b$ from those effects which survive in the $m_b \to \infty$ limit being of $O((m_s - m_{u,d})/M)$, where M is a large scale independent of the heavy quark mass.

In this paper we investigate the flavour SU(3)- symmetry violation in charmless $B \to P_1 P_2$ decays in the framework of QCD sum rules. Within this method, the ratios of hadronic matrix elements are calculated in terms of the quark mass difference $m_s - m_{u,d}$ and the ratios of universal nonperturbative parameters, the strange- and nonstrange-quark condensates.

The content of the paper is as follows: In Sect. 2, we demonstrate how SU(3)-violation reveals itself in QCD sum rules. As a study case we discuss the f_K/f_π ratio estimated from two-point QCD(SVZ) sum rules. In Sect. 3 we employ light-cone sum rules (LCSR) and update some previous calculations obtaining the differences between the relevant $B \to P$ ($B_{u,d} \to \pi$, $B_{u,d} \to K$, $B_s \to K$) form factors. In Sect. 4 we comment on the heavy-mass limit of the SU(3) violation effects in heavy-to-light form factors. Sect. 5 contains the analysis of nonfactorizable corrections in $B \to P_1 P_2$ with kaons and pions, employing LCSR and QCD factorization. In Sect. 6, we calculate the parameter $\delta_{SU(3)}$ in the relation (1) and analyse two other SU(3)-relations.

2 The f_K/f_π ratio from SVZ sum rules

We begin by reminding how the decay constants of pseudoscalar mesons are calculated from QCD sum rules [4]. Comparing the sum rules for f_K and f_π allows to quantify the SU(3) violation.

In the case of the pion, the starting point is the correlation function

$$\begin{aligned}\Pi^{(\pi)}_{\mu\nu}(q) &= i\int d^4x e^{iqx}\langle 0 \mid T\{j^{(\pi)}_\mu(x) j^{(\pi)\dagger}_\nu(0)\} \mid 0\rangle \\ &= -\Pi^{(\pi)}_1(q^2) g_{\mu\nu} + \Pi^{(\pi)}_2(q^2) q_\mu q_\nu, \end{aligned} \tag{4}$$

of two axial-vector quark currents $j^{(\pi)}_\mu = \bar{u}\gamma_\mu\gamma_5 d$. We use the standard definition of the pion decay constant, $\langle 0|j^{(\pi)}_\mu|\pi(q)\rangle = iq_\mu f_\pi$.

One possible way to obtain f_π is to employ the invariant function

$$\Pi^{(\pi)}(q^2) \equiv -\frac{q^\mu q^\nu}{q^2}\Pi^{(\pi)}_{\mu\nu} = \Pi^{(\pi)}_1(q^2) - q^2\Pi^{(\pi)}_2(q^2)\,, \tag{5}$$

and write down the dispersion relation for it:

$$\Pi^{(\pi)}(q^2) = \frac{f_\pi^2 m_\pi^2}{m_\pi^2 - q^2} + \sum_{\pi'} \frac{f_{\pi'}^2 m_{\pi'}^2}{m_{\pi'}^2 - q^2}\,. \tag{6}$$

The r.h.s. contains the ground-state pion contribution proportional to f_π^2; the sum over π' represents, in a simplified form, the dispersion integral over the excited states with the pion quantum numbers. Note that the axial meson $a_1(1260)$ and other hadronic states with $J^P = 1^+$ do not contribute to Eq. (6). The amplitude $\Pi^{(\pi)}(q^2)$ is calculated from Eq. (4) using $\partial_\mu j^{(\pi)}_\mu = i(m_u + m_d)\bar{u}\gamma_5 d$ and employing the standard tools of current algebra. At $O(m_{u,d})$ only the contact term proportional to the quark condensate contributes:

$$\Pi^{(\pi)}(q^2) = -\frac{(m_u + m_d)\langle 0|\bar{u}u + \bar{d}d|0\rangle}{q^2} + O(m_q^2) \tag{7}$$

In order to match Eqs. (6) and (7), one has to admit that the decay constants of excited states decouple in the chiral limit ($f_{\pi'} \sim m_q$). As a result the well-known Gell-Mann-Oakes-Renner relation [5] is reproduced:

$$f_\pi^2 m_\pi^2 = -(m_u + m_d)\langle 0|\bar{u}u + \bar{d}d|0\rangle + O(m_q^2)\,. \tag{8}$$

The analogous relation for the kaon is obtained by replacing $d \to s$ everywhere in this derivation:

$$f_K^2 m_K^2 = -(m_u + m_s)\langle 0|\bar{u}u + \bar{s}s|0\rangle + O(m_s^2)\,. \tag{9}$$

It is important that the light-quark masses are independently extracted from various QCD sum rules. Knowing the value of $m_u + m_d$ one calculates the nonstrange quark condensate density from Eq. (8). We take

$$\langle \bar{q}q\rangle \equiv \langle 0|\bar{u}u|0\rangle \simeq \langle 0|\bar{d}d|0\rangle = -(240 \pm 10 \text{ MeV})^3 \tag{10}$$

in the isospin symmetry limit and at the renormalization scale $\mu = 1$ GeV. In what follows we adopt the chiral limit for the u, d quarks having in mind that $m_{u,d} \ll m_s$. The interval for the strange quark mass is taken

$$m_s(1\text{GeV}) = 130 \pm 20 \text{ MeV}\,, \tag{11}$$

corresponding to $m_s(2\text{ GeV}) = 100 \pm 15$ MeV, obtained in the two recent sum rule analyses [6], in a good agreement with the lattice QCD results. For the strange/nonstrange condensate ratio we adopt

$$\langle \bar{s}s\rangle = (0.8 \pm 0.3)\langle \bar{q}q\rangle\,, \tag{12}$$

in accordance with the early sum rule analyses for strange baryons [7]. This interval also agrees with more recent estimates [8]. We assume that the intervals in Eqs. (11) and (12) are independent from each other [1]. It is well known that a numerical comparison of two sides in Eq. (9) reveals a rather large $O(m_s^2)$ correction to the r.h.s.. Importantly, the latter correction can also be estimated using QCD sum rules for the correlation function $\Pi^{(K)}$ at the $O(m_s^2)$ level [9, 10]. The calculated $O(m_s^2)$ terms bring r.h.s. of Eq. (9) to an agreement with the experimental value of its l.h.s..

In this paper we use an alternative way to calculate f_K and f_π, employing QCD (SVZ) sum rules [4] derived from the invariant amplitude Π_2 in Eq. (4). Taking into account the condensates up to dimension 6 and subtracting the sum rule for f_π^2 from the one for f_K^2 one obtains for the ratio:

$$\frac{f_K^2}{f_\pi^2} = \exp\left(\frac{m_K^2 - m_\pi^2}{M^2}\right)\left\{1 + \left(\frac{M^2}{4\pi^2 f_\pi^2}\left[\exp\left(-\frac{s_0^\pi}{M^2}\right) - \exp\left(-\frac{s_0^K}{M^2}\right)\right]\left(1 + \frac{\alpha_s(M)}{\pi}\right)\right.\right.$$
$$\left.\left. + \frac{m_s\langle\bar{s}s\rangle - m_d\langle\bar{q}q\rangle}{f_\pi^2 M^2} + \frac{16\pi\alpha_s(M)}{81 f_\pi^2 M^4}\left(9\langle\bar{q}q\rangle\langle\bar{s}s\rangle + \langle\bar{s}s\rangle^2 - 10\langle\bar{q}q\rangle^2\right)\right)\exp\left(\frac{m_\pi^2}{M^2}\right)\right\}, \tag{13}$$

where the $O(m_s^2)$ effects are neglected. In this approximation the gluon-condensate contributions cancel in the difference of two sum rules, and the quark-gluon condensate

[1] In QCD the ratio of strange and nonstrange condensates should be correlated with the mass difference of s and u, d quarks. However, it is difficult to trace this correlation within the current accuracy of the sum rules used to estimate these input parameters.

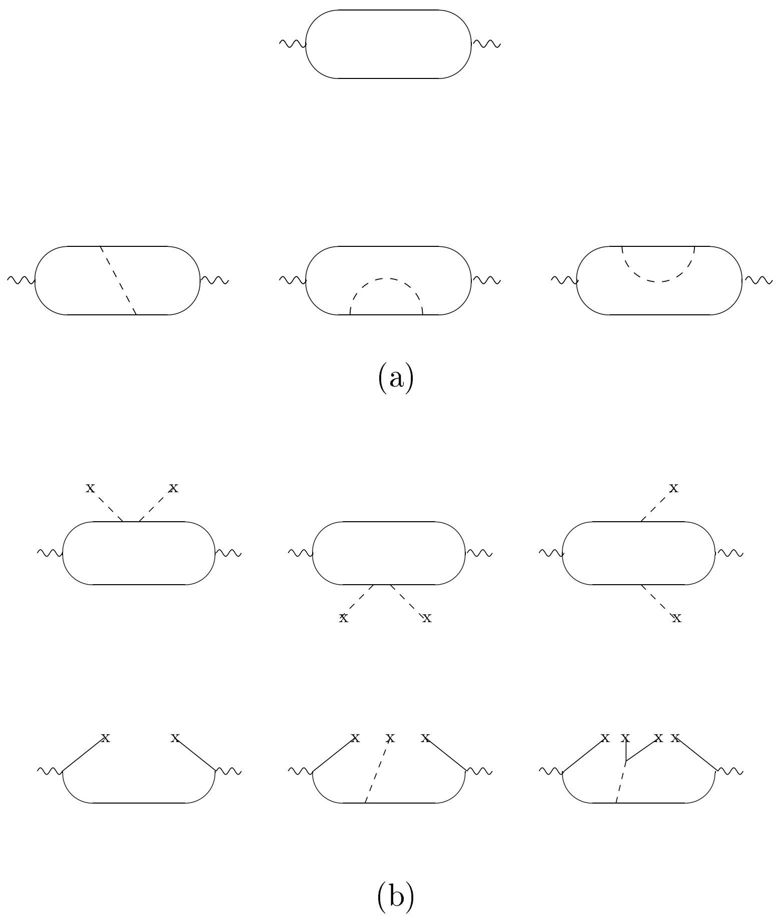

Figure 1: *The diagrams corresponding to the OPE for the correlation function (4): (a) the loop and $O(\alpha_s)$ corrections; (b) the condensate contributions.*

terms vanish. In the above relation the duality threshold parameter $s_0^\pi = 0.7$ GeV^2 and the range of the Borel parameter $0.5 < M^2 < 1.2$ GeV^2 are fixed from the SVZ sum rule for the pion decay constant [4]. The corresponding parameter for the kaon, s_0^K, is fitted, to achieve the maximal stability of the r.h.s. in Eq. (13). We obtain $s_0^K = 1.05 \mp 0.1\ GeV^2$. In Fig. 1 the ratio f_K/f_π is plotted, quite stable with respect to M^2 and in a good agreement with experiment. As expected, the resulting interval $f_K/f_\pi = 1.20 \pm 0.04$ is mainly caused by the uncertainties in m_s and $\langle \bar{s}s\rangle/\langle \bar{q}q\rangle$. The sum rule relation (13) can be further improved by including higher powers of the s quark mass in the sum rule for f_K^2. To give an impression of their magnitude we write down the complete answer for the loop diagram in this sum rule:

$$[f_K^2]_{loop} = \frac{1}{4\pi^2}\int_{m_s^2}^{s_0^K} e^{(m_K^2-s)/M^2}\left(1 - \frac{3m_s^4}{s^2} + \frac{2m_s^6}{s^3}\right) ds\,. \tag{14}$$

Interestingly, the main contribution to the ratio (13) originates from the difference in the threshold parameters for the kaon and pion channel, whereas the quark-condensate term contributes with about 40 %. The 4-quark condensate contribution (factorized [4] into the square of the quark condensates) is small. Note that parametrically $s_0^K \simeq s_0^\pi + 2\sqrt{s_0^\pi}m_s$, i.e., the difference between the threshold parameters is of $O(m_s)$. One can easily expand

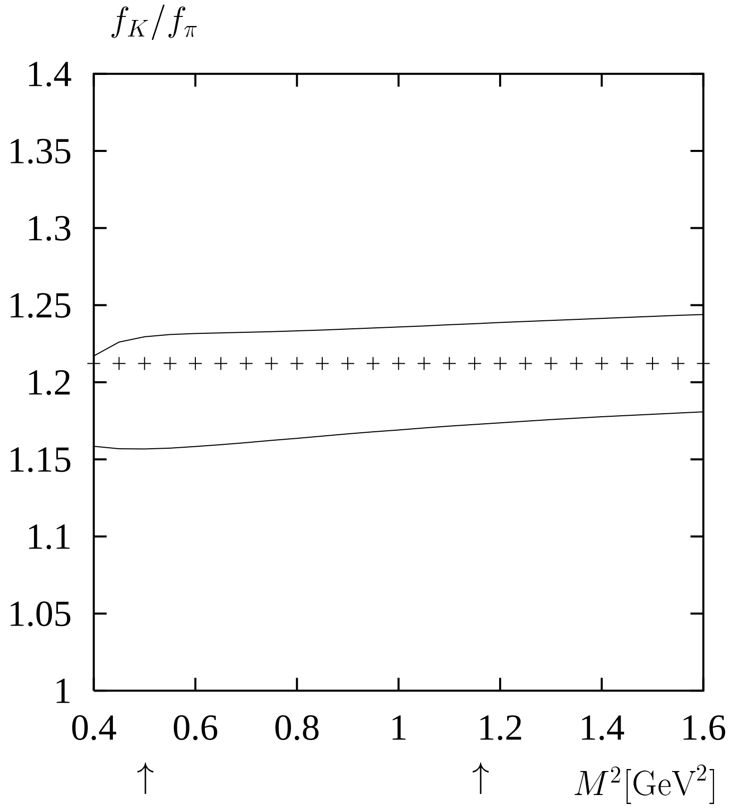

Figure 2: *The ratio f_K/f_π calculated from QCD sum rule (13) as a function of the Borel parameter, in comparison with the experimental value (crosses). The upper and lower solid curves indicate the interval of theoretical uncertainties. The arrows indicate the relevant interval of M^2.*

the ratio (13) in SU(3)-violating quantities m_s and $\langle \bar{s}s\rangle - \langle \bar{q}q\rangle$ obtaining:

$$f_K/f_\pi \simeq 1 + m_s\Big[\frac{\sqrt{s_0^\pi}}{4\pi^2 f_\pi^2}e^{-s_0^\pi/M^2}\left(1+\frac{\alpha_s(M)}{\pi}\right) - \frac{\langle \bar{q}q\rangle}{2f_\pi^2 M^2}\Big] + \\ +\Big(\langle \bar{q}q\rangle - \langle \bar{s}s\rangle\Big)\frac{88\pi\alpha_s\langle \bar{q}q\rangle}{81 M^4 f_\pi^2} + O(m_s^2)\,. \tag{15}$$

The above analysis clearly demonstrates that QCD sum rules directly relate the ratio f_K/f_π with the differences between strange and nonstrange quark masses and condensates. This example justifies the use of sum rules for other $SU(3)$-violating ratios considered below.

3 SU(3) violation in heavy-to-light form factors from LCSR

To obtain the factorizable part of a given $B \to P_1P_2$ amplitude ($B = B_{u,d,s}$; $P_{1,2} = \pi, K$) one needs, in addition to f_π and f_K, the $B \to P$ form factors at the momentum transfer squared $q^2=m_\pi^2\simeq 0$ or $q^2=m_K^2$. We define these form factors in a standard way:

$$\langle P(p)|\bar u\gamma_\mu b|B(p+q)\rangle = f^+_{BP}(q^2)\left[(2p+q)_\mu-\frac{m_B^2-m_P^2}{q^2}q_\mu\right]+f^0_{BP}(q^2)\frac{m_B^2-m_P^2}{q^2}q_\mu\,. \quad (16)$$

In the isospin symmetry limit there are only three flavour combinations: $B_{u,d}\to\pi$, $B_{u,d}\to K$ and $B_s\to K$. Hereafter we drop the flavour index u,d at $B_{u,d}$ retaining it only for B_s. The method of QCD light-cone sum rules (LCSR) [11, 12, 13] is used to calculate the heavy-to-light form factors including SU(3)-violating effects. Here we will concentrate on the latter aspect of this calculation. Recent LCSR determinations of $f^+_{B\pi}(q^2)$ can be found in [14, 15], $f^0_{B\pi}$ was calculated in [16, 17], f^+_{BK} in [18, 17, 14], and $f^+_{B_sK}$ in [19].

Let us recall the basic steps of the LCSR derivation. The correlation function used to calculate the $B\to\pi$ form factors is

$$\begin{aligned}F_\mu(p,q) &= i\int d^4x e^{iqx}\langle\pi^+(p)\mid T\{\bar u\gamma_\mu b(x), m_b\bar b i\gamma_5 d(0)\}\mid 0\rangle\\ &= p_\mu F((p+q)^2,q^2)+q_\mu\widetilde{F}((p+q)^2,q^2)\,. \end{aligned}\quad (17)$$

At large spacelike $(p+q)^2$ and at $q^2\ll m_b^2$ the operator-product expansion (OPE) around the light-cone is used for the product of two currents in Eq. (17). The virtual heavy-quark fields are contracted whereas the light quarks form the light-cone distribution amplitudes (DA) of the pion, e.g., the lowest twist-2 pion DA defined in a standard way:

$$\langle\pi^+(p)\mid\bar u(x)\gamma_\mu\gamma_5 d(0)\mid 0\rangle = -ip_\mu f_\pi\int_0^1 du\, e^{iupx}\,\varphi_\pi(u)\,. \quad (18)$$

The sum rule for $f^+_{B\pi}(q^2)$ is obtained by equating the OPE result for the invariant amplitude F to the dispersion relation in the B-meson channel:

$$F((p+q)^2,q^2)=\frac{2f_Bf^+_{B\pi}(q^2)m_B^2}{m_B^2-(p+q)^2}+\sum_{B_h}\frac{2f_{B_h}f^+_{B_h\pi}(q^2)m^2_{B_h}}{m^2_{B_h}-(p+q)^2}\,, \quad (19)$$

where the ground-state contribution contains the form factor multiplied by the B-meson decay constant f_B. The remaining standard steps of the derivation are: the quark-hadron duality approximation for the sum over higher states in Eq. (19) and the Borel transformation $(p+q)^2\to M'^2$. The resulting LCSR reads:

$$\begin{aligned}f^+_{B\pi}(q^2) &= \frac{f_\pi m_b^2}{2m_B^2 f_B}\int_{u_0}^1\frac{du}{u}\exp\left(\frac{m_B^2}{M'^2}-\frac{m_b^2-q^2\bar u}{uM'^2}\right)\Bigg(\varphi_\pi(u,\mu)+\\ &\frac{\mu_\pi}{m_b}\left[u\varphi_p^{(\pi)}(u,\mu)+\frac{\varphi_\sigma^{(\pi)}(u,\mu)}{3}-\frac{u\varphi_\sigma^{(\pi)\prime}(u,\mu)}{6}\right]\Bigg)+\ldots\,,\end{aligned}\quad (20)$$

where $\bar u=1-u$, $\varphi_\sigma^{(\pi)\prime}(u)=d\varphi_\sigma^{(\pi)}(u)/du$, $u_0=(m_b^2-q^2)/(s_0^B-q^2)$ and s_0^B is the duality-threshold parameter in the B channel. The typical values of the Borel parameter are $M'^2\sim m_B^2-m_b^2$, the same for the normalization scale μ. The twist 3 DA's $\varphi_{p,\sigma}$ are normalized with $\mu_\pi = m_\pi^2/(m_u+m_d)$, nonvanishing in the chiral limit. Additional twist 3 contributions of quark-antiquark-gluon DA, twist 4 effects [20] and $O(\alpha_s)$ corrections [21] are not shown in the above expression but will be taken into account in the numerical calculation.

For the $B \to K$ form factor, one has to simply adjust the quark flavours in the correlation function (17) replacing $u \to s$ in the vector heavy-light current. Accordingly, the sum rule for f^+_{BK} is obtained from Eq. (20) by replacing DA's: $\varphi_\pi \to \varphi_K$, $\varphi^{(\pi)}_{p.\sigma} \to \varphi^{(K)}_{p.\sigma}$, etc. In addition there are trivial "kinematical effects" caused by the shift of the variable $p^2 = m_\pi^2 \simeq 0 \to p^2 = m_K^2$, yielding very small $O(m_K^2/m_b^2)$ variations in the exponent and in the threshold u_0 in Eq. (20). Effects of the same order originate from the variation of the momentum transfer from $q^2 = 0$ to $q^2 = m_K^2$.

Similarly, the correlation function for the $B_s \to K$ transition is obtained by replacing $d \to s$ in the pseudoscalar heavy-light current in Eq. (17). In this case one also has to replace $m_B \to m_{B_s}$ and $f_B \to f_{B_s}$. Note that the 2-point sum rule calculation of the B decay constants includes SU(3)-violation, similar to the case of f_K/f_π. We will use the most recent estimate [22]:

$$\frac{f_{B_s}}{f_B} = 1.16 \pm 0.05, \tag{21}$$

where the uncertainty originates mainly from $\langle \bar{s}s\rangle/\langle \bar{q}q\rangle$ and m_s.

In the following, we will discuss the SU(3) violation in LCSR caused by the differences between the kaon and pion DA's. It is possible to classify and estimate these effects by expanding DA's in the asymptotic and nonasymptotic parts. One then uses two-point QCD sum rules to calculate the relevant nonperturbative parameters entering these expansions. The latter include the normalization factors and coefficients of the nonasymptotic terms at a low normalization scale. The twist-2 DA normalization factors are simply f_π and f_K, so that one does not need a new calculation. The twist-2 pion DA defined in Eq. (18) is symmetric with respect to $u \to \bar{u}$ transformation (in the isospin limit), and the expansion goes over the even Gegenbauer polynomials:

$$\varphi_\pi(u,\mu) = 6u(1-u)\Big[1 + \sum_{n=2,4,6,...} a_n^\pi(\mu) C_n^{3/2}(2u-1)\Big], \tag{22}$$

whereas the kaon DA contains also the odd polynomials

$$\varphi_K(u,\mu) = 6u(1-u)\Big[1 + a_1^K(\mu) C_1^{3/2}(2u-1) + \sum_{n=2,3,4,..} a_n^K C_n^{3/2}(2u-1)\Big]. \tag{23}$$

In the convention adopted here, u is the longitudinal momentum fraction of the strange quark in the kaon.

The coefficient a_1^K is related to the difference between the average momentum fractions of s and $\bar{d}(\bar{u})$ quarks in $\bar{K}^0(K^-)$: $a_1^K = 5/3\langle x_s - x_{u,d}\rangle = 5/3\int_0^1 du(2u-1)\varphi_K(u)$. The parameter a_1 was originally estimated [23] using 2-point sum rules for the kaon-interpolating currents. Recently, the sum rule based on the nondiagonal correlator of pseudoscalar and axial-vector currents was reanalysed in [24] where a sign error in the previous answer [23] for the loop-diagram was found and the important $O(\alpha_s)$ correction was calculated. We will use the numerical estimate obtained in [24] [2]:

$$a_1^K(1\text{GeV}) = -0.18 \pm 0.09\,. \tag{24}$$

[2] We have checked that the signs found in [24] are indeed correct. Note that according to this result

In our numerical analysis the asymptotic DA is taken for φ_π. In order to investigate the uncertainties caused by possible nonasymptotic effects we allow for a nonvanishing coefficient a_2^π . With this simple ansatz, the comparison [25] of the LCSR for the pion e.m. form factor with experiment yields the interval $0 < a_2^\pi(1GeV) < 0.4$. In order to estimate the corresponding a_2^K, we use the relation [23, 24] obtained by subtracting the QCD sum rule for a_2^π from the one for a_2^K (neglecting the $O(\alpha_s)$ parts) :

$$a_2^K = \frac{e^{m_K^2/M^2}}{f_K^2}\left[a_2^\pi f_\pi^2 + \frac{14}{3}\left(\frac{m_s\langle\bar{s}s\rangle}{2M^2} - \frac{5m_s\langle\bar{s}\sigma_{\mu\nu}G^{\mu\nu}s\rangle}{12M^4}\right.\right.$$
$$\left.\left. + \frac{8\pi\alpha_s}{27M^4}\left[3\langle\bar{q}q\rangle\langle\bar{s}s\rangle - 5\langle\bar{q}q\rangle^2 + 2\langle\bar{s}s\rangle^2\right]\right)\right]. \quad (25)$$

In the above $G^{\mu\nu} \equiv g_s G^{a\mu\nu}\frac{\lambda^a}{2}$. The input is the same as in the sum rule for f_K^2/f_π^2 considered in Sect. 2, in addition only the quark-gluon condensate densities have to be specified. For them we adopt:

$$\langle\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle = \left[(0.8 \pm 0.2)\mathrm{GeV}^2\right]\langle\bar{q}q\rangle\,, \quad \frac{\langle\bar{s}\sigma_{\mu\nu}G^{\mu\nu}s\rangle}{\langle\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle} = \frac{\langle\bar{s}s\rangle}{\langle\bar{q}q\rangle}\,. \quad (26)$$

The sum rule (25) yields for the above interval of a_2^π :

$$-0.11 < a_2^K < 0.26\,. \quad (27)$$

which includes the interval obtained in [24]. Note that according to the sum rules the SU(3)-symmetry breaking generates a nonasymptotic part of the kaon DA (both $a_{1,2} \neq 0$) even if the pion DA is purely asymptotic.

Concerning higher twist DA's entering LCSR [3] we first determine the normalization factors. The twist 3 quark-antiquark DA's $\varphi_{p,\sigma}^{(\pi)}$ and $\varphi_{p,\sigma}^{(K)}$ are normalized by $\mu_\pi = m_\pi^2/(m_u+m_d) = -2\langle\bar{q}q\rangle/f_\pi^2$ and $\mu_K = m_K^2/(m_u+m_s)$, fixed by our choice of the quark condensate density and m_s, respectively. The remaining input parameters are the normalization factors $f_{3\pi,3K}$ and $\delta^2_{\pi,K}$ of the twist-3 quark-antiquark-gluon and twist-4 DA's, respectively, as defined in [26, 27]. We use $f_{3\pi} = 0.0035$ GeV^2 and $\delta_\pi^2 = 0.17 \pm 0.05$ GeV^2 determined from the two-point QCD sum rules [28, 23]. To assess the level of SU(3)-violation in these parameters we present in the Appendix a new sum rule calculation of δ_K^2, yielding

$$\frac{\delta_K^2 f_K}{\delta_\pi^2 f_\pi} = 1.07^{+0.14}_{-0.12}\,. \quad (28)$$

For f_{3K}, the sum rule calculation is more complicated and we postpone it to the future. Having in mind the result above, we assume:

$$\frac{f_{3K}}{f_{3\pi}} = 1.0 \pm 0.2\,. \quad (29)$$

the sign of the asymmetry is negative, opposite to the naive expectation for the heavier s-quark to have, in average, a larger longitudinal momentum fraction. To finally establish this important parameter of the kaon DA it is desirable to recalculate it with the same accuracy as in [24] also from the diagonal correlator of the two axial-vector currents, a study which is beyound the scope of this work. So far, only the quark-condensate term of the diagonal sum rule is known [23] yielding a positive sign for a_1.

[3] The complete set of the twist 3,4 DA's of pseudoscalar mesons worked out in [26, 27] can be found, e.g. in the Appendix B of [25].

We also adopt purely asymptotic higher twist DA's, in particular we neglect possible asymmetries in the kaon twist 3,4 DA's analogous to $a_1^K \neq 0$. At the same time, we take into account the mass corrections to the twist 3,4 kaon DA's [27], due to the mixing of various twists at the $O(m_K^2)$ level.

Having specified the DA parameters we are able to calculate the form factors numerically, using LCSR (20) and the analogous sum rules for $B \to K$ and $B_s \to K$ form factors. The remaining input parameters are the same as in [14]: $m_b = 4.7 \pm 0.1$ GeV (the one-loop b-quark pole mass), $s_0^B = 35 \mp 2$ GeV^2 and $M'^2 =$ 8-12 GeV^2. The normalization scale is $\mu_b = m_B^2 - m_b^2$. With the above input we predict $f^+_{B\pi}(0) = 0.25^{+0.05}_{-0.02}$, an interval close to the ones obtained in [14, 15]. Simultaneously, the following ratio of the $B \to K$ and $B \to \pi$ form factors is obtained:

$$f^+_{BK}(0)/f^+_{B\pi}(0) = 1.08^{+0.19}_{-0.17}\,, \tag{30}$$

where the separate uncertainties due to the spread of the independent input parameters are added in quadrature. Note that the s-quark mass and the condensate-ratio dependence of all input parameters in LCSR is taken into account in a correlated way. Numerically, the SU(3) violation effect originates mainly from the ratio of the twist 2 normalization factors f_K/f_π and from the asymmetry $a_1^{(K)} \neq 0$. Both quantities are calculable from 2-point sum rules, as we have seen above. We can thus trace the origin of the ratio (30) to m_s and the ratio of strange and nonstrange condensates. Moreover, the uncertainty of our predictions is to a large extent due to the variation of m_s and of the condensate ratio. The remaining uncertainties in both sum rules, such as the ones caused by the intervals of m_b and M'^2 largely cancel in the ratio.

Turning to the $B_s \to K$ -transition, we note that here the strange-nonstrange asymmetry in the kaon DA has effectively an opposite sign to the $B \to K$ case, because the s quark is now a "spectator". In other words, we can use in LCSR the same DA $\varphi_K(u)$ but with a_1 having an opposite sign. We obtain

$$f^+_{B_sK}(0)/f^+_{B\pi}(0) = 1.40^{+0.12}_{-0.13}\,, \tag{31}$$

quite a substantial effect. Our numerical results (30) and (31) are different from the ones obtained earlier in [18, 17, 14, 19] because of the sign change of the parameter a_1. Note that LCSR predict substantial magnitudes of SU(3) violation also for the $B \to \rho, K^*, \phi$ form factors [30].

In addition, we have checked numerically that the change of the kinematical variable p^2 from zero to m_K^2 in the correlation function as well as the switch to the momentum transfer $q^2 = m_K^2$, being both $O(m_K^2/m_b^2)$ are $\leq 1\%$. Having in mind uncertainties of our calculation we neglect the latter small changes and use in all amplitude relations $f^0_{BP}(m_K^2) \simeq f^0_{BP}(m_\pi^2) = f^0_{BP}(0) = f^+_{BP}(0)$, so that the factorizable amplitudes defined in Eq. (3) are:

$$A_{fact}(B \to P_1P_2) \simeq i\frac{G_F}{\sqrt{2}} m_B^2 f_{P_2} f^+_{BP_1}(0)\,. \tag{32}$$

Finally, using Eqs. (30) and (31) we predict SU(3) violation in the factorizable $B \to$

P_1P_2 amplitudes, for all possible flavour configurations (in the isospin limit):

$$\left.\begin{array}{rcl} A_{fact}(B\to\pi K) &=& \frac{f_K}{f_\pi}A_{fact}(B\to\pi\pi)=1.22 \\ A_{fact}(B\to K\pi) &=& 1.08^{+0.19}_{-0.17} \\ A_{fact}(B\to K\bar{K}) &=& 1.35^{+0.23}_{-0.19} \\ A_{fact}(B_s\to K\bar{K}) &=& 1.74^{+0.15}_{-0.16} \\ A_{fact}(B_s\to K\pi) &=& 1.42^{+0.12}_{-0.13} \end{array}\right\}\times A_{fact}(B\to\pi\pi)\,. \qquad (33)$$

We conclude that in certain cases flavour SU(3) is not a reliable symmetry for charmless B decays. Instead of using SU(3) relations one should better rely on the QCD calculation of separate decay amplitudes.

4 Heavy quark limit of SU(3) violation

With the help of LCSR it is possible to study the $m_b\to\infty$ behaviour of the $B\to P$ form factors. Making the standard substitutions: $m_B^2=m_b^2+2m_b\Lambda$, $s_0^B=m_b^2+2\omega_0 m_b$, so that $u_0^B\simeq 1-\omega_0/m_b$, $M'^2=2m_b\tau$ and $f_B=m_b^{-1/2}\hat{f}_B$ one extracts the heavy mass scale in all m_b-dependent parameters in the sum rule (20), obtaining

$$\lim_{m_b\to\infty} f^+_{B\pi}(0)=m_b^{-3/2}\left\{\frac{f_\pi}{2\hat{f}_B}exp\left(\frac{\Lambda}{\tau}\right)\int_0^{2\omega_0}d\rho\; exp\left(-\frac{\rho}{\tau}\right)\right\}\Big[-\rho\varphi'_\pi(1) \\ +\mu_\pi\left(\varphi_p^{(\pi)}(1)-\frac{\varphi_\sigma^{(\pi)\prime}(1)}{6}\right)\Big]+O(m_b^{-5/2})\,. \qquad (34)$$

Replacing $\pi\to K$ with our choice of twist 2 DA we get $\varphi_\pi\to\varphi_K(u)=6u(1-u)(1+a_1(2u-1))$, so that $-\varphi'_K(1)=1+a_1\sim 1+O\left(m_s/M\right)$ with the scale $M\sim 1$ GeV. We immediately notice that certain SU(3) violating effects survive in the ratio $f_{BK}/f_{B\pi}$ at $m_b\to\infty$. The fact that the flavour SU(3)-symmetry remains broken in the heavy-quark limit seems quite natural. Even if the light quarks in the $B\to P$ transition originate from the decay of a very heavy b quark, there is always a long-distance part of SU(3)-violation manifesting itself in the ratios of normalization constants f_K/f_π, μ_K/μ_π and in the asymmetry in the kaon twist-2 DA.

5 SU(3) violation in nonfactorizable amplitudes

After having calculated the magnitude of SU(3) violation in the factorizable $B\to P_1P_2$ amplitudes, the remaining task is to investigate the SU(3) effects in the process-dependent nonfactorizable contributions. We will mainly concentrate on the charmless decay amplitudes entering the relation (1). The effective weak Hamiltonian is given by

$$H_W=\frac{G_F}{\sqrt{2}}\sum_i\lambda_i c_i O_i\,, \qquad (35)$$

where λ_i , c_i and O_i are the CKM factors, Wilson coefficients and effective operators, respectively. Each decay amplitude can be represented as a decomposition in the hadronic matrix elements of O_i with different contractions of quark lines (topologies) [31]:

$$\begin{aligned}
&A(B \to P_1 P_2) \equiv \langle P_1 P_2 | H_W | B \rangle \\
&= \sum_{T=E,P,A,..} A_T(B \to P_1 P_2) = \frac{G_F}{\sqrt{2}} \sum_{T=E,P,A,..} \sum_i \lambda_i c_i \langle P_1 P_2 | O_i | B \rangle_T \,.
\end{aligned} \tag{36}$$

For the decay channels involved in the relation (1) it is sufficient to consider the hadronic matrix elements of the current-current operators $O_{1,2}$ in the emission topology. These matrix elements are the only ones which enter $A(B^- \to \pi^- \pi^0)$. The additional annihilation and penguin contributions to $A(B^- \to \pi^0 K^-)$ cancel (in the isospin symmetry limit) with the amplitude $A(B^- \to \pi^- \bar{K}^0)$ which contains only annihilation and penguin terms (remember that we neglect electroweak penguins), so that

$$\sqrt{2} A(B^- \to \pi^0 K^-) + A(B^- \to \pi^- \bar{K}^0) = \sqrt{2} A_E(B^- \to \pi^0 K^-) \,. \tag{37}$$

The two relevant amplitudes are given by the following combinations of hadronic matrix elements:

$$\begin{aligned}
A_E(B^- \to \pi^0 K^-) &= \frac{G_F}{\sqrt{2}} V_{us} V_{ub}^* \Bigg\{ \left(c_1 + \frac{c_2}{3} \right) \langle \pi^0 K^- | O_1^{(s)} | B^- \rangle_E + 2 c_2 \langle \pi^0 K^- | \tilde{O}_1^{(s)} | B^- \rangle_E \\
&\quad + \left(c_2 + \frac{c_1}{3} \right) \langle K^- \pi^0 | O_2^{(s)} | B^- \rangle_E + 2 c_1 \langle K^- \pi^0 | \tilde{O}_2^{(s)} | B^- \rangle_E \Bigg\} \\
= \frac{V_{us} V_{ub}^*}{\sqrt{2}} &\left[A_{fact}(B \to \pi K) \left(c_1 + \frac{c_2}{3} + 2 c_2 r_E^{B\pi K} \right) + A_{fact}(B \to K\pi) \left(c_2 + \frac{c_1}{3} + 2 c_1 r_E^{BK\pi} \right) \right],
\end{aligned} \tag{38}$$

$$\begin{aligned}
A(B^- \to \pi^- \pi^0) &= \frac{G_F}{\sqrt{2}} V_{ud} V_{ub}^* \Bigg\{ \left(c_1 + \frac{c_2}{3} \right) \langle \pi^0 \pi^- | O_1^{(d)} | B^- \rangle_E + 2 c_2 \langle \pi^0 \pi^- | \tilde{O}_1^{(d)} | B^- \rangle_E \\
&\quad \left(c_2 + \frac{c_1}{3} \right) \langle \pi^- \pi^0 | O_2^{(d)} | B^- \rangle_E + 2 c_1 \langle \pi^- \pi^0 | \tilde{O}_2^{(d)} | B^- \rangle_E \Bigg\} \\
&= \frac{V_{ud} V_{ub}^*}{\sqrt{2}} A_{fact}(B \to \pi\pi) \left[\frac{4}{3}(c_1 + c_2) + 2(c_1 + c_2) r_E^{B\pi\pi} \right],
\end{aligned} \tag{39}$$

where the current-current operators are $O_1^{(n)} = (\bar{n}\Gamma_\mu u)(\bar{u}\Gamma^\mu b)$, and $O_2^{(n)} = (\bar{u}\Gamma_\mu u)(\bar{n}\Gamma^\mu b)$, ($n = s, d$; $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$) and we used Fierz transformations: $O_{1,2}^{(n)} = \frac{1}{3} O_{2,1}^{(n)} + 2\tilde{O}_{2,1}^{(n)}$, so that $\tilde{O}_1^{(n)} = (\bar{n}\Gamma_\mu \frac{\lambda^a}{2} u)(\bar{u}\Gamma^\mu \frac{\lambda^a}{2} b)$ and $\tilde{O}_2^{(n)} = (\bar{u}\Gamma_\mu \frac{\lambda^a}{2} u)(\bar{n}\Gamma^\mu \frac{\lambda^a}{2} b)$. In the relations (38) and (39) we introduced the ratios of matrix elements in the emission topology:

$$r_E^{(BP_1P_2)} = \frac{\langle P_1 P_2 | \tilde{O}_i^{(n)} | B \rangle_E}{\langle P_1 P_2 | O_i^{(n)} | B \rangle_E} \,, \tag{40}$$

where $i = 1$ or 2 and P_2 is the emitted meson. In the third lines in Eqs. (38),(39) we take into account that, in first approximation, the matrix elements of $O_{1,2}$ coincide with the corresponding factorizable amplitudes. The matrix elements of $\tilde{O}_{1,2}$ accumulate nonfactorizable effects originating from the hard- and soft-gluon exchanges. We will take them into account in $O(\alpha_s)$ and $O(1/m_b)$, respectively. Using the notation introduced in Eq. (2), we separate these two effects:

$$r_E^{(BP_1P_2)} = \frac{\alpha_s C_F}{\pi}\delta_E^{(BP_1P_2)} + \frac{\lambda_E^{(BP_1P_2)}}{m_B}. \tag{41}$$

The hadronic matrix elements of $\tilde{O}_{1,2}$ and correspondingly the ratios $r_E^{BP_1P_2}$ are calculable from LCSR using the method suggested in [32].

To exemplify the LCSR calculation we consider the matrix element $\langle \pi^+ K^- |\tilde{O}_1^{(s)}|\bar{B}^0\rangle_E = r^{(B\pi K)} A_{fact}(B \to \pi K)$. The starting point is the correlation function

$$F_\alpha^{(B\pi K)}(p,q,k) = -\int d^4x\ e^{-i(p-q)x}\int d^4y\ e^{i(p-k)y}\langle 0|T\left\{ j_\alpha^{(K)}(y)\tilde{O}_1^{(s)}(0)j_5^{(B)}(x)\right\}|\pi^-(q)\rangle$$
$$= (p-k)_\alpha F^{(B\pi K)} + ... \tag{42}$$

where $j_\alpha^{(K)} = \bar{u}\gamma_\alpha\gamma_5 s$ and $j_5^{(B)} = im_b\bar{b}\gamma_5 d$ are the quark currents interpolating kaon and B meson, respectively. We only need the invariant amplitude $F^{(B\pi K)}$ which depends on the kinematical invariants $(p-q)^2$, $(p-k)^2$ and $P^2 \equiv (p-q-k)^2$, the other amplitudes in Eq. (42) are denoted by ellipses. Following the derivation in [32], one uses dispersion relations, quark-hadron duality and Borel transformation in both kaon and B meson channels characterized by the variables $(p-k)^2$ and $(p-q)^2$, respectively. The variable P^2 is analytically continued to the physical point m_B^2, so that the artificial momentum k vanishes in the resulting LCSR for the hadronic matrix element:

$$\langle K^-(p)\pi^+(-q)|\tilde{O}_1|\bar{B}^0(p-q)\rangle = \frac{-i}{\pi^2 f_B f_K m_B^2}\int_{m_b^2}^{s_0^B} ds_2\ e^{(m_B^2-s')/M'^2}$$
$$\times \int_{m_s^2}^{s_0^K} ds_1\ e^{(m_K^2-s)/M^2}\mathrm{Im}_{s_2}\ \mathrm{Im}_{s_1}\ F^{(B\pi K)}(s_1,s_2,m_B^2)\,. \tag{43}$$

The amplitude $F^{(B\pi K)}$ and its imaginary part are calculated using light-cone OPE in the domain $(p-k)^2, (p-q)^2, P^2 < 0, |(p-q)^2|, |(p-q)^2|, |P^2| \gg \Lambda_{QCD}$. It is important for the consistency of the method that the factorizable amplitude containing the product of f_K and the LCSR for $B \to \pi$ form factor can be restored [32] from the correlation function similar to Eq. (42) but with the operator O_1. The corresponding tree-level diagram is shown in Fig. 3.

The QCD result for $F^{(B\pi K)}$ is determined by the diagrams shown in Fig. 4,5 which contain an additional gluon exchange that violates factorization. These diagrams represent convolutions of hard-scattering amplitudes formed by virtual quarks and gluons at light-cone separations, with the pion light-cone DA's of growing twist accumulating the long-distance dynamics.

So far, only the soft-gluon part of the sum rule for $B \to \pi\pi$ was obtained [32] resulting in the estimate for $\lambda_E^{(B\pi\pi)}$. Here we will extend this calculation to the channels with kaons in order to obtain $\lambda_E^{(B\pi K)}$ and $\lambda_E^{(BK\pi)}$. The soft-gluon contribution to $B \to \pi K$ originates from the diagram in Fig. 4a,b which are similar to the diagrams determining the LCSR relation for $\lambda_E^{B\pi\pi}$ obtained in [32]. In addition, for the correlation function (42) there are

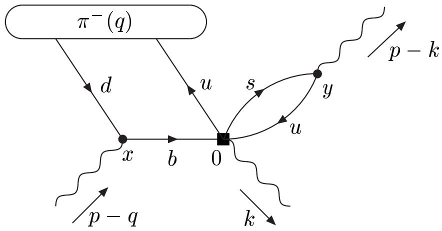

Figure 3: Tree-level diagram corresponding to the correlation function similar to with operator O_1

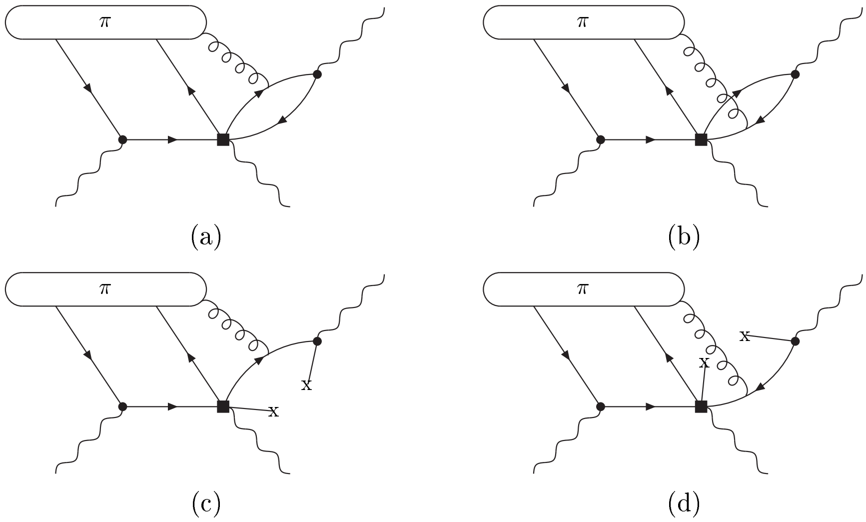

Figure 4: Diagram corresponding to the soft-gluon contributions in the correlation function.

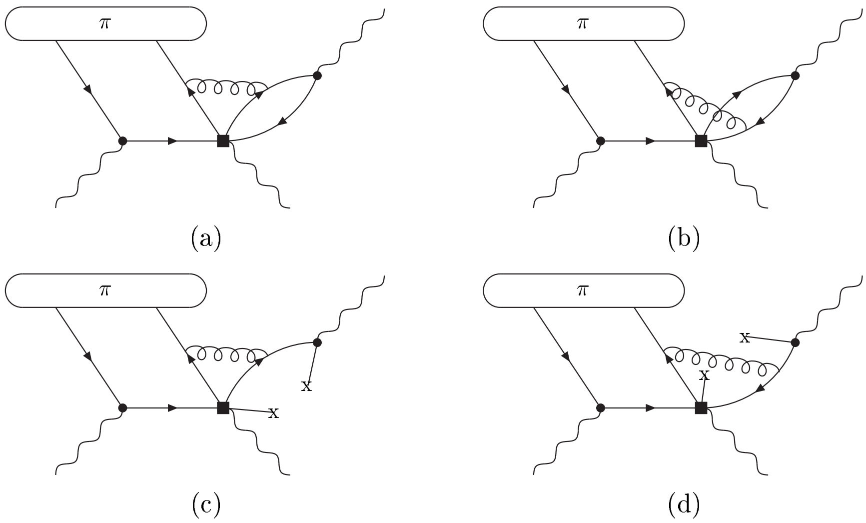

Figure 5: Diagrams corresponding to the $O(\alpha_s)$ contributions in the correlation function:(a,b) hard-gluon (c,d) hard-gluon and quark condensate

new diagrams shown in Fig. 4b,c which are absent in the case of $B \to \pi\pi$ (in the chiral limit). These diagrams correspond to the four-quark-gluon contributions to the pion DA and are factorized in terms of the quark condensate and quark-antiquark-gluon DA. Similar condensate contributions have been taken into account in LCSR for the penguin matrix elements in $B \to \pi\pi$ [33] where one can find a more detailed discussion. The sum rule relation obtained from Eq. (43) reads:

$$\lambda_E^{(B\pi K)} = \frac{m_B}{f^+_{B\pi}(0)}\left(\frac{1}{4\pi^2 f_K^2}\int\limits_0^{s_0^K} ds\; e^{-s/M^2}\right)\left(\frac{m_b^2}{2f_B m_B^4}\int\limits_{u_0^B}^{1}\frac{du}{u^2}e^{m_B^2/M'^2 - m_b^2/uM'^2}\right.$$
$$\times\left[m_b f_{3\pi}\left(1+\frac{4\pi^2 m_s\langle\bar{q}q\rangle}{3M^4}-\frac{4m_s^2}{M^2}\right)\int\limits_0^u\frac{dv}{v}\varphi_{3\pi}(1-u,u-v,v)\right.$$
$$\left.\left.+f_\pi\delta_\pi^2\Big(1+O(m_s\langle\bar{q}q\rangle)\Big)\widetilde{\varphi}_\pi^{tw4}(u)\right]\right)\,, \tag{44}$$

where $\varphi_{3\pi}(\alpha_i) = 360\alpha_1\alpha_2\alpha_3^2$ is the twist-3 quark-antiquark-gluon DA taken in the asymptotic form, $f_{3\pi}$ is the corresponding normalization constant. We have also taken into account the $O(m_s^2)$ correction to the perturbative loop and the twist-3 part of the quark-condensate term. Since this term turned out to be numerically extremely small we have neglected the corresponding $O(m_s\langle\bar{q}q\rangle)$ twist-4 contribution indicated in Eq. (44). The same argument holds for the corrections of order m_s^2/M^2 to the twist-4 part which we calculated but found to be negligible. Consequently, in Eq. (44), $\widetilde{\varphi}^{tw4}(u)$ denotes the same combination of twist 4 quark-antiquark-gluon DA's which enters LCSR for $B \to \pi\pi$, and can be easily read off from Eq.(30) in [32]. Finally, for $f^+_{B\pi}(0)$ we use LCSR (20). Comparing the sum rule for $\lambda^{(B\pi K)}$ with the one for $\lambda^{(B\pi\pi)}$ one immediately recognizes that SU(3) violation originates from the differences in the emitted meson channels: f_K vs f_π , s_0^K vs s_0^π and the absence of the quark-condensate and $O(m_q^2)$ terms in $\lambda^{(B\pi\pi)}$. In the $B \to K\pi$ channel (with the emitted pion) the SU(3) violation with respect to $B \to \pi\pi$ has another origin and is due to the differences between the kaon and pion DA's which were already discussed in the previous section. Thus, in order to obtain the sum rule for $\lambda^{(BK\pi)}$ one has to replace in Eq. (44) $f^+_{B\pi}(0) \to f^+_{BK}(0)$, $f_{K(\pi)} \to f_{\pi(K)}$, $s_0^K \to s_0^\pi$, $m_s \to 0$ (quark condensate terms vanish), $f_{3\pi} \to f_{3K}$, $\varphi_{3\pi} \to \varphi_{3K}$, $\delta_\pi^2 \to \delta_K^2$, $\tilde{\varphi}_\pi^{tw4} \to \tilde{\varphi}_K^{tw4}$. Numerically, we obtain:

$$\lambda_E^{(B\pi\pi)} = 110 \pm 40\text{MeV}\,,\quad \lambda_E^{(B\pi K)} = 120^{+34}_{-43}\text{ MeV}\,,\quad \lambda_E^{(BK\pi)} = 109^{+39}_{-45}\text{ MeV}\,, \tag{45}$$

where the uncertainties are correlated. We find that the magnitude of SU(3)-breaking in $\lambda_E^{(BP_1P_2)}$ is generally smaller than in the form factors revealing that the effects due to m_s and $\langle\bar{s}s\rangle/\langle\bar{q}q\rangle$ largely cancel in the ratios of nonfactorizable and factorizable amplitudes.

The two-loop diagrams in Fig. 5 have not been calculated yet, nevertheless in order to clarify the origin of SU(3) effects it is sufficient to write down the answer for these diagrams in a generic form:

$$\mathrm{Im}_{s_2}\ \mathrm{Im}_{s_1}\ F^{(B\pi K)}(s_1,s_2,m_B^2)^{(Fig.5)} = \frac{\alpha_s C_F}{\pi}\Bigg[T_{5a,b}(s_1,s_2,m_b,m_s^2) + m_s\langle\bar{q}q\rangle T_{5c,d}(s_1,s_2,m_b)\Bigg]\varphi_\pi(s_2/m_b^2)\,, \tag{46}$$

where, for simplicity only the leading twist-2 part is shown. In the above, the indices at the hard amplitudes T denote the corresponding diagrams. Substituting Eq. (47) in Eq. (43) we observe that SU(3)-violation with respect to $B\to\pi\pi$ is again due to the differences in the channel of the emitted meson: 1) $f_K \neq f_\pi$, $s_0^K \neq s_0^\pi$; 2) quark condensate $O(m_s)$ contributions; 3) $O(m_s^2)$ effects. The analogous expression for $B\to K\pi$ is obtained by the following replacements in Eqs. (47),(43): $\varphi_\pi \to \varphi_K$, $s_0^K \to s_0^\pi$, $f_K\to f_\pi$, $m_s\to 0$. As in the case of the soft contribution, now the differences between DA's of kaon and pion determine the SU(3)-violation.

After this qualitative discussion we still need to estimate the hard-gluon contribution numerically. For that we employ QCD factorization. The expressions for the matrix elements can be found in [3] and we will not repeat them here. As an input in this calculation we use the LCSR form factors, and adopt the normalization scale μ_b. In addition we take from [3] the inverse moments of the B meson DA and of the pion twist 3 DA: $\lambda_B = 0.35\pm 0.15$ GeV and $X_\pi^H = 2.4\pm 2.4$ GeV, respectively. The numerical result is:

$$\begin{aligned}
\frac{\alpha_s C_F}{\pi}\delta_E^{(B\pi\pi)} &= (-0.025)\text{-}(+0.044) - 0.045i\,,\\
\frac{\alpha_s C_F}{\pi}\delta_E^{(B\pi K)} &= (-0.035)\text{-}(+0.032) - (0.040\pm 0.002)i\,,\\
\frac{\alpha_s C_F}{\pi}\delta_E^{(BK\pi)} &= (-0.029)\text{-}(+0.055) - 0.045i\,.
\end{aligned} \tag{47}$$

The uncertainties in the real parts are due to the spread in λ_B and $a_2^{\pi,K}$, and the small uncertainty in the imaginary part of $\delta_E^{(B\pi K)}$ is due to a_1^K. Altogether the uncertainties in the real parts overshoot the ones related to the SU(3) breaking. Combining Eqs. (45) and (47) we obtain the parameters $r^{(BP_1P_2)}$ that are needed to complete the calculation of the matrix elements (38) and (39).

Before closing this section, let us mention that the LCSR analysis of nonfactorizable contributions can easily be extended to the matrix elements of the quark-penguin operators O_{3-6} as far as the emission topology is concerned. Because of the $(V+A)$ structure of the operator O_5 (which becomes $(S+P)$ after Fierz transformation), we expect the result to change qualitatively: First, in the chiral limit $m_q\to 0$, the Fig. 4a,b diagrams have to vanish due to chiral symmetry. Consequently, the loop diagram is proportional to m_s if the emitted particle is a kaon, and vanishes if it is a pion. Second, due to the changed Dirac structure of the correlator, the leading twist is 4. This implies that the soft-gluon nonfactorizable correction is suppressed by $1/m_b^2$. In total, we get a result of the form

$$\frac{\lambda_{V+A}^{BP_1K}}{m_B} \sim O\left(\frac{\Lambda_{QCD}}{m_b^2}\right)\left(O(m_s) + O\left(\frac{\langle\bar{s}s\rangle - \langle\bar{u}u\rangle}{M^2}\right)\right)\,. \tag{48}$$

It is interesting to note that also the quark condensate term vanishes if the emitted particle is a pion, as long as we rely on isospin symmetry. We postpone a more detailed study of these contributions, as well as the analysis of the SU(3)-violation in the penguin-topology contributions (generated by current-current and penguin operators) to the future. In fact, some results can already be read off from the LCSR estimates for gluonic penguins and charming penguins [33] replacing pions by kaons. However, in most of $B \to PP$ decay amplitudes, the penguin effects are accompanied by annihilation contributions. The latter have not yet been analysed within the LCSR approach. The annihilation amplitudes with hard-gluon exchanges are also problematic for the QCD factorization approach. Therefore the uncertainties caused by annihilation effects are at the moment certainly larger than any SU(3)-breaking in the penguin amplitudes.

6 How accurate are the SU(3) relations?

After analysing the rate of the SU(3) violation for different elements of the $B \to PP$ amplitudes we are now in a position to return to the relation (1) and calculate the magnitude of its violation representing the individual amplitudes in terms of the factorizable parts and nonfactorizable corrections. As we already mentioned, in this particular relation the penguin and annihilation contributions are absent. We obtain:

$$\delta_{SU(3)} = \frac{(c_1 + c_2/3 + 2c_2 r_E^{B\pi K}) f_K/f_\pi + (c_2 + c_1/3 + 2c_1 r_E^{BK\pi}) f_{BK}/f_{B\pi}}{[(c_1 + c_2/3 + 2c_2 r_E^{B\pi\pi}) + (c_2 + c_1/3 + 2c_1 r_E^{B\pi\pi})]} - 1\,. \tag{49}$$

Using the numerical results for $r_E^{B\pi\pi}$, $r_E^{B\pi K}$ and $r_E^{BK\pi}$ obtained in the previous section and the ratio of form factors (30) we obtain

$$\delta_{SU(3)} = (0.21^{+0.015}_{-0.014}) + \left(0.008^{+0.013}_{-0.015}\right) i. \tag{50}$$

For consistency the Wilson coefficients $c_{1,2}$ have been taken at the same scale μ_b at which the hadronic matrix elements have been calculated from LCSR. Importantly, our result for $\delta_{SU(3)}$ has a rather small uncertainty indicating a moderate SU(3)-breaking in the relation (1) which can be taken into account in the applications of this relation.

To demonstrate that the situation is not always like that, let us consider the U-spin relation

$$A(B_s \to K^+K^-) \simeq A(B_d \to \pi^+\pi^-) \tag{51}$$

which is employed in certain CP-violation studies [1]. From the results obtained above we are able to predict the ratio of factorizable hadronic matrix elements of O_1 for these channels (written without CKM factors).

$$\frac{A_{fact}(B_s \to K^+K^-)}{A_{fact}(B_d \to \pi^+\pi^-)} = \left(\frac{f_K}{f_\pi}\right)\left(\frac{f_{B_sK}(0)}{f_{B\pi}(0)}\right)\frac{m_{B_s}^2 - m_K^2}{m_B^2 - m_\pi^2} = 1.74^{+0.15}_{-0.16}. \tag{52}$$

The nonfactorizable corrections to these relations are more complicated and include annihilation and penguin contributions which are not discussed here. We only notice that the predicted violation of the U-spin is quite substantial. Note that on general grounds there is actually no preference for U-spin symmmetry with respect to the general SU(3). Finally, with our results one can also estimate the accuracy of the other relation

$$A(B_s \to K^+K^-) \simeq A(B_d \to \pi^+K^-) \tag{53}$$

suggested [1] as an estimate for the $B_s \to K^+K^-$ amplitude. We get (neglecting nonfactorizable corrections):

$$\frac{A(B_s \to K^+K^-)_{fact}}{A(B_d \to \pi^+K^-)_{fact}} = \left(\frac{f_{B_sK}}{f_{B\pi}}\right)\frac{m_{B_s}^2 - m_K^2}{m_B^2 - m_\pi^2} = 1.40^{+0.12}_{-0.13}, \tag{54}$$

again, a rather large SU(3)-violation effect.

7 Conclusion

We have demonstrated that QCD sum rules provide *quantitative* estimates of $SU(3)$-violating corrections to the amplitude relations for charmless B decays. Our main goal was to formulate a consistent approach where all relevant hadronic matrix elements (decay constants, form factors and hadronic decay amplitudes) are calculated with the same method (a combination of two-point and light-cone sum rules) and using a universal input (quark masses, condensates, meson distribition amplitudes). The clear advantage of this approach is the possibility to calculate the flavour symmetry-violating corrections in terms of the differences between the s and u, d quark masses and condensates.

For the SU(3) relation we have taken as a study case we predict a moderate correction, with small uncertainties, indicating that the method works, despite the fact that QCD sum rules have limited accuracy. Simultaneosly, we have demonstrated that, according to LCSR, SU(3) violating effects in the heavy-light form factors are not suppressed in the $m_b \to \infty$ limit. Furthermore, the sum rule approach is able to identify the cases where accumulation of several effects leads to a large SU(3)-breaking, such as in the U-spin relation between the factorizable amplitudes $B_s \to K^+K^-$ and $B \to \pi^+\pi^-$. In such cases flavour symmetry is not reliable and an actual QCD calculation for separate decay amplitudes is preferable.

The accuracy of our calculation can still be improved, with a better knowledge of m_s and the nonperturbative parameters of the kaon DA's (a_1^K , a_2^K, δ_K^2 etc.). Note that having at hand precise measurements of $D \to K$ and kaon electromagnetic form factors and comparing the sum rule predictions for these form factors with the data, one may gain a lot of important constraints on these parameters and improve the accuracy of the results obtained above.

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A Normalization parameter of the twist 4 kaon DA

The normalization parameter of the twist-four DA's of the kaon has not been calculated yet. The corresponding normalization for the pion is given by the nonperturbative quantity δ_π^2, defined by the matrix element

$$\langle 0|\tilde{a}_\mu|\pi^+(p)\rangle = -if_\pi \delta_\pi^2 p_\mu \tag{55}$$

of the current

$$\tilde{a}_\mu = \bar{d}\gamma_\rho \tilde{G}_{\rho\mu} u, \tag{56}$$

where $\tilde{G}_{\rho\mu} = \frac{1}{2}\epsilon_{\rho\mu\alpha\beta}G^{\alpha\beta}$. It was determined in [28] using standard two-point sum rules. Two different approaches were used, the results of which where shown to be in a good agreement. The first one is based on the non-diagonal correlator of $\tilde{a}_\mu$ with $j_\nu^{(\pi)}$ and is sensitive to the gluon condensate density. We prefer to use the diagonal correlator

$$\tilde{\pi}_{\mu\nu}(q) = i\int e^{iqx}d^4x\langle 0|T\{\tilde{a}^\dagger_\mu(x), \tilde{a}_\nu(0)\}|0\rangle. \tag{57}$$

In order to calculate δ_K^2 one simply has to replace $d \to s$ in the currents. The correlator consists of two independent structures, $\sim q_\mu q_\nu$ and $\sim g_{\mu\nu}$, of which only the first one, denoted as $\tilde{\pi}(q^2)$, is of interest. Following the standard procedure with dispersion relation, quark-hadron duality and Borel transformation, the sum rule is obtained:

$$\delta_K^4 f_K^2 = \frac{1}{\pi}\int_0^{s_0^K} ds\ \mathrm{Im}_s\tilde{\pi}^{QCD}(s)e^{(m_K^2-s)/M^2}. \tag{58}$$

The intermediate hadronic states are the same as in the sum rule for f_K, so that the hadronic threshold parameter s_0^K and the Borel window are fixed: $s_0^K = 1.05$ GeV^2, 0.5 GeV$^2 < M^2 < 1.2$ GeV^2. For the calculation of the correlator in QCD, we take into account condensates up to dimension 6, except the $d = 5$ quark-gluon condensate which is suppressed. Also the perturbative part shown to be negligible in [28] is left out. Our result reads

$$\delta_K^4 f_K^2 = e^{m_K^2/M^2}\left\{ M^2\left[\frac{\alpha_s m_s}{6\pi}\left(\langle\bar{s}s\rangle - \frac{4}{3}\langle\bar{u}u\rangle\right)\right.\right. \tag{59}$$

$$\left.\left. +\frac{1}{72}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\right] + \frac{8}{9}\pi\alpha_s\langle\bar{s}s\rangle\langle\bar{u}u\rangle\right\} + O\left(m_s^2\right) + O\left(m_s\left\langle\bar{q}Gq\right\rangle\right). \tag{60}$$

In the limit $m_s \to 0$, $m_K \to m_\pi \approx 0$, the quark condensate does not contribute and this expression agrees with the original result for $\delta_\pi^4 f_\pi^2$.

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