# **RESULTS ON** $\blacksquare$ (1232) **RESONANCE PARAMETERS:** A NEW $\pi N$ **PARTIAL WAVE ANALYSIS**

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The residue of the  $\Delta(1232)$  pole derived from a speed plot for the VPI-GWU solution SP00 differs considerably from the value given by the Particle Data Group. An updated version of KH80 is in preparation.

#### 1 Determination of Resonance Parameters from Speed Plots

The  $\Delta(1232)$  can be treated as a resonance in the elastic region. Then the *speed* is defined by (W=total energy in the c.m. frame,  $s = W^2$ )

$$SP(W) = |dT(W)/dW|, \qquad T(W) = \frac{1}{2i} [exp(2i\delta(W) - 1]].$$
(1)

T(W) is the dimensionless P33 partial wave amplitude and  $\delta(W)$  its phase.

It is 'noncontroversial among theorists' (see  $Chew^1$  and the references in my 'pole-emics', p.697 in Ref.<sup>2</sup> that in S-matrix theory the effects of resonances follow from first order poles in the 2nd sheet. Following many other authors, we consider the pole in the *W*-plane nearest to the physical real axis.

The resonant parts of T(W) and of  $\delta(W)$  are

$$T_R(W) = \frac{\Gamma/2}{M - W - i\Gamma/2}, \qquad \tan \delta_R(W) = \frac{\Gamma/2}{M - W}.$$
 (2)

It follows for the speed of the resonant part of the P33 amplitude

$$SP(M) \equiv H = 2/\Gamma, \qquad SP(M \pm \Gamma/2) = H/2.$$
 (3)

Fig.1 shows SP(W) from the VPI-GWU solution SP00. The height *H* and the mass *M* are well defined, whereas the speed at half height shows a small asymmetry due to the background,

$$M = 1210.8 MeV, \quad H = 20.2 GeV^{-1}, \quad \Gamma = 99 MeV.$$
(4)

A comparison with the table of the PDG (p.725 in Ref.<sup>2</sup>) shows an agreement with all earlier determinations of M and  $\Gamma$ . But we obtain  $\Gamma/2 = 49.5 MeV$  for the residue, whereas the value in Ref.<sup>2</sup> is 38 MeV. This is not due to a difference of the P33 phases but to the new method used in the determination from SM95. Our value for

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the residue is in reasonable agreement with the values derived from SM90, KH80 and CMB80.

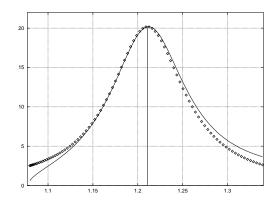


Figure 1. Speedplot for the Resonance  $\Delta(1232)$ . Solid line: from SP00, dots: from Eq.(2) using Eq.(4).

Fig.1 shows that SP(W) calculated from the P33 phase (SP00) at  $W = M \pm \Gamma/2$  almost agrees with the speed calculated from the resonant part of P33 alone. This confirms our result in Eq.(4). Next the background will be discussed <sup>*a*</sup>.

# 2 The Background in the P33 Partial Wave

# 2.1 Contribution of the background to the phase

In the elastic region the background can be described by its phase

$$\delta_B(W) = \delta(W) - \delta_R(W). \tag{5}$$

At W = M we have  $\delta_R = 90^\circ$  whereas the total P33 phase  $\delta$  is much smaller:  $\delta = 66^\circ$ , so  $\delta_B = -24^\circ$ . Fig. 2 (left panel) shows that the *W*-dependence of  $\delta_B$  is almost negligible in the range  $W = M \pm \Gamma/2$ .

If the background is taken into account, the T-matrix element for elastic scattering can be written

$$T(W) = T_B(W) + T_R(W) \exp(i\phi(W)), \quad T_B(W) = \sin(\delta_B) \exp(i\delta_B).$$
(6)

Elastic unitarity demands that  $\phi(W) = 2\delta_B(W)$ . The residue of the pole term is now complex-valued. Our calculation gives  $\phi(M) = -48^\circ$ . A determination of  $\phi$  from an *Argand plot of the speed vector dT/dW* gives the same result<sup>4</sup>. Again we find a

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<sup>&</sup>lt;sup>a</sup>see Ref.<sup>6</sup> for details and further references

large discrepancy with the value of the PDG<sup>2</sup> from SM95  $\phi = -22^{\circ}$  and also with the value from SM90, but agreement with KH80 and CMB80.

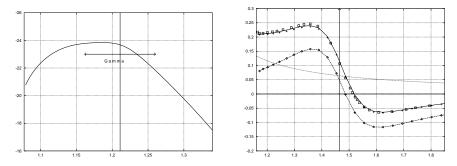


Figure 2. Left: Background phase in degrees vs. W in GeV. Right: Contributions to  $ReT/q^3 * m_{\pi}^{-3}$  vs. s in  $(GeV^2)$ .

Upper solid line: from SP00 (nearby squares from KA85), nearby circles from the r.h. side of Eq.(7). Background L(s): decreasing line, solid line with circles: dispersion integral, evaluated with ImT(SP00).

## 2.2 The dispersion relation for the P33 partial wave

The dispersion relation was studied in great detail by J. Hamilton et al. who showed that an approximation led to a *relativistic Chew-Low plot*<sup>3</sup>. An improved version was evaluated by R. Koch et al.<sup>7,8</sup>, using KH80 and t-channel partial waves of our group<sup>5</sup> as input.

The relation is written for the *reduced amplitude*  $F(s) = T(s)/q^3$  in order to suppress the contributions of distant sigularities in the s-plane which are neglected in our simplified calculation

$$ReF(s) = L(s) + \frac{1}{\pi} \int_{sth}^{\infty} \frac{ImF(s')}{s' - s} ds' .$$
(7)

According to table 2 in Ref.<sup>8</sup>, the dominant contributions to L(s) come from the uchannel nucleon Born term (Chew-Low) and the t-channel S-wave. The sum can be approximated by an effective pole (m=nucleon mass)

$$L(s) = \frac{0.037}{s - m^2} \quad \text{units:} \ m_{\pi}^{-3}, s \text{ in } GeV^2$$
(8)

An accurate evaluation has recently been made by J. Stahov.

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# 3 An Updated Version of the KH80 Partial Wave Analysis

Since fixed-t analyticity can be proven within the framework of QCD<sup>9</sup>, *it is necessary* to include this constraint in  $\pi N$  partial wave analysis. Using the main part of a version of E. Pietarinen's program rewritten for a PC in 1992, H.M. Staudenmaier, C. Hansch and G. H. have produced a program which is running on our workstation alpha, including the graphics and a new data base. Since KH80 has a problem with new spin-rotation data<sup>11</sup>, our earlier study of the zero trajectories has been taken up again, taking into account the important papers by I.S. Stefanescu. They include consequences of two-variable analyticity and questions of uniqueness and stability (see<sup>10</sup> for a review). - We hope that the test runs can be finished in 2001.

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