TTP96-35
DTP/96/80
hep-ph/9609411
February 1997

Extracting α_s from Electron–Positron Annihilation around 10 GeV

K.G. Chetyrkin[a], J.H. Kühn[b], T. Teubner[c]

[a] Max-Planck-Institut für Physik, Werner-Heisenberg-Institut,
Föhringer Ring 6, D-80805 Munich, Germany

[b] Institut für Theoretische Teilchenphysik,
Universität Karlsruhe, D-76128 Karlsruhe, Germany

[c] Department of Physics,
University of Durham, Durham, DH1 3LE, UK

Abstract

The total cross section for electron–positron annihilation into hadrons is calculated in the region just below the B meson threshold. QCD corrections up to third order, quark mass effects, initial state radiation, the running of $\alpha_{\rm QED}$ and the tails of the Υ resonances are included in the prediction. For given α_s the prediction is accurate to 0.5 percent. An experimental measurement with the corresponding precision would allow to determine α_s with high accuracy at intermediate energies.

1. Introduction

One of the most precise and from the theoretical point unambiguous determinations of the strong coupling constant α_s has been obtained from the hadronic decay rate of the Z boson. Ingredients are the large event rate which allows to reduce the statistical uncertainty in α_s down to 0.003, the precise calibration through the accurate measurement of the luminosity or the leptonic rate [1] and, last but not least, the elaborate theoretical calculations [2, 3]. This high energy determination of α_s should and could be complemented by a similar measurement of $R_{\rm had}$ in the energy region just below the B meson threshold: large event rates are available at the e^+e^- storage ring CESR and an experimental analysis is currently underway [4]. Theoretical predictions for $R_{\rm had}$ below the $B\overline{B}$ threshold have been presented earlier [5]. These include terms up to order α_s^3 for the massless limit and for m_c^2/s corrections as well, terms of order α_s^2 for the quartic mass terms $(m_c^2/s)^2$, terms of order $\alpha_s(m_c^2/s)^3$ and contributions from virtual bottom quarks up to order $\alpha_s^2(s/m_b^2)$. Recently it has been demonstrated that this expansion in (m_c^2/s) provides an excellent approximation not only around $\sqrt{s} = 10$ GeV, but also down to 6 or even 5 GeV.

The forthcoming experimental analysis necessitates on the one hand a thorough understanding of the background, e.g. from two photon events or the feedthrough from τ pairs into the hadronic cross section, and, on the other hand, a realistic prediction for the annihilation channel. The former is outside the scope of this paper, the latter is the subject of this work. It requires to incorporate the effect of the running QED coupling constant and initial state radiation. The photon vacuum polarization leads to an increase of the cross section by about 7–8%. Magnitude and even sign of initial state corrections depend on the experimental procedure, in particular on the minimal mass of the hadronic system accepted for the event sample. Another important issue is the treatment of the tails of the Υ resonances, which contribute coherently through mixing with the photon and incoherently through their radiative tails. In order to study these effects and the associated theoretical uncertainties separately the corresponding analysis for the reaction $e^+e^- \to \tau^+\tau^-$ is presented in section 2. These results may also serve as an independent experimental calibration of the cross section. The corresponding predictions for the hadronic cross section are discussed in section 3. In section 4 we comment on the accuracy of the presented approach and section 5 contains our summary and conclusions.

2. Lepton Pair Production

To calibrate the predictions for the hadron production cross section it seems appropriate and useful to calculate in a first step the cross section for lepton pair production. To arrive at a reliable prediction, initial and final state radiation, the effect of vacuum polarization and the influence of the nearby Υ resonances must be included. In this section results are presented for τ pair production.

(a) Initial state radiation

The most important correction to the total cross section is introduced by initial state radiation. It leads to a reduction of the invariant mass of the produced lepton pair or hadronic system and, for fairly loose cuts, to a significant enhancement of the cross section, albeit with events of significantly lower invariant mass of the system of interest. For the precise determination of R_{had} discussed below it is advisable to exclude the bulk of these low mass events. This reduces the size of the correction and at the same time the dependence on the input for R_{had} from the lower energy region.

The treatment of initial state radiation has advanced significantly as a consequence of the detailed calculations performed for the analysis of the Z line shape. The result for the inclusive cross section can be written in the form

$$\sigma(s) = \int_{z_0}^{1} \mathrm{d}z\, \sigma_0(sz)\, G(z)\,. \tag{1}$$

The cross section including photon vacuum polarization (running α) is denoted by σ_0. The invariant mass of the produced fermion pair is given by $s\,z$, where

$$\frac{m_{\mathrm{min}}^2}{s} \leq z_0 \leq z \leq 1 \tag{2}$$

and $m_{\mathrm{min}} = 2m_\ell$ for lepton pairs and $m_{\mathrm{min}} = 2m_\pi$ for hadron production. In the cases of interest for this paper z_0 will have to be adopted to the experimental setup. Typically it is significantly larger than the theoretically allowed minimal value. For $E_{\mathrm{cm}} = 10.52$ GeV a cut in z around 0.25 corresponding to roughly 5 GeV in the minimal mass will in the case of hadronic final states exclude the charmonium resonance region and the charm threshold region as well, thus limiting the hadron analysis to truely multihadronic final states.

The complete radiator function $G(z)$ up to order α^2 has been calculated in [6, 7]. The resummation of leading logarithms is discussed in [8] (see also [9]). For the present purpose an approximation is adequate which is exact in order α and which includes the dominant terms of order α^2 plus leading logarithms. For the radiator function G we thus take [10]

$$G(z) = \beta(1-z)^{\beta-1}\, e^{\delta_{yfs}}\, F\, \left(\delta_C^{V+S} + \delta_C^H\right), \tag{3}$$

with

$$\begin{aligned}
\beta &= \frac{2\alpha}{\pi}(L-1), \\
L &= \ln \frac{s}{m_e^2}, \\
\delta_{yfs} &= \frac{\alpha}{\pi}\left(\frac{L}{2} - 1 + 2\zeta(2)\right), \\
\delta_c^{V+S} &= 1 + \frac{\alpha}{\pi}(L-1) + \frac{1}{2}\left(\frac{\alpha}{\pi}\right)^2 L^2, \\
\delta_C^H &= -\frac{1-z^2}{2} + \frac{\alpha}{\pi} L \left[-\frac{1}{4}\left(1+3z^2\right)\ln z - 1 + z\right], \\
F &= \frac{e^{-\beta\gamma_E}}{\Gamma(1+\beta)},
\end{aligned}$$

where $\gamma_E = 0.5772\ldots$ is Euler's constant. Eq. (3) is suited for quick numerical integration. The difference to the initial state radiation convolution using the complete order α^2 result (eq (3.12) of [8]) is below one permille. The predictions for the reference energy of 10.52 GeV and a variety of cuts z_0 are listed in Table 1. The differential distribution $d\sigma/dz$ and the cutoff dependent cross section

Table 1: Dependence of the cross section $e^+e^- \to \tau^+\tau^-$ on the cutoff z_0 for $E_1 = 10.52$ GeV.

z_0	0.114	0.20	0.25	0.5	0.8	0.9	0.95
m_{cut} $[GeV]$	$2m_\tau$	4.705	5.260	7.439	9.409	9.980	10.254
σ $[nb]$	0.9269	0.9101	0.8998	0.8543	0.7831	0.7359	0.6922
σ/σ_{pt}	1.1822	1.1608	1.1477	1.0896	0.9988	0.9386	0.8829

$\sigma(z > z_0)$ are displayed in Fig. 1a, b, respectively, as functions of z, z_0. For the cross section before convolution the corrections from the vacuum polarization, the tails of the Υ resonances and final state radiation are included. Initial state radiation of lepton pairs and of hadrons has been treated in [7, 11]. These could easily be incorporated in the present formalism. The resulting corrections are at the permille level and will be ignored for the moment. Corrections from initial state radiation are evidently huge for loose or extremely tight cutoffs, with opposite sign. The strong cutoff dependence of the result is apparent from Table 1 and Fig. 1 as well. The result is relatively stable against variations of z_0 for values of z_0 below 0.75. At the same time the sensitivity to the input for σ_0 from smaller $s' = s\,z$ is eliminated for this choice. From this viewpoint a value around 0.7 to 0.8 seems optimal for the experimental determination of R_{had}. On the other hand, if one allows to use input from smaller s' from other experiments or from the measured events with photons from initial state radiation, then also lower values of z_0 down to about 0.25 are equally acceptable.

The distribution $d\sigma/dz$ is flat in the region $0.25 < z < 0.75$ and numerically small. An experimental analysis based on a cutoff $z_0 = s'/s$ with a reasonably symmetric resolution in s' is thus insensitive towards the details of the resolution function. In fact, even an uncontrolled shift of the central value by $\Delta z_0 = 0.05$ would lead to a 1% deviation of the cross section only.

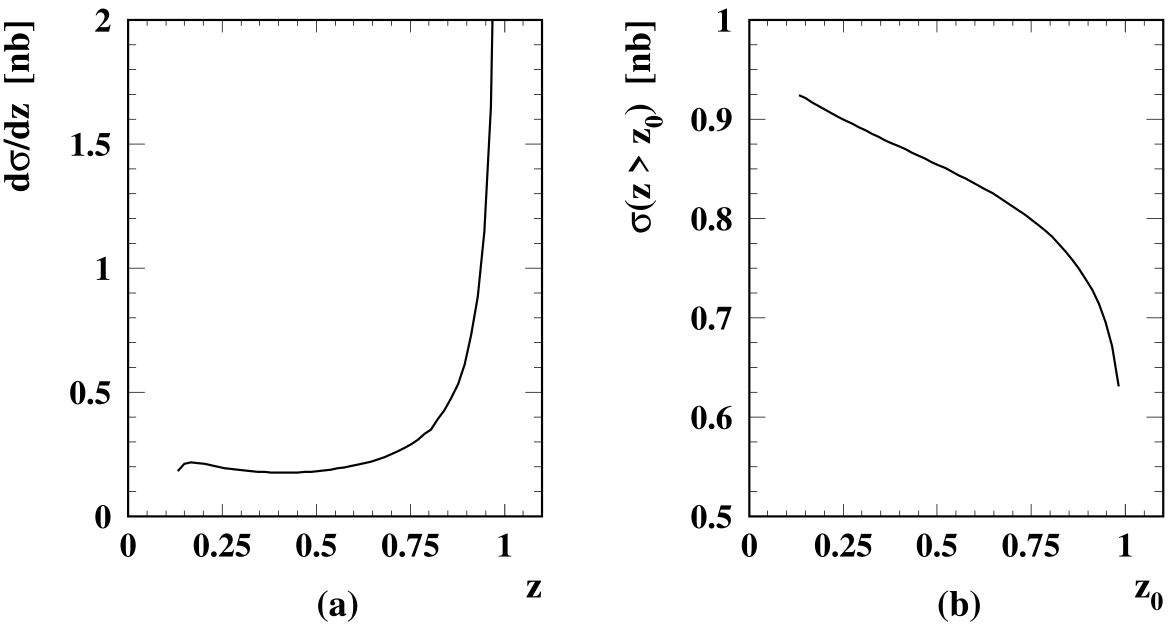

Figure 1: *(a) The differential cross section* $\mathrm{d}\sigma(e^+e^- \to \tau^+\tau^-)/\mathrm{d}z$ *and (b) the cutoff dependent cross section* $\sigma(z > z_0)$ *in nb for the centre of mass energy* $E_1 = 10.52$ *GeV as functions of* z *and* z_0*, respectively.*

(b) Final state radiation

Initial state radiation is strongly enhanced, a consequence of the inherently large logarithm in the correction function (cf. eq. (3)). Final state radiation with loose cuts, in contrast, is typically of order α/π without a large logarithm. In the massless limit and without a cutoff on the invariant fermion pair mass the correction factor $(1 + 3/4\,\alpha/\pi)$ amounts to two permille only. It depends, however, on the lepton mass and the cutoff. For the relatively loose cuts with z_0 between 0.25 and 0.8 one may well use the totally inclusive correction function $r^{(1)}$ defined by

$$R_{f\bar{f}} := \frac{\sigma(e^+e^- \to \gamma^* \to f\bar{f}\ldots)}{\sigma_{pt}} \quad = \quad r^{(0)} + \left(\frac{\alpha}{\pi}\right) r^{(1)} + \ldots\,, \tag{4}$$

with $\sigma_{pt} = 4\pi\alpha^2/3s$ and

$$r^{(0)} = \frac{\beta}{2}(3-\beta^2)\,, \tag{5}$$

$$\begin{aligned} r^{(1)} = {} & \frac{(3-\beta^2)\,(1+\beta^2)}{2}\left[2\,\mathrm{Li}_2(p) + \mathrm{Li}_2(p^2) + \ln p\,\Big(2\,\ln(1-p) + \ln(1+p)\Big)\right] \\ & - \beta\,(3-\beta^2)\,\Big(2\,\ln(1-p) + \ln(1+p)\Big) - \frac{(1-\beta)\,(33 - 39\,\beta - 17\,\beta^2 + 7\,\beta^3)}{16}\,\ln p \\ & + \frac{3\,\beta\,(5 - 3\,\beta^2)}{8}\,, \end{aligned} \tag{6}$$

where

$$p = \frac{1-\beta}{1+\beta}\,, \qquad \beta = \sqrt{1 - 4m_f^2/s}\,. \tag{7}$$

In the case of hadron production, the main motivation of this investigation, photons from final state radiation will in general anyhow be included in the hadronic invariant mass.

(c) Leptonic and hadronic vacuum polarization

The leptonic vacuum polarization in one loop approximation is given by

$$\widehat{\Pi}_{\gamma\gamma}(s) := \Pi_{\gamma\gamma}(s) - \Pi_{\gamma\gamma}(0) = \frac{\alpha}{3\pi} \sum_f N_c Q_f^2 \, P(s, m_f) \,, \tag{8}$$

with

$$P(s, m_f) = \frac{1}{3} - \left(1 + \frac{2m_f^2}{s}\right) \left(2 + \beta \ln \frac{\beta - 1}{\beta + 1}\right) . \tag{9}$$

For $|s| \gg m_\ell^2$ the function $P(s, m_f)$ is well approximated by

$$P_{\text{asymp.}}(s, m_f) = -\frac{5}{3} + \ln\left(- \frac{s}{m_f^2} + i\,\epsilon\right) . \tag{10}$$

The hadronic vacuum polarization is obtained from it's imaginary part via a dispersion relation

$$\text{Re}\widehat{\Pi}_{\text{had}}(q^2) = \frac{\alpha q^2}{3\pi} \, \text{P} \int_{m_\pi^2}^{\infty} \frac{R_{\text{had}}(s')}{s'(s' - q^2)} \, \text{d}s' \, . \tag{11}$$

To avoid complications that arise from the numerical evaluation of the dispersion integral over the data in the timelike region, the integral is evaluated at the corresponding value of s in the spacelike region. We use parametrisations from the evaluation of eq. (11) in the spacelike region as provided by [12, 13]. The results for the complete cross sections calculated with the different parametrisations from [12] and [13] agree to better than 10^{-4} in the region of interest. The error from the identification of spacelike and timelike $\widehat{\Pi}_{\text{had}}(q^2)$ should be small except for the b quark contribution, where the threshold is fairly close to the q^2 values of interest. Therefore we subtract the perturbative b quark contribution evaluated for spacelike q^2 and add the corresponding value for timelike $q^2 = s$. We have checked that this ansatz is in excellent agreement with the numerical evaluation* of (11) in the timelike region. Effects due to Υ resonances are described in detail below.

The running α is then obtained from the real part of $\widehat{\Pi}$ through

$$\alpha(s) = \frac{\alpha}{1 - \text{Re}\widehat{\Pi}(s)} \, . \tag{12}$$

The relative shift in $\alpha(s)$ from the hadronic plus leptonic vacuum polarization is shown in Fig. 2 as a function of $\sqrt{s}$. The size of the individual contributions is listed in Table 2 for the reference energy $E_1 = 10.52$ GeV and for a few selected lower energies. The uncertainty in the cross section from our treatment of the hadronic vacuum polarization in the timelike region is estimated to be below two permille.

(d) Narrow Υ resonances

Up to this point we have not included the contributions from the Υ resonances. As a consequence of their close proximity their effect will be enhanced and thus should be discussed separately. The Breit–Wigner amplitudes from the Υ resonances above and below the $B\overline{B}$ threshold interfere with the virtual photon amplitude and thus enhance or decrease the cross section by

$$\delta\sigma_{\text{int}} = \frac{6\,\Gamma_e}{\alpha(s)\,s} \frac{M^3(s - M^2)}{(s - M^2)^2 + \Gamma^2 M^2} \, \sigma_0 \, . \tag{13}$$

---

*We thank H. Burkhardt for providing the numerical evaluation of (11) required for this comparison.

Table 2: Individual contributions to $\mathrm{Re}\widehat{\Pi}(s) \cdot 10^2$.

$\sqrt{s}$/GeV	$2m_\tau$	5	7	9	10.52
e	1.241	1.294	1.346	1.385	1.409
μ	0.415	0.468	0.520	0.559	0.583
τ	-0.207	-0.049	0.047	0.101	0.132
had	0.923	1.096	1.269	1.379	1.456
$1/\alpha(s)$	133.79	133.19	132.68	132.34	132.13

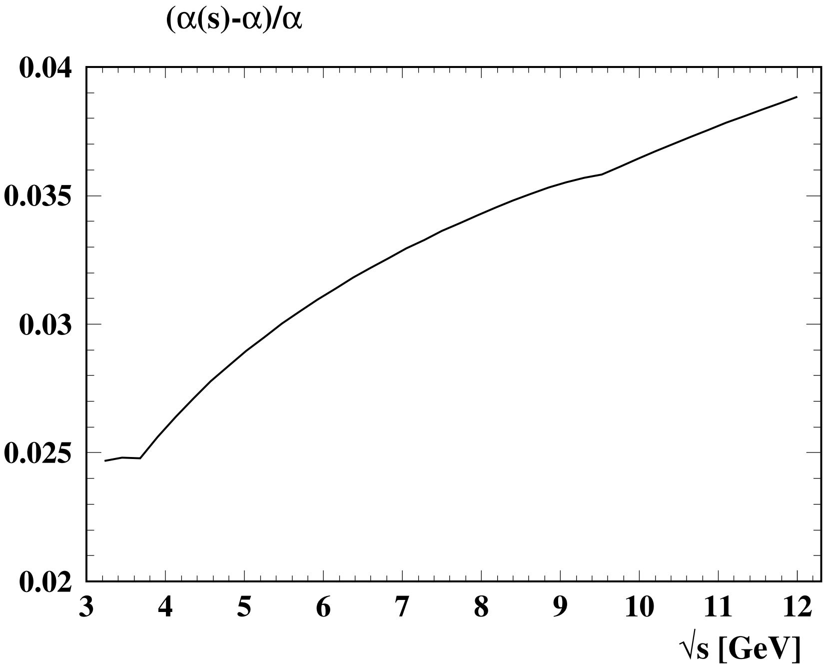

Figure 2: *The relative shift in $\alpha(s)$ as a function of $\sqrt{s}$ from the hadronic plus leptonic vacuum polarization as described in the text.*

The individual contributions are listed in Table 3 for the two cms energies $E_1 = 10.52$ GeV and $E_2 = 9.98$ GeV after the convolution with initial state radiation. For E_1 these interference terms happen to cancel to a large extent, leaving a minute correction with a relative magnitude around 10^{-4}. For E_2, however, due to the close proximity of $\Upsilon(2S)$ one receives a non–negligible negative contribution of about -0.4%. This effect could become relevant in precision tests.

Also the radiative tails of the resonances with $M < \sqrt{s}$ have to be taken into consideration. These can be easily calculated with the radiator function eq. (3), taking either the full Breit–Wigner resonance

$$\delta\sigma_{\mathrm{R}} = \left(\frac{3\,\Gamma_e\,M}{\alpha(s)\,s}\right)^2 \frac{M^4}{(s-M^2)^2+\Gamma^2 M^2}\,\sigma_0 \tag{14}$$

or the narrow width approximation

$$\delta\sigma_{\mathrm{R}}\Big|_{\mathrm{NW}} = \frac{9\,\Gamma_e^2}{\alpha(s)^2}\frac{M}{\Gamma}\,\pi\,\delta(s-M^2)\,\sigma_0\,. \tag{15}$$

Both formulae lead to nearly identical results. The individual contributions are also listed in Table 3.

Table 3: Relative contributions from the Υ resonances to the leptonic cross section $\sigma(e^+e^- \to \tau^+\tau^-)$ for the energies $E_1 = 10.52$ GeV and $E_2 = 9.98$ GeV and two different values of the cutoff $m_{\rm min}$. Interference terms and radiative tails are listed separately. Also given are the continuum contributions and the resulting predictions for the total cross section.

	$\Upsilon(1)$	$\Upsilon(2)$	$\Upsilon(3)$	$\Upsilon(4)$	$\Upsilon(5)$	$\Upsilon(6)$
M $[GeV]$	9.460	10.023	10.355	10.580	10.865	11.019
Γ_e $[keV]$	1.32	0.576	0.476	0.24	0.31	0.13
$\Gamma_{\rm tot}$ $[MeV]$	0.0525	0.044	0.0263	23.8	110	79
$E_1 = 10.52$ GeV, $m_{\rm min} = 2m_\tau = 3.554$ GeV:						
Continuum:	0.9256 nb					
Interf.:	$2.7 \cdot 10^{-4}$	$2.9 \cdot 10^{-4}$	$7.5 \cdot 10^{-4}$	$-1.04 \cdot 10^{-3}$	$-3.0 \cdot 10^{-4}$	$-9.7 \cdot 10^{-5}$
$\sum$ interf.:	$-1.3 \cdot 10^{-4}$					
Rad. tails :	$5.5 \cdot 10^{-4}$	$2.5 \cdot 10^{-4}$	$8.1 \cdot 10^{-4}$	–	–	–
$\sum$ rad. tails:	$1.61 \cdot 10^{-3}$					
Total:	0.9269 nb					
$E_1 = 10.52$ GeV, $m_{\rm min} = 5$ GeV:						
Continuum:	0.9033 nb					
Interf.:	$2.9 \cdot 10^{-4}$	$3.1 \cdot 10^{-4}$	$7.7 \cdot 10^{-4}$	$-1.07 \cdot 10^{-3}$	$-3.1 \cdot 10^{-4}$	$-9.8 \cdot 10^{-5}$
$\sum$ interf.:	$-9.7 \cdot 10^{-5}$					
Rad. tails :	$5.7 \cdot 10^{-4}$	$2.6 \cdot 10^{-4}$	$8.3 \cdot 10^{-4}$	–	–	–
$\sum$ rad. tails:	$1.65 \cdot 10^{-3}$					
Total:	0.9047 nb					
$E_2 = 9.98$ GeV, $m_{\rm min} = 2m_\tau = 3.554$ GeV:						
Continuum:	1.0200 nb					
Interf.:	$6.4 \cdot 10^{-4}$	$-3.51 \cdot 10^{-3}$	$-4.5 \cdot 10^{-4}$	$-1.6 \cdot 10^{-4}$	$-1.6 \cdot 10^{-4}$	$-5.9 \cdot 10^{-5}$
$\sum$ interf.:	$-3.70 \cdot 10^{-3}$					
Rad. tails:	$1.08 \cdot 10^{-3}$	–	–	–	–	–
$\sum$ rad. tails:	$1.08 \cdot 10^{-3}$					
Total:	1.0173 nb					
$E_2 = 9.98$ GeV, $m_{\rm min} = 5$ GeV:						
Continuum:	0.9947 nb					
Interf.:	$6.8 \cdot 10^{-4}$	$-3.60 \cdot 10^{-3}$	$-4.6 \cdot 10^{-4}$	$-1.6 \cdot 10^{-4}$	$-1.6 \cdot 10^{-4}$	$-5.8 \cdot 10^{-5}$
$\sum$ interf.:	$-3.75 \cdot 10^{-3}$					
Rad. tails:	$1.11 \cdot 10^{-3}$	–	–	–	–	–
$\sum$ rad. tails:	$1.11 \cdot 10^{-3}$					
Total:	0.9920 nb					

In total they raise the lepton pair cross section by about 0.16% or 0.11% for E_1 and E_2, respectively. At this point a brief comment ought to be made concerning the treatment of $\Upsilon(4S)$. The approximation of neglecting the energy dependence of the width of the resonance is justified even for E_1, since the nominal width $\Gamma = 24$ MeV is significantly smaller than the difference between the cms energy E_1 and the mass of the resonance: $\Delta E = 60$ MeV.

3. The Total Cross Section for Hadron Production

As stated already in the introduction the emphasis of this work is on the energy just below the bottom meson threshold. Virtual b quark loops can be easily taken into account, and in fact it has been demonstrated that the hard mass expansion works surprisingly well for these diagrams — not only far below but even close to the nominal b quark threshold. On the other hand this energy is sufficiently far above the open charm threshold such that charm quark mass effects can be included through an expansion in powers of m_c, if quadratic and quartic terms are incorporated. With this method the production cross section can be predicted reliably not only in the high energy region but even relatively close to threshold through an expansion in m^2/s. This approach was suggested originally in [14, 5]. The calculation of $R_{\rm had}$ to second order, including the full mass dependence [15] has demonstrated the nearly perfect agreement between approximate and exact result for the coefficient of the α_s^2 term for $E_{\rm cm}$ above $4m$. In fact, even for $E_{\rm cm}$ around $3m$, which is around the lowest advisable value of the cutoff $m_{\rm min} = 5$ GeV, the deviation of the α_s^2 coefficient from the complete result leads to a difference of less than five percent in the rate. For the energy around 10 GeV this region contributes through the radiative tail only, and the approximation is thus adequate throughout. In total we thus use the following individual contributions

$$R = R_{\rm NS} + R_{\rm S} + \delta R_{m_b} + \delta R_{m_c} + \delta_{\rm QED} \,, \tag{16}$$

where

$$\begin{aligned}
R_{\rm NS} &= \sum_{f=u,d,s,c} 3\,Q_f^2 \left[1 + \frac{\alpha_s}{\pi} + 1.5245\left(\frac{\alpha_s}{\pi}\right)^2 - 11.52033\left(\frac{\alpha_s}{\pi}\right)^3\right] , \\
R_{\rm S} &= -\left(\frac{\alpha_s}{\pi}\right)^3 \Big(\sum_{u,d,s,c} Q_f\Big)^2 1.239 \;=\; -0.55091\left(\frac{\alpha_s}{\pi}\right)^3 , \\
\delta R_{m_b} &= \sum_{f=u,d,s,c} 3\,Q_f^2 \left(\frac{\alpha_s}{\pi}\right)^2 \frac{s}{\overline{m}_b^2}\left[\frac{44}{675} + \frac{2}{135}\log\frac{\overline{m}_b^2}{s}\right] , \\
\delta R_{m_c} &= 3\,Q_c^2\, 12\,\frac{m_c^2}{s}\frac{\alpha_s}{\pi}\left[1 + 9.097\frac{\alpha_s}{\pi} + 53.453\left(\frac{\alpha_s}{\pi}\right)^2\right] - 3\sum_{f=u,d,s,c} Q_f^2 \frac{m_c^2}{s}\left(\frac{\alpha_s}{\pi}\right)^3 6.476 \\
&\quad + 3\,Q_c^2\frac{m_c^4}{s^2}\left[-6 - 22\frac{\alpha_s}{\pi} + \left(141.329 - \frac{25}{6}\ln\frac{m_c^2}{s}\right)\left(\frac{\alpha_s}{\pi}\right)^2\right] \\
&\quad + 3\sum_{f=u,d,s,c} Q_f^2\frac{m_c^4}{s^2}\left(\frac{\alpha_s}{\pi}\right)^2\left[-0.4749 - \ln\frac{m_c^2}{s}\right] - 3\,Q_c^2\frac{m_c^6}{s^3}\left[8 + \frac{16}{27}\frac{\alpha_s}{\pi}\left(6\ln\frac{m_c^2}{s} + 155\right)\right] , \\
\delta_{\rm QED} &= \sum_{f=u,d,s,c} 3\,Q_f^4 \frac{\alpha}{\pi}\frac{3}{4} .
\end{aligned}$$

The formulae are evaluated for $n_f = 4$ with α_s and the charm quark mass interpreted accordingly. For the massless case and the m^2/s terms the results are available in third order, for the quartic and m^6/s^3 terms in second and first order α_s, respectively. Tables which list the numerical values of the running quark masses and the magnitude of the individual contributions can be found in [5, 3] together with the details of the matching between the theories with $n_F = 4$ and 5.

The predictions for $R_{\rm had}$ as a function of $E_{\rm cm}$ are shown in Fig. 3 for three values of the strong coupling constant, $\alpha_s(M_Z^2) = 0.115$, 0.120 and 0.125. Fig. 3a displays the contributions induced by the light plus charm quark currents (as defined in eq. (16)), whereas in Fig. 3b only the light (u, d and s) quark current contributions are shown. Note, that this figure by definition does not contain QED–corrections from initial state radiation and vacuum polarization, but includes the tiny singlet terms,

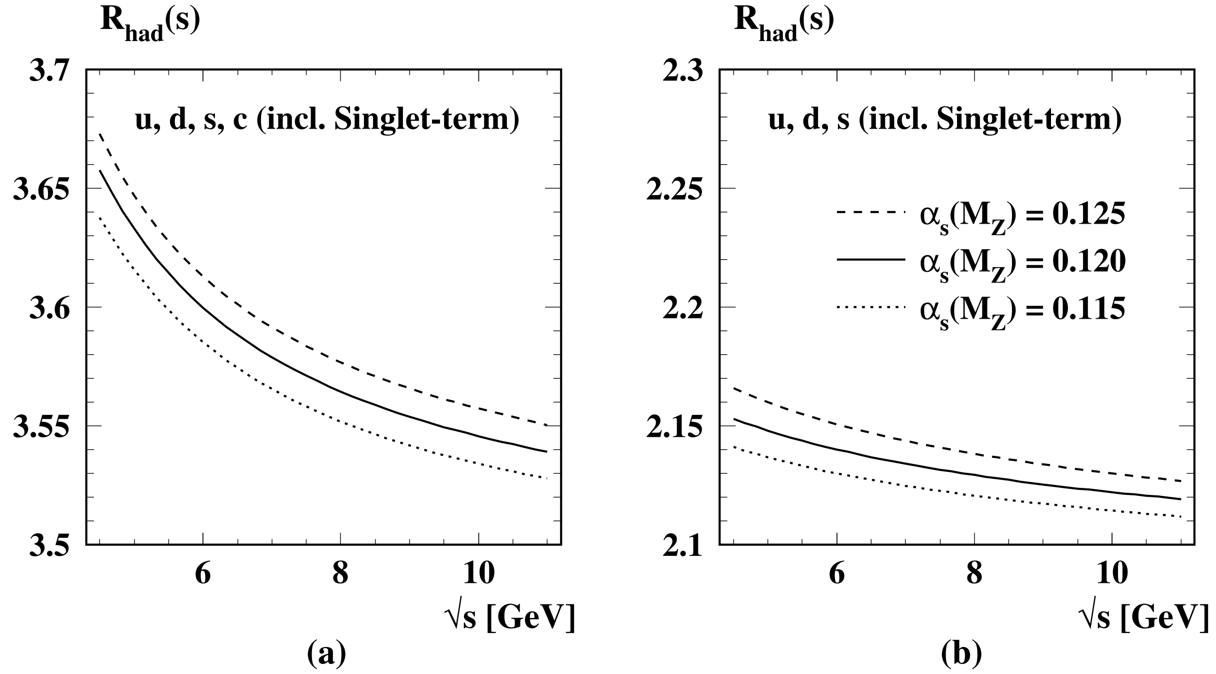

Figure 3: *(a) $R_{\rm had}(s)$ as defined in eq. (16), and (b) only the contributions from the light (u, d and s) quark currents for different values of α_s.*

which cannot be attributed to one individual quark species and the final state QED–corrections $\delta_{\rm QED}$. The quark masses are chosen to be $\overline{m}_c(\overline{m}_c) = 1.24$ GeV ($n_f = 4$) and $\overline{m}_b(\overline{m}_b) = 4.1$ GeV ($n_f = 5$) corresponding to pole masses of 1.6 GeV and 4.7 GeV, respectively, if $\alpha_s^{(n_f=5)}(M_Z) = 0.120$ is adopted. A variation of the charm quark mass around the default value by +300 MeV/−300 MeV changes $R_{\rm had}$ only by $+0.006/-0.004$ for $\sqrt{s} = 10.52$ GeV and by $+0.031/-0.028$ for $\sqrt{s} = 5$ GeV.

Throughout this paper the QCD results are interpreted in fixed order α_s^3 without any attempt to improve the formulae through the inclusion of guesses for higher order coefficients. An estimate of the scale dependence is easily obtained through the evaluation of a variant of eq. (16) where $R_{\rm NS}$ is calculated for a general t'Hooft scale μ^2. Adopting $\alpha_s(10.5\text{ GeV}) = 0.177$ corresponding to $\alpha_s(M_Z) = 0.12$ and varying μ^2 between $s/4$ and $4s$ the predicted value of R varies by -2 and +0.2 permille. This is well below the anticipated experimental precision. Alternatively we may include in eq. (16) an α_s^4 term with the coefficient based on a recent estimate in [16]. This would lead to a decrease in R by 1.7 permille, again far below the forseeable experimental accuracy.

Let us now discuss the impact of initial state radiation, the running α and the Υ resonances on the hadronic cross section, as observed at around 10 GeV under realistic experimental conditions. Lower energies contribute again through initial state radiation (eq. (1)). If we restrict the cutoff z_0 to a value of 0.25 which corresponds to a cut on the mass of the hadronic system of around 5 GeV, eq. (16) for $R_{\rm had}$ can be applied also for $\sigma_0(zs')$ which appears in the integrand of (1). At the same time this cutoff excludes the region of the broad charmonium resonances which have not been well explored up today. A cutoff around $z_0 = 0.7$ reduces the initial state radiation corrections further and eliminates contributions from the lower energy range completely. The vacuum polarization of the virtual photon has been discussed before for the case of the τ lepton production and is identical for the hadronic cross section. The interference between the Υ resonances and the virtual photon is completely analogous to the leptonic case and leads to the same relative corrections. However,

Table 4: Relative contributions from the Υ resonances to the hadronic cross section $\sigma(e^+e^- \to \text{hadrons})$ for the energies $E_1 = 10.52$ GeV and $E_2 = 9.98$ GeV and the two different values of the cutoff $m_{\text{min}} = 5$ GeV and 9 GeV. Interference terms and radiative tails are listed separately. Also given are the continuum contributions and the resulting predictions for the total cross section.

	$\Upsilon(1)$	$\Upsilon(2)$	$\Upsilon(3)$	$\Upsilon(4)$	$\Upsilon(5)$	$\Upsilon(6)$
$E_1 = 10.52$ GeV, $m_{\text{min}} = 5$ GeV:						
Continuum:	3.2228 nb					
Interf.:	$2.9 \cdot 10^{-4}$	$3.1 \cdot 10^{-4}$	$7.7 \cdot 10^{-4}$	$-1.06 \cdot 10^{-3}$	$-3.1 \cdot 10^{-4}$	$-9.7 \cdot 10^{-5}$
$\sum$ interf.:	$-1.0 \cdot 10^{-4}$					
Rad. tails:	$5.88 \cdot 10^{-3}$	$5.38 \cdot 10^{-3}$	$1.212 \cdot 10^{-2}$	–	–	–
$\sum$ rad. tails:	$2.343 \cdot 10^{-2}$					
Total:	3.2980 nb					
$E_1 = 10.52$ GeV, $m_{\text{min}} = 9$ GeV:						
Continuum:	2.8515 nb					
Interf.:	$4.0 \cdot 10^{-4}$	$3.7 \cdot 10^{-4}$	$8.9 \cdot 10^{-4}$	$-1.19 \cdot 10^{-3}$	$-3.3 \cdot 10^{-4}$	$-1.1 \cdot 10^{-4}$
$\sum$ interf.:	$3.9 \cdot 10^{-5}$					
Rad. tails:	$6.64 \cdot 10^{-3}$	$6.08 \cdot 10^{-3}$	$1.370 \cdot 10^{-2}$	$5 \cdot 10^{-5}$	–	–
$\sum$ rad. tails:	$2.648 \cdot 10^{-2}$					
Total:	2.9272 nb					
$E_2 = 9.98$ GeV, $m_{\text{min}} = 5$ GeV:						
Continuum:	3.5568 nb					
Interf.:	$6.7 \cdot 10^{-4}$	$-3.58 \cdot 10^{-3}$	$-4.6 \cdot 10^{-4}$	$-1.6 \cdot 10^{-4}$	$-1.6 \cdot 10^{-4}$	$-5.8 \cdot 10^{-5}$
$\sum$ interf.:	$-3.74 \cdot 10^{-3}$					
Rad. tails:	$1.145 \cdot 10^{-2}$	$9 \cdot 10^{-5}$	–	–	–	–
$\sum$ rad. tails:	$1.154 \cdot 10^{-2}$					
Total:	3.5845 nb					
$E_2 = 9.98$ GeV, $m_{\text{min}} = 9$ GeV:						
Continuum:	3.0666 nb					
Interf.:	$8.8 \cdot 10^{-4}$	$-4.1 \cdot 10^{-3}$	$-5.0 \cdot 10^{-4}$	$-1.7 \cdot 10^{-4}$	$-1.6 \cdot 10^{-4}$	$-6.1 \cdot 10^{-5}$
$\sum$ interf.:	$-4.14 \cdot 10^{-3}$					
Rad. tails:	$1.328 \cdot 10^{-2}$	$1.04 \cdot 10^{-4}$	–	–	–	–
$\sum$ rad. tails:	$1.338 \cdot 10^{-2}$					
Total:	3.0949 nb					

the contributions from the radiative tails of the Breit–Wigner amplitudes are distinctively different. Without initial state radiation the tail from one resonance is given by

$$\sigma_{\text{had,R}} = \frac{12\,\pi\,\Gamma_e\,\Gamma_{\text{had}}\,M^4}{s^3}\,\frac{M^2}{(s-M^2)^2+\Gamma^2 M^2}\,. \tag{17}$$

For the $\Upsilon(4,5,6)$ resonances an energy dependent width $\Gamma(s)$ has to be used in (17), which varies rapidly in the region of interest. For s below the $B\overline{B}$ threshold this leads to a drastic suppression of these contributions. In the narrow width approximation eq. (17) implies

$$\sigma_{\text{had,R}}\Big|_{\text{NW}} = \frac{12\,\pi^2\,\Gamma_e\,\Gamma_{\text{had}}}{s}\,\frac{M}{\Gamma}\,\delta(s-M^2)\,. \tag{18}$$

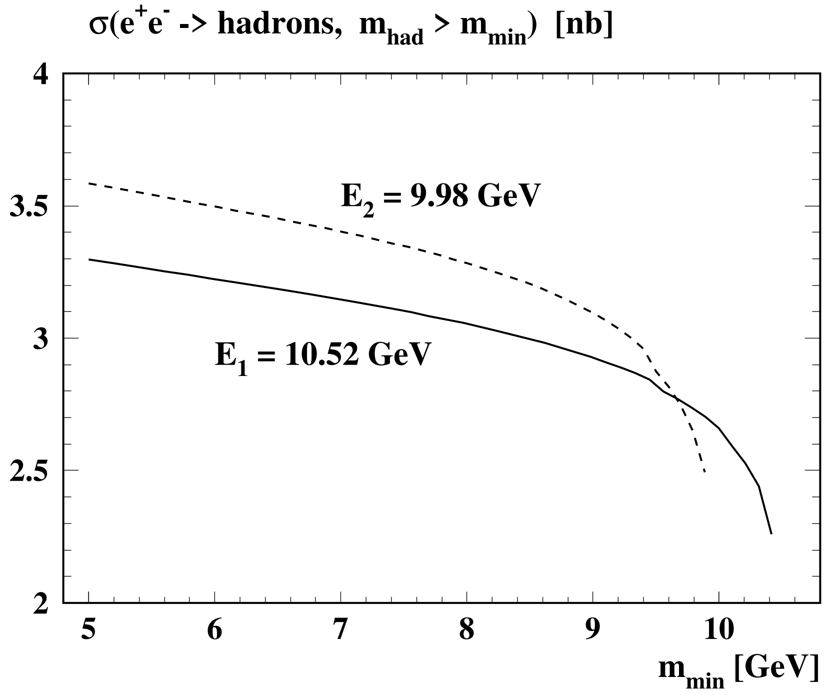

Figure 4: *Dependence of the hadronic cross section* $\sigma(e^+e^- \to \text{hadrons})$ *on the cutoff in the minimal invariant mass of the hadronic system,* $m_{\rm min}$*, for the two energies* $E_1 = 10.52$ *GeV,* $E_2 = 9.98$ *GeV and* $\alpha_s(M_Z) = 0.120$.

The contributions of interference terms and radiative tails from the individual resonances to the hadronic cross section are listed in Table 4 for the energies $E_1 = 10.52$ GeV and $E_2 = 9.98$ GeV and $\alpha_s(M_Z^2) = 0.120$. Also given are the continuum contribution and the sum. The cutoffs $m_{\rm min} = 5$ GeV and 9 GeV are adopted. These correspond to the minimal and maximal values recommended for a precision measurement of $R_{\rm had}$. The relative strength of the interference terms is identical for hadronic and leptonic final states. Again one observes the accidental cancellation at 10.52 GeV and the dominance of the (negative) $\Upsilon(2)$ contribution at the energy $E_2 = 9.98$ GeV just below this resonance. Compared to the leptonic cross section (Table 3) the radiative tails are significantly more prominent — a consequence of the relatively large direct, non–QED–mediated hadronic decay rates of the Υ resonances. This leads to a positive correction of 2.3% and 1.2% for E_1 and E_2, if $m_{\rm min} = 5$ GeV and to 2.7% and 1.3% if $m_{\rm min} = 9$ GeV, respectively. The difference in the strength of the Υ tails leads to an apparent variation of $R_{\rm had}$ around 1.3%. The cutoff dependence of the cross section is illustrated in Fig. 4. The strong dependence of the cross section for tight cuts is again clearly visible, suggesting a cut between about 5 and 9 GeV for E_1 and 5 and 8.5 GeV for E_2.

4. Uncertainties

From the comparison of different radiator functions for initial state radiation the theoretical uncertainty from this source can be estimated to be below one permille. The error induced through the present simplified treatment of the hadronic vacuum polarization in the timelike region is estimated around two permille and could easily be reduced even further, if required. The combined theoretical uncertainty from these and other effects is generously estimated below five permille. In addition photons from initial state radiation might be included in the invariant mass of the hadronic system or, conversely, photons from final state radiation may escape the detection. This $\mathcal{O}(\alpha)$ effect can only be evaluated for the concrete experimental analysis with the help of a Monte Carlo simulation. These theoretical uncertainties are significantly below the expected experimental error of roughly two percent, which is dominated by systematical uncertainties [4].

5. Summary and Conclusions

Precise predictions have been presented for the total cross section in the energy region explored presently by the CLEO experiment and at a future B meson factory. The present sample of nearly one million hadronic events allows for a small statistical error. These measurements will determine the value for α_s under particularly clean conditions similar to the Z line shape measurements but at a different energy. When compared to the experimental results from Z decays, a determination of R with a precision of 2.5% would evidently demonstrate the running of α_s between 90 and 10 GeV. A precision of 0.3% would be competitive with the α_s measurements from the Z line shape which are based on the combined results of all four LEP experiments.

Acknowledgements

We thank Fred Jegerlehner and Helmut Burkhardt for providing their programs and Fred Jegerlehner for important discussions concerning the hadronic vacuum polarization. The interest of Dave Besson in this study was essential for its completion. TT thanks the UK Particle Physics and Astronomy Research Council and the Royal Society for support. This work was supported by BMFT under Contract 057KA92P(0), and INTAS under Contract INTAS-93-0744.

References

[1] A. Blondel, plenary talk presented at the *28th International Conference on High Energy Physics*, 25–31 July 1996, Warsaw, Poland.

[2] *Reports of the Working Group on Precision Calculations on the Z Resonance*, D. Bardin, W. Hollik, G. Passarino (eds.), Yellow Report CERN 95-03 (1995).

[3] K. G. Chetyrkin, J. H. Kühn and A. Kwiatkowski, *Phys. Reports* **277(4)** (1996) 189.

[4] D. Besson, private communications.

[5] K. G. Chetyrkin and J. H. Kühn, *Phys. Lett.* **B 342** (1995) 356.

[6] G. J. H. Burgers, *Phys. Lett.* **B 164** (1985) 167.

[7] V. A. Berends, W. L. Van Neerven and G. J. H. Burgers, *Nucl. Phys.* **B 297** (1988) 429; *Phys. Lett.* **B 185** (1987) 395.

[8] V. A. Berends, *Z Line Shape*, in *Z Physics at LEP 1*, Yellow Report, G. Altarelli, R. Kleiss and C. Verzegnassi (eds.), CERN 89-08 (1989).

[9] E. A. Kuraev and V. S. Fadin, *Sov. J. Nucl. Phys.* **41** (1985) 466.

[10] S. Jadach and B. F. L. Ward, TPJU–15/88 (1988).

[11] B. A. Kniehl, M. Krawczyk, J. H. Kühn and R. G. Stuart, *Phys. Lett.* **B 209** (1988) 337.

[12] S. Eidelman and F. Jegerlehner, *Z. Phys.* **C 67** (1995) 585.

[13] H. Burkhardt and B. Pietrzyk, *Phys. Lett.* **B 356** (1995) 398.

[14] K. G. Chetyrkin and J. H. Kühn, *Nucl. Phys.* **B 432** (1994) 337.

[15] A. H. Hoang, J. H. Kühn and T. Teubner, *Nucl. Phys.* **B 452** (1995) 173;
K. G. Chetyrkin, J. H. Kühn and M. Steinhauser, *Nucl. Phys.* **B 482** (1996) 213.

[16] A. L. Kataev and V. V. Starshenko, *Mod. Phys. Lett.* **A10** (1995) 235.