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KIT, 6-10 February '12

Beyond the Standard Model

Neutrino Masses & Mixings 2012

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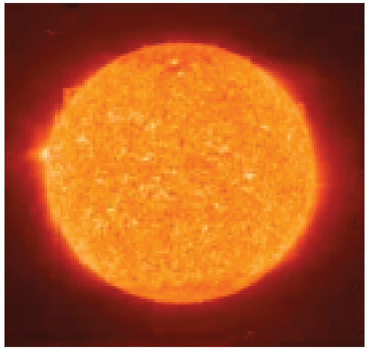
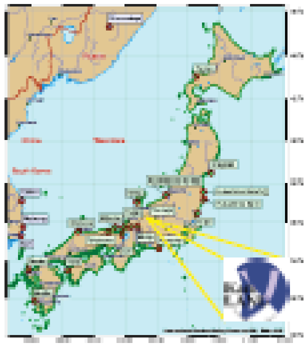
In the last 2 decades data on ν oscillations have added some (badly needed) fresh experimental input to particle physics

sun



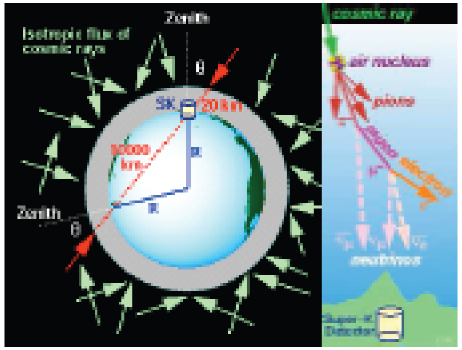
Homestake, SAGE, GALLEX
SuperK, SNO, Borexino

reactors



KamLAND, CHOOZ

atmosphere



SuperKamiokande

accelerators

K2K, MINOS, T2K

Schwetz

ν masses are not all vanishing but they are very small

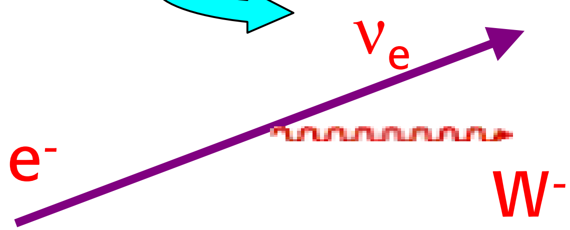
This suggests that ν's are Majorana particles and that the lepton number L is not conserved

ν mixing angles follow a different pattern from quark mixings

This also is probably related to the Majorana nature of ν's

ν Oscillations Imply Different ν Masses

ν_e: same weak isospin doublet as e^-



flavour   mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U^+ \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

U: mixing matrix

$U = U_{PMNS}$

Pontecorvo
Maki, Nakagawa, Sakata

$\nu_e = \cos\theta\,\nu_1 + \sin\theta\,\nu_2$

$\nu_\mu = -\sin\theta\,\nu_1 + \cos\theta\,\nu_2$

e.g 2 flav.

Stationary source:

Stodolsky

$\nu_{1,2}$: different mass, different x-dep:

$\nu_a(x) = e^{ip_a x}\,\nu_a$ $p_a^2 = E^2 - m_a^2$

$$P(\nu_e <-> \nu_\mu) = |< \nu_\mu(L)|\,\nu_e>|^2 = \sin^2(2\theta)\cdot\sin^2(\Delta m^2 L/4E)$$

At a distance L, ν_μ from μ^- decay can produce e^- via charged weak interact's

Evidence for solar and atmosph. ν oscillations confirmed on earth by K2K, KamLAND, MINOS, T2K...

Δm^2 values:

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$,

$\Delta m^2_{sol} \sim 8\ 10^{-5}\ eV^2$

and mixing angles measur'd:

θ_{12} (solar) large

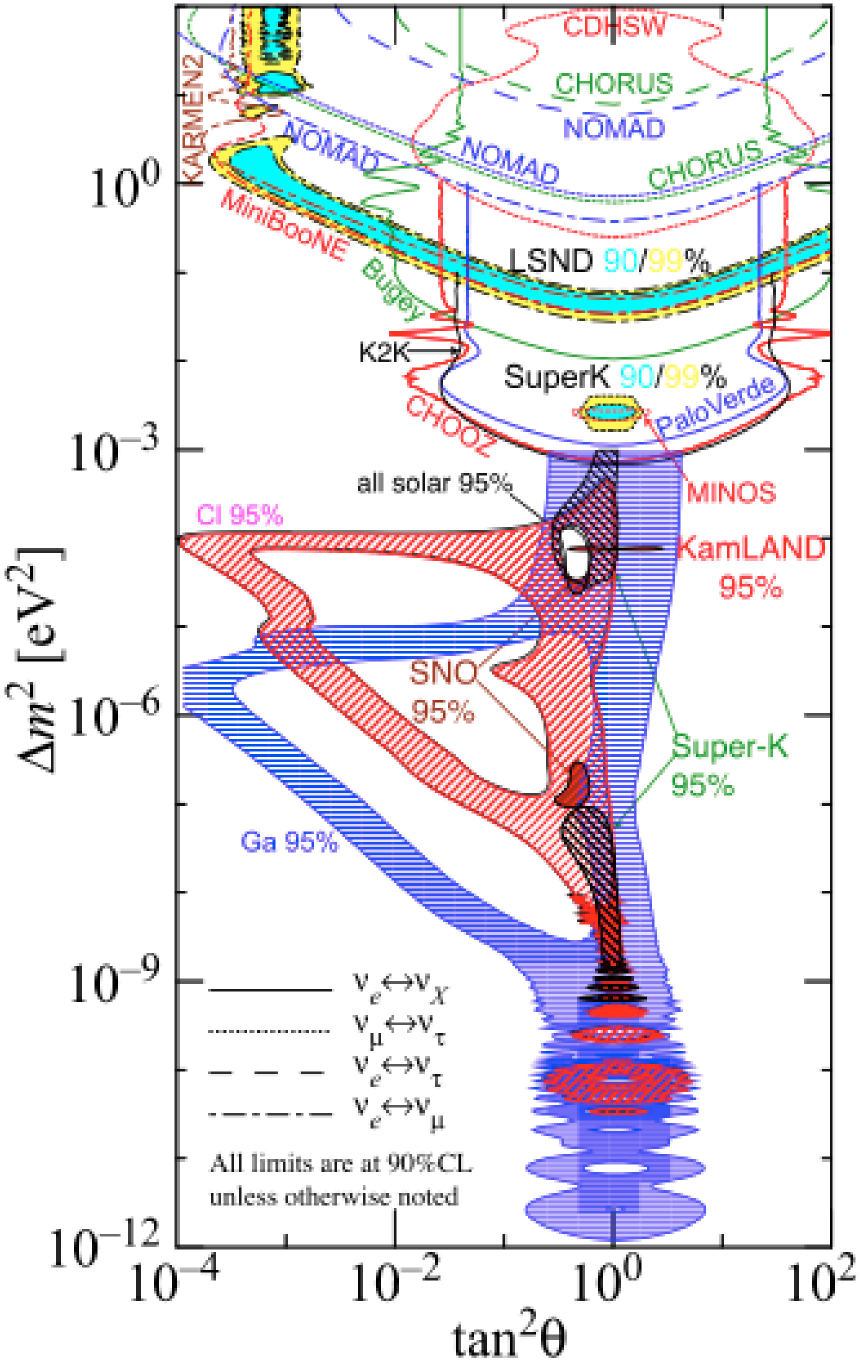
θ_{23} (atm) large~ maximal

θ_{13} (T2K, MINOS, DOUBLE CHOOZ) small

A 3rd frequency?

A persisting confusion: LSND+MiniBooNE

Sterile (no weak int's) neutrino



Are sterile ν's coming back? A number of "hints"

(they do not make an evidence but
pose an experimental problem that needs clarification)

- LSND and MiniBoone
- Reactor flux & anomaly
- Gallium ν_e disappearance vs ν_e^{bar} reactor limits

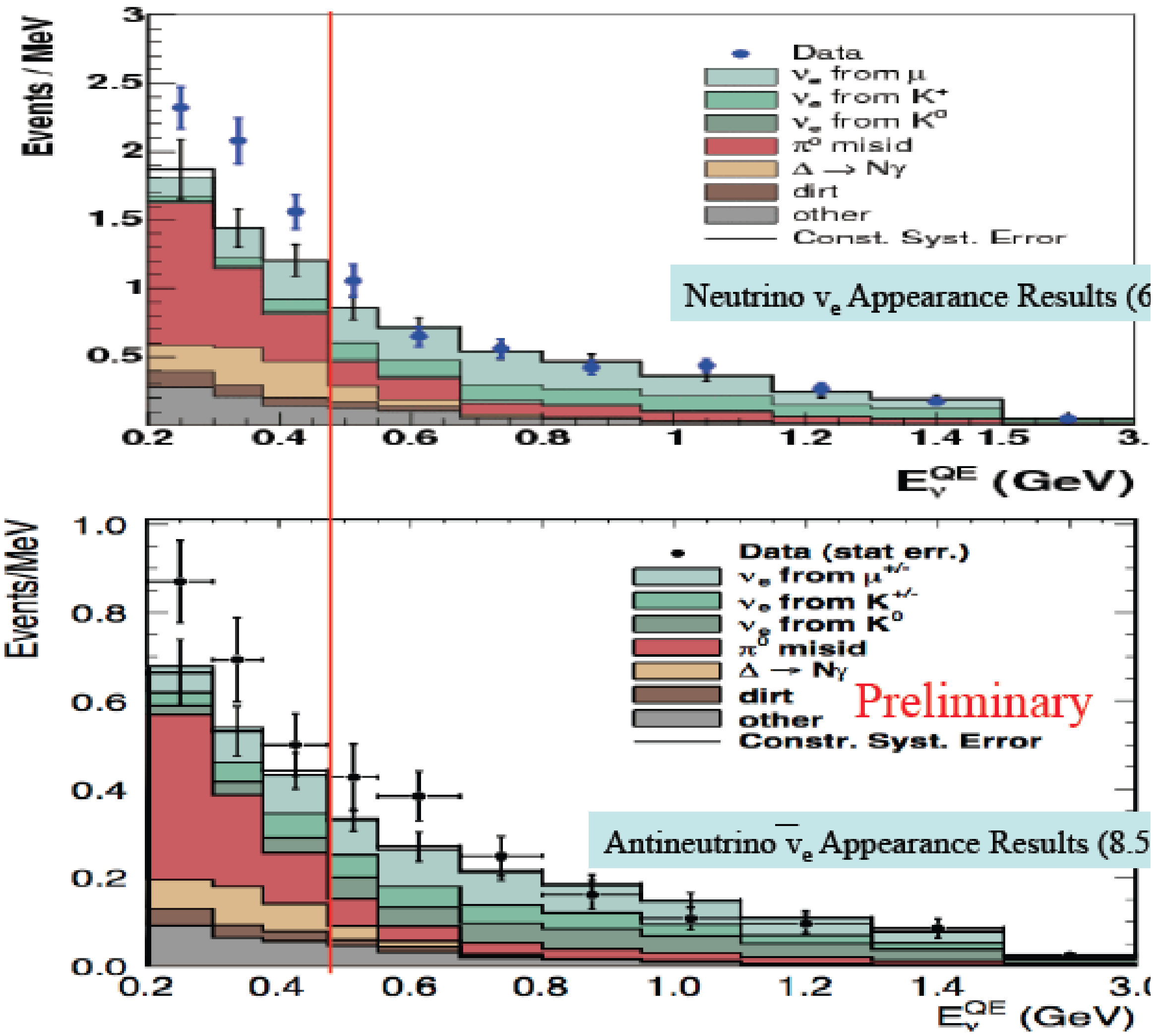
If all true (unlikely) then need at least 2 sterile ν's

Important information also from

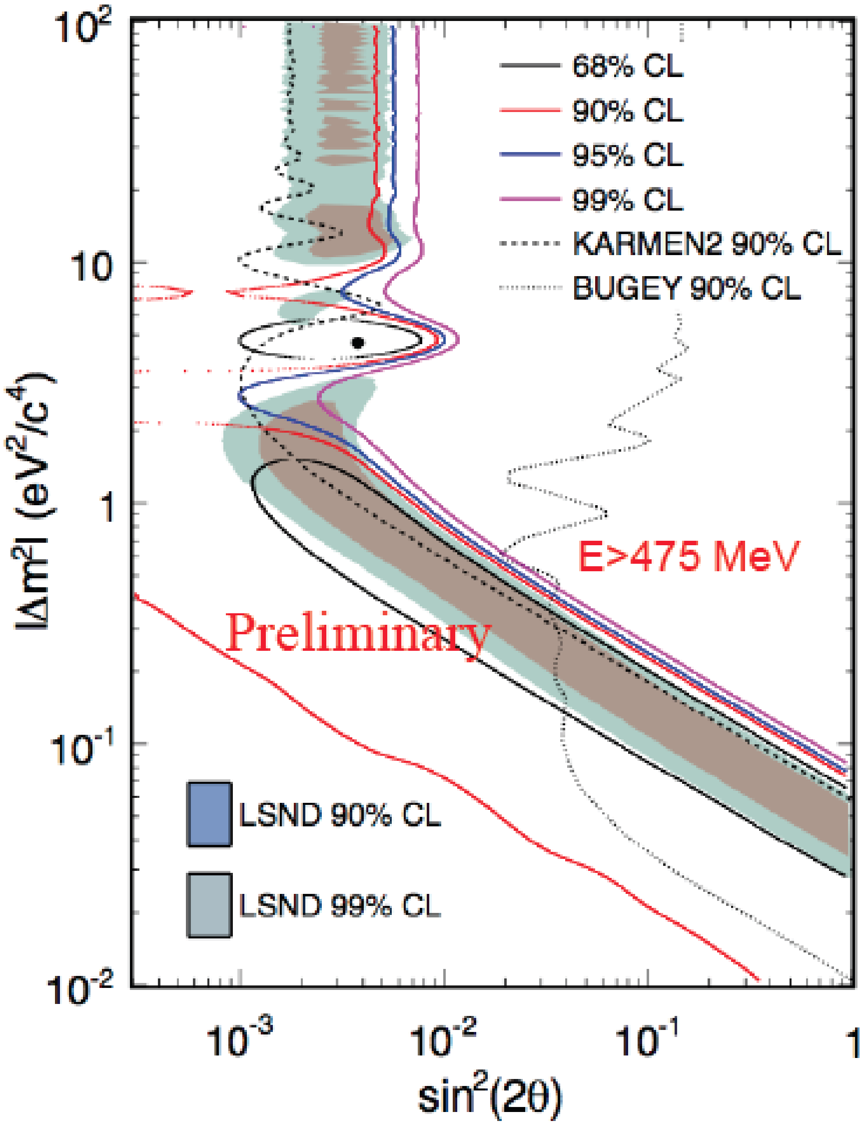
- Neutrino counting from cosmology

Unidentified excess at low energy

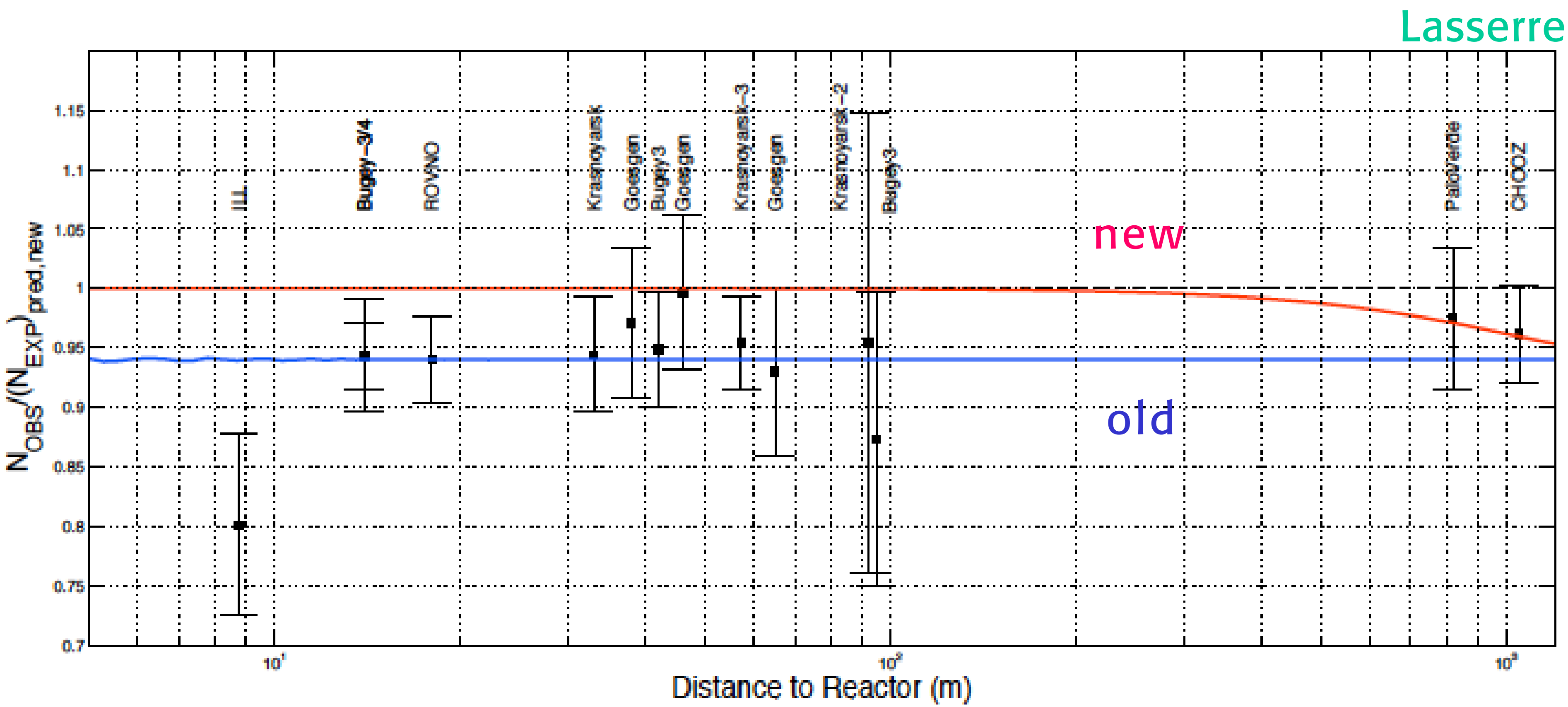
MiniBooNE



$(\Delta m^2, \sin^2 2\theta) = (4.6\ eV^2, 0.0045)$

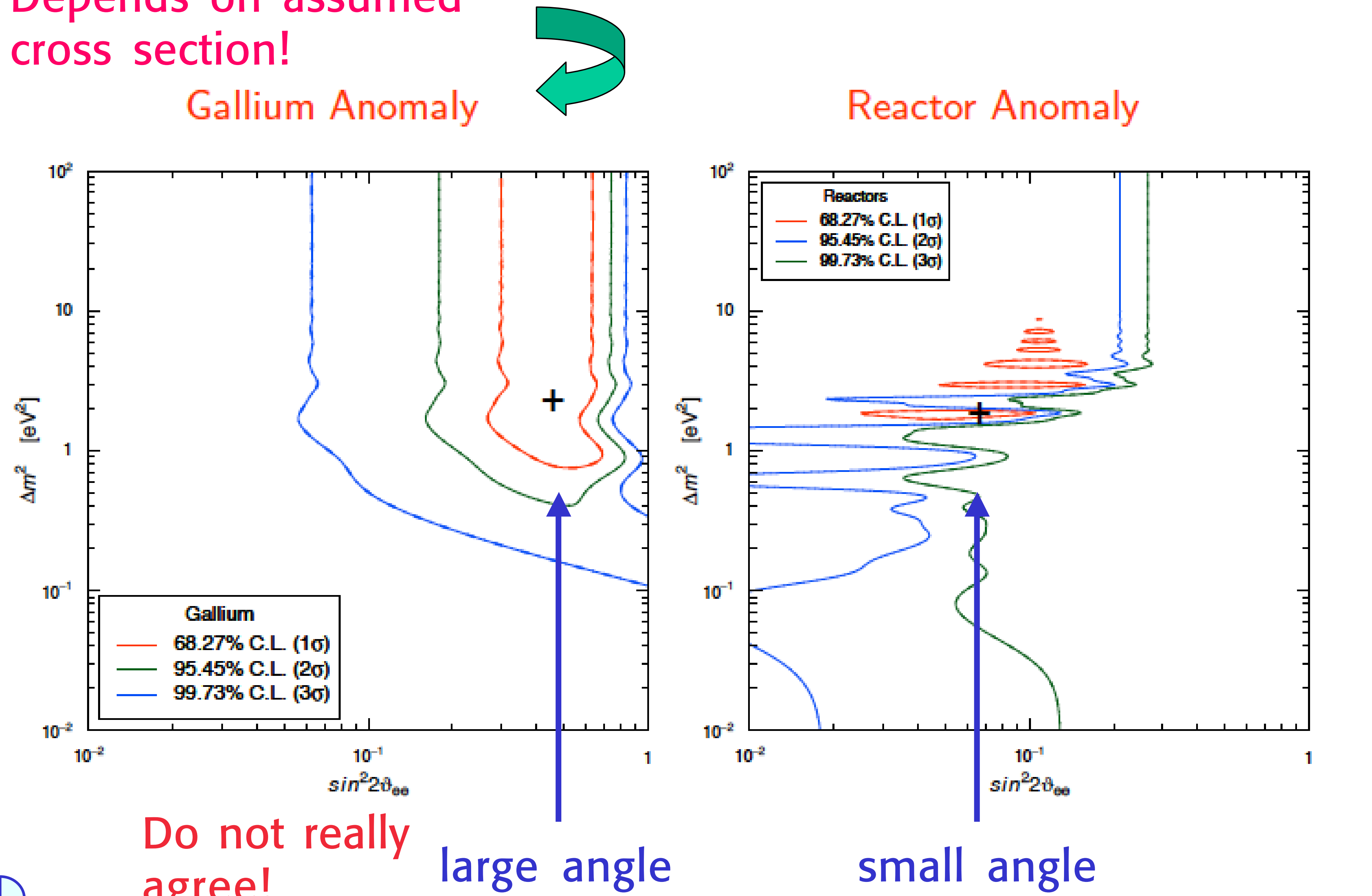


The reactor anomaly



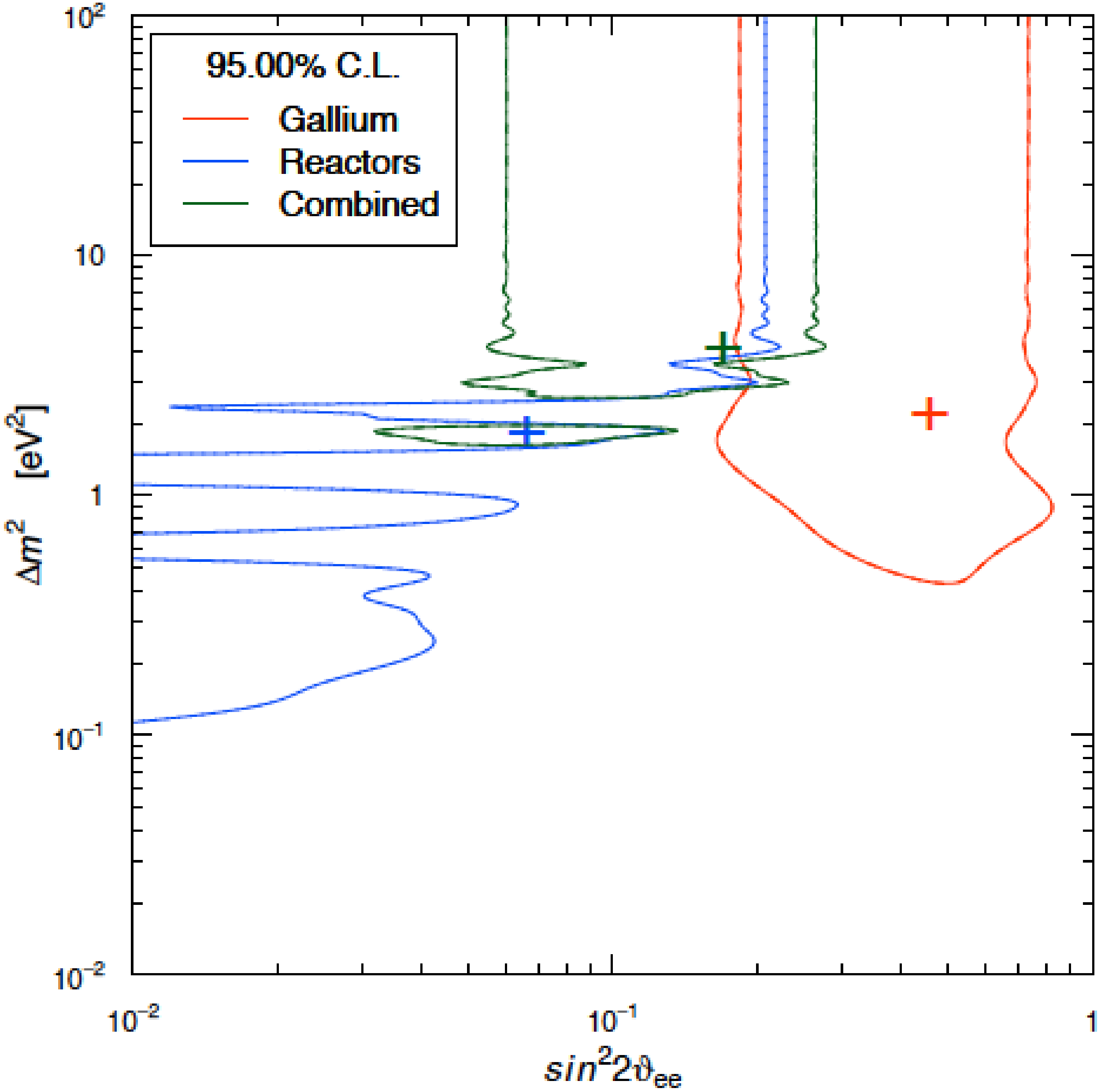
Systematic errors not shown in this figure (estimated in paper)!
Certainly of the same order of the shift.
They could well be larger than estimated

Depends on assumed cross section!



Do not really agree!

This is the compromise realized in the fit



$\chi^2_{\min} = 59.8$

$\mathrm{NdF} = 65$

$\mathrm{GoF} = 66\%$

$\sin^2 2\vartheta = 0.17$

$\Delta m^2 = 4.17\,\mathrm{eV}^2$

$\mathrm{PGoF} = 1.1\%$

Cosmology could accept one sterile neutrino

The bound from nucleosynthesis is the most stringent (assuming thermal properties at decoupling)

- BBN:
  $N_s = 0.22 \pm 0.59$ [Cyburt, Fields, Olive, Skillman, AP 23 (2005) 313, astro-ph/0408033]
  $N_s = 0.64^{+0.40}_{-0.35}$ [Izotov, Thuan, ApJL 710 (2010) L67, arXiv:1001.4440]

N_s

- BBN: $N_s < 1.2$ (95% CL) Mangano, Serpico, 1103.1261

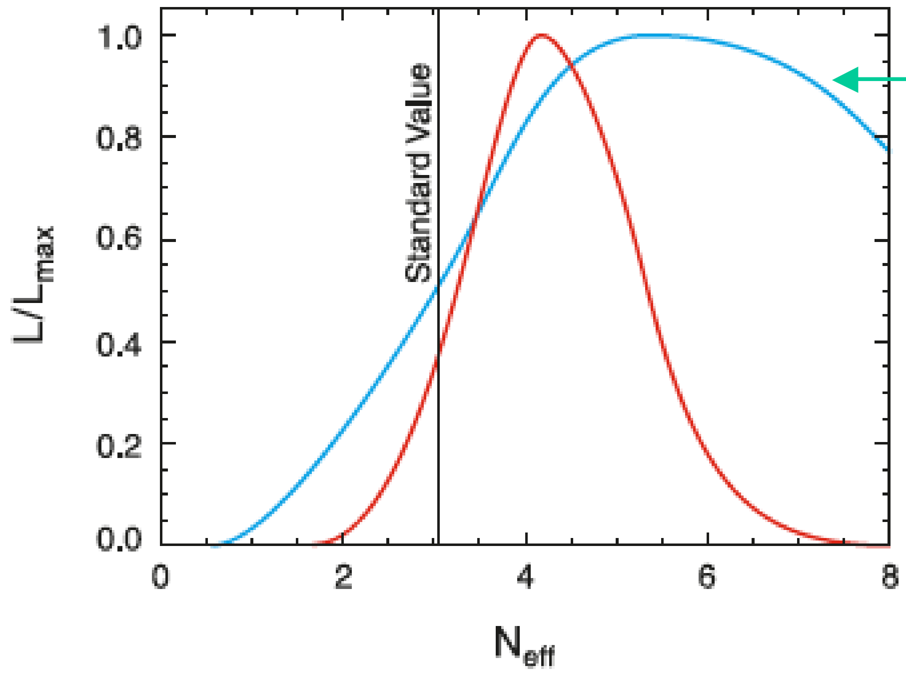
- BBN: $N_s < 1.54$ (95% CL) [M. Pettini, et al, arXiv:0805.0594]

From other than nucleosynthesis:

WMAP+BAO+H_0

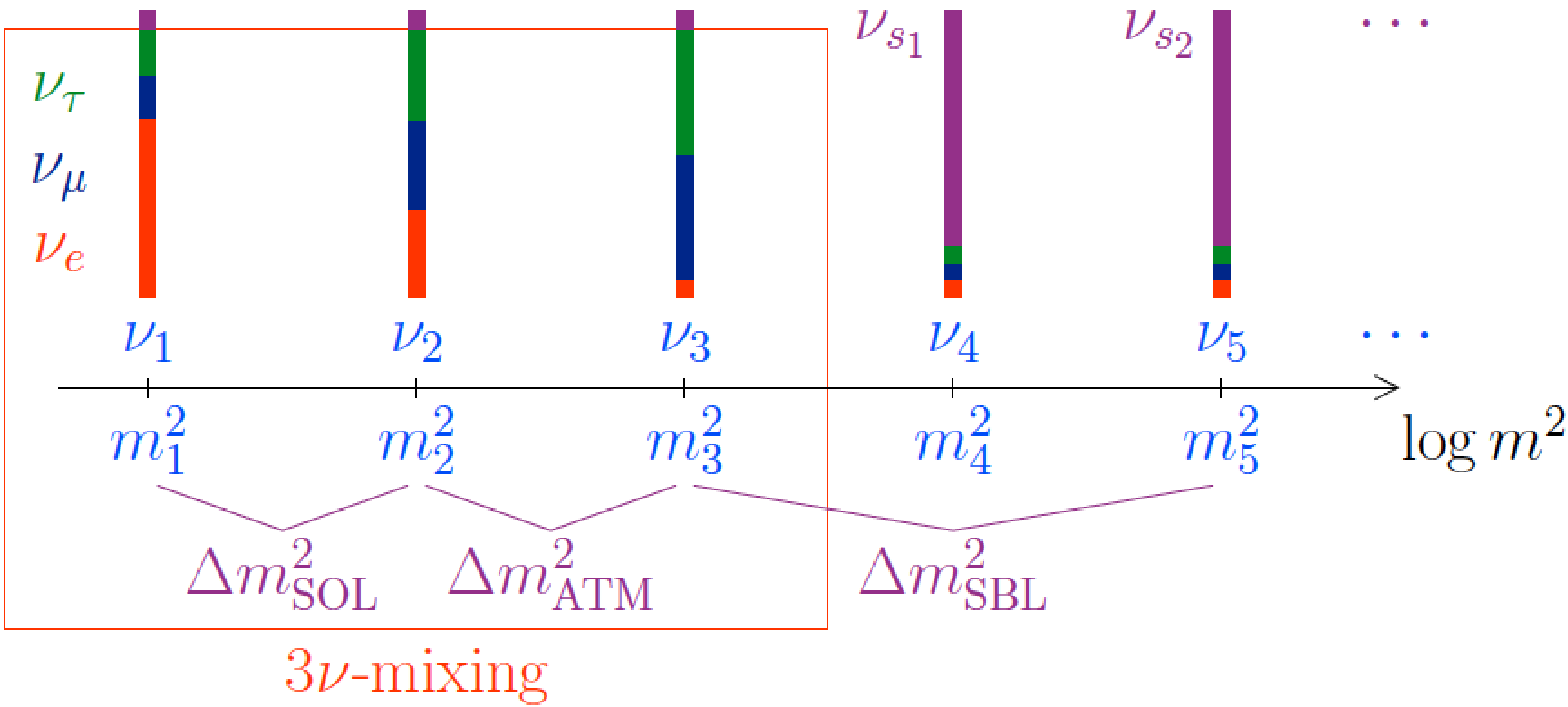
$N_s = 1.34 \pm 0.87$

Komatsu et al



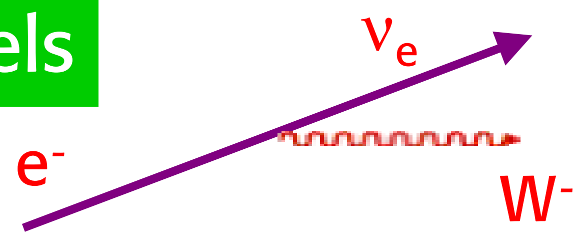
WMAP only

In any case only a small leakage from active to sterile neutrinos is allowed by present data



Most common EW scale BSM do not contain sterile neutrinos. A sterile neutrino would probably be a remnant of some hidden sector or of gravity. So would be a great discovery.

Still the main framework: 3-ν Models



$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U^+ \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavour mass

$U = U_{PMNS}$

Pontecorvo
Maki, Nakagawa, Sakata

In basis where e^-, μ^-, τ^- are diagonal:

δ: CP violation

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim$$

s = solar: large

CHOOZ: $|s_{13}|$ small

$$\sim \begin{bmatrix} c_{13}\,c_{12} & c_{13}\,s_{12} & s_{13}e^{-i\delta} \\ \ldots & \ldots & c_{13}\,s_{23} \\ \ldots & \ldots & c_{13}\,c_{23} \end{bmatrix}$$

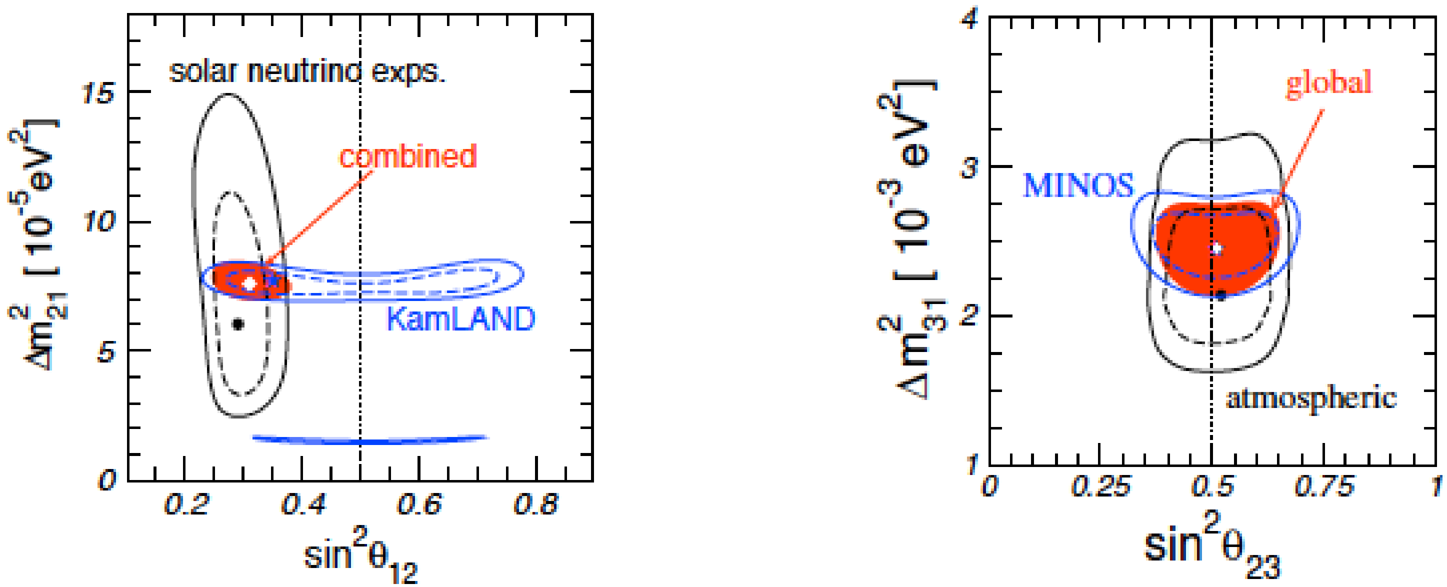
atm.: ~ max

(some signs are conventional)

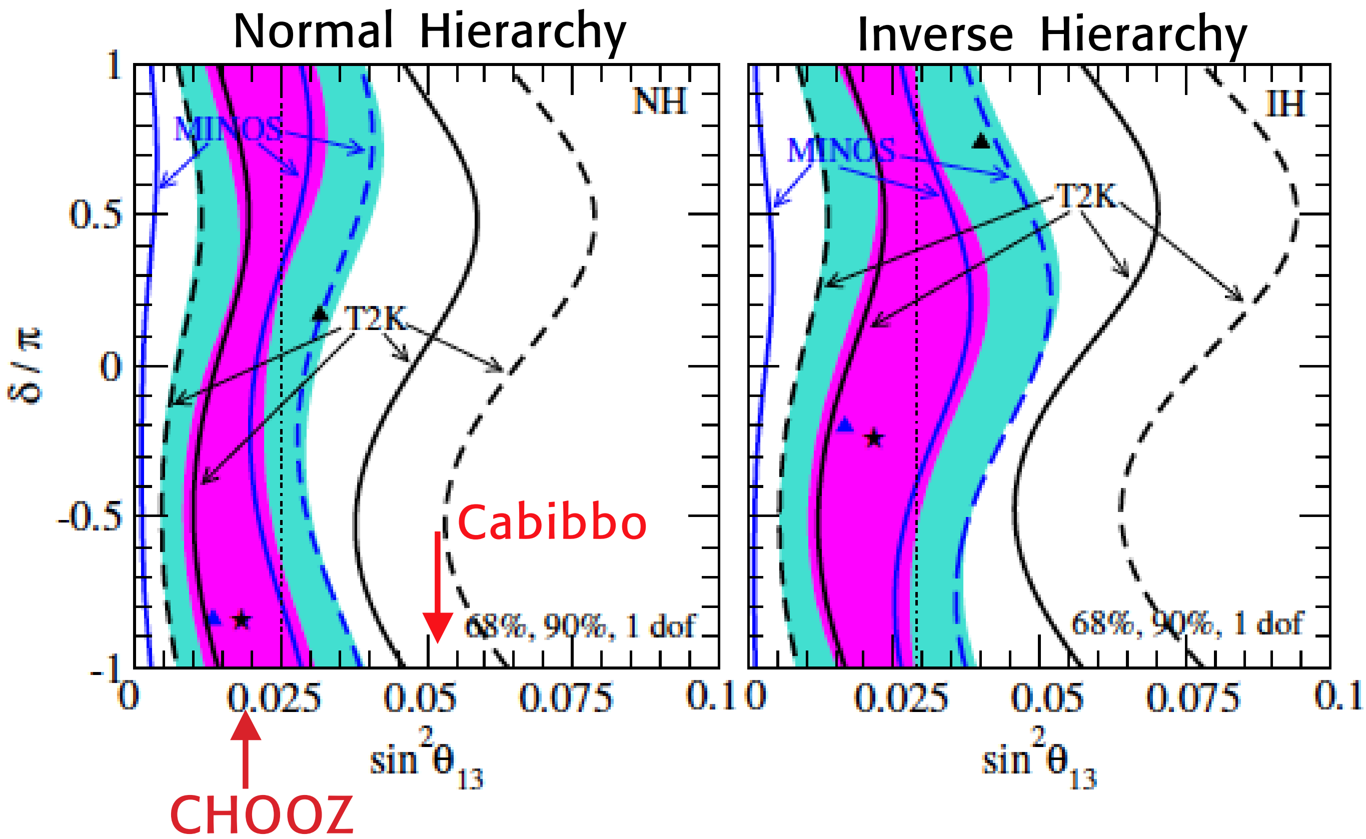
In general: $U = U^+_e U_\nu$

Recent Fits (2011)

Quantity	Fogli et al [1)]	Schwetz et al [2)]
Δm^2_{sun} $(10^{-5}$ eV$^2)$	$7.58^{+0.22}_{-0.26}$	$7.59^{+0.20}_{-0.18}$
Δm^2_{atm} $(10^{-3}$ eV$^2)$	$2.35^{+0.12}_{-0.09}$	$2.50^{+0.09}_{-0.16}$
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.016}$	$0.312^{+0.017}_{-0.015}$
$\sin^2\theta_{23}$	$0.42^{+0.08}_{-0.03}$	$0.52^{+0.06}_{-0.07}$
$\sin^2\theta_{13}$	0.025 ± 0.007	$0.013^{+0.007}_{-0.005}$



Recent results on θ_{13} (T2K, MINOS, DOOBLE CHOOZ)



T2K: 6 $\nu_\mu \rightarrow \nu_e$ events seen 1.5 ± 0.3 expected

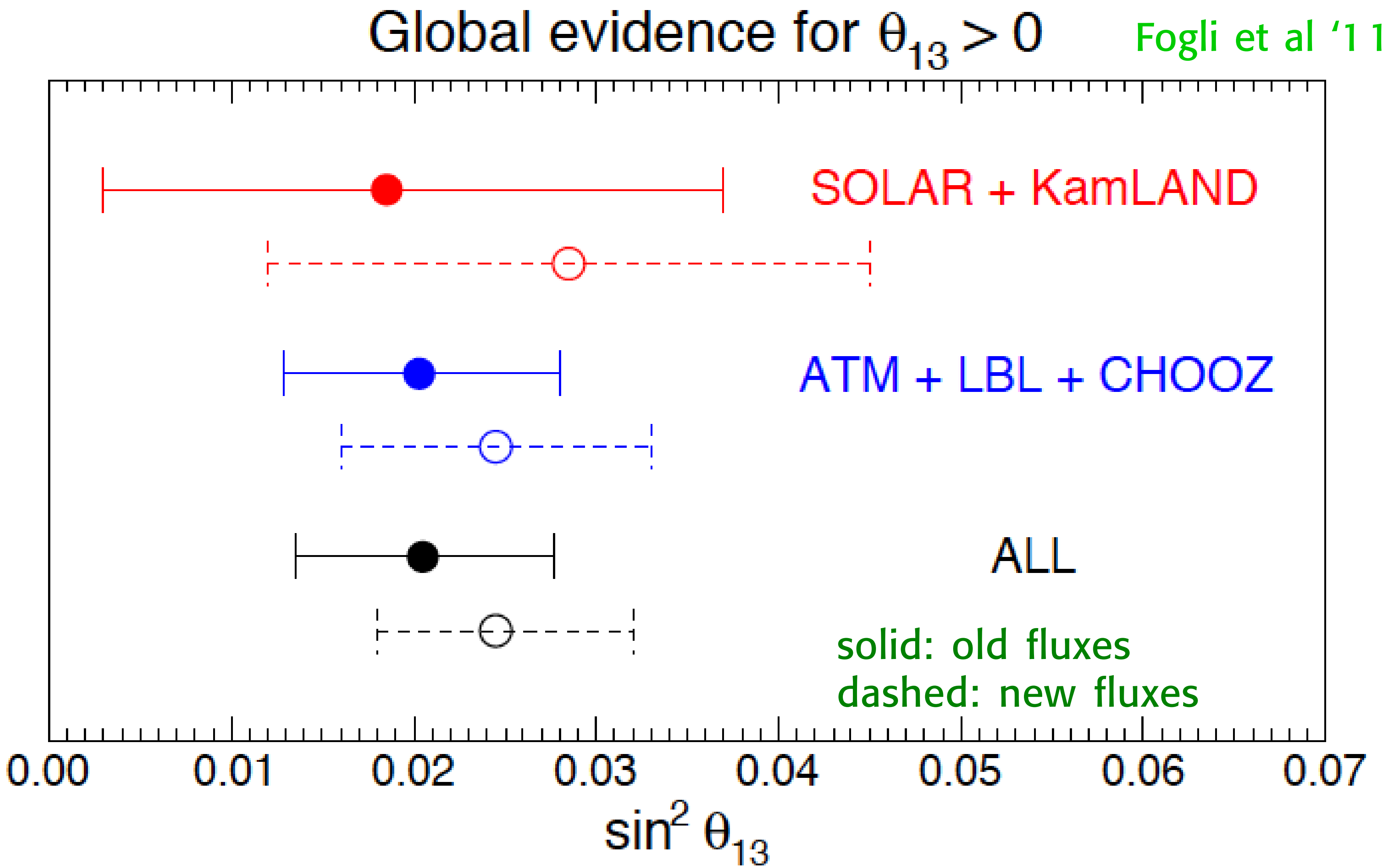
$0.03 < \sin^2 2\theta_{13} < 0.28$ for NH, 90%cl

for $\theta_{13} = 0$

MINOS: 62 $\nu_\mu \rightarrow \nu_e$ events seen 49.6 ± 7.5 expected

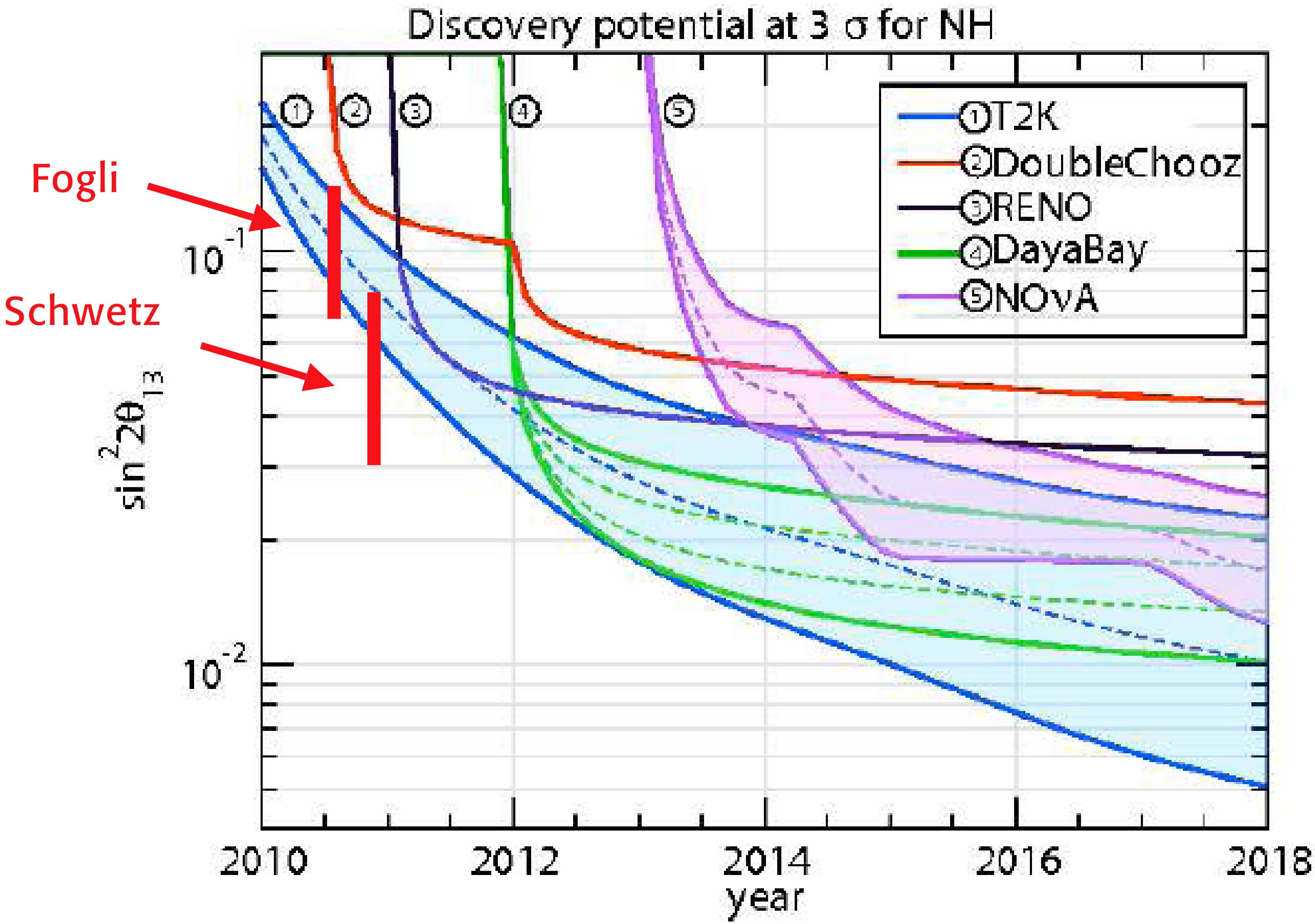
$0 < \sin^2 2\theta_{13} < 0.12$ for NH, 90%cl

DOUBLE CHOOZ: $\sin^2 2\theta_{13} = 0.085 \pm 0.051$



Global evidence for $\theta_{13} > 0$
Fogli et al '11
SOLAR + KamLAND
ATM + LBL + CHOOZ
ALL
solid: old fluxes
dashed: new fluxes
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
$\sin^2\theta_{13}$

The near future of θ_{13}



ν oscillations measure Δm^2. What is m^2?

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2 = (0.05\ eV)^2$; $\Delta m^2_{sun} \sim 8\ 10^{-5}\ eV^2 = (0.009\ eV)^2$

- Direct limits

$m_{"\nu e"} < 2.2$ eV

$m_{"\nu\mu"} < 170$ KeV

$m_{"\nu\tau"} < 18.2$ MeV

End-point tritium β decay (Mainz, Troitsk)
Future: Katrin, MARE
0.2 eV sensitivity
(Karsruhe)

$m_{ee} = |\Sigma\, U_{ei}^2\, m_i|$

- $0\nu\beta\beta$ $\quad m_{ee} < 0.2 - 0.7 - ?$ eV (nucl. matrix elmnts)

Evidence of signal? Klapdor-Kleingrothaus

- Cosmology $\quad \Omega_\nu\, h^2 \sim \Sigma_i m_i\ /94\text{eV}$ $\quad (h^2 \sim 1/2)$

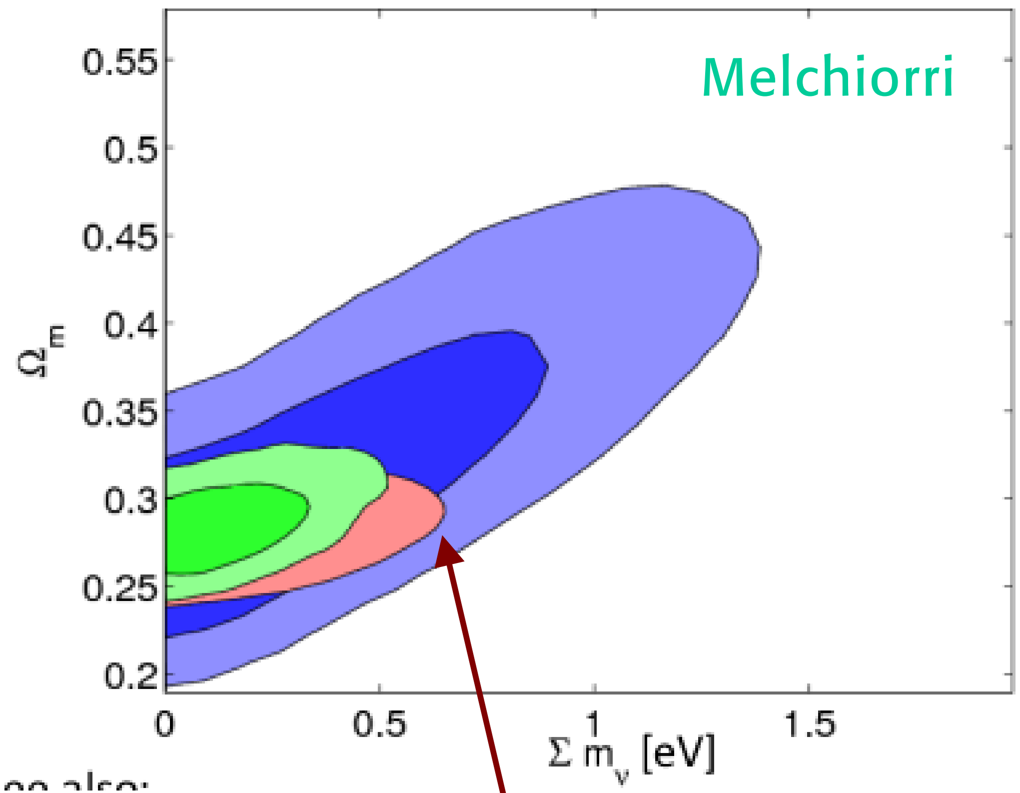
$\Sigma_i m_i < 0.2\text{-}0.7$ eV (dep. on data&priors)

WMAP, SDSS, 2dFGRS, Ly-α

$\Rightarrow$ Any ν mass $< 0.06 - 0.23 - 2.2$ eV

Current constraints on neutrino mass from Cosmology

By itself CMB (eg WMAP) is only mildly sensitive to $\Sigma_i m_i$
Only with Large Scale Structure the limit becomes stronger.



Blue: WMAP-7
Red: w7+SN+Bao+H0
Green: w7+CMBsuborb+SN+LRG+H0

Current constraints (assuming ΛCDM):

$\Sigma m_\nu < 1.3$ [eV] CMB

$\Sigma m_\nu < 0.7\text{-}0.5$ [eV] CMB+other

$\Sigma m_\nu < 0.3$ [eV] CMB+LSS (extreme)

best estimate

$\Sigma m_\nu < 0.58$ eV (95% CL) WMAP +BAO+ Hubble constant

Komatsu et al, 2009

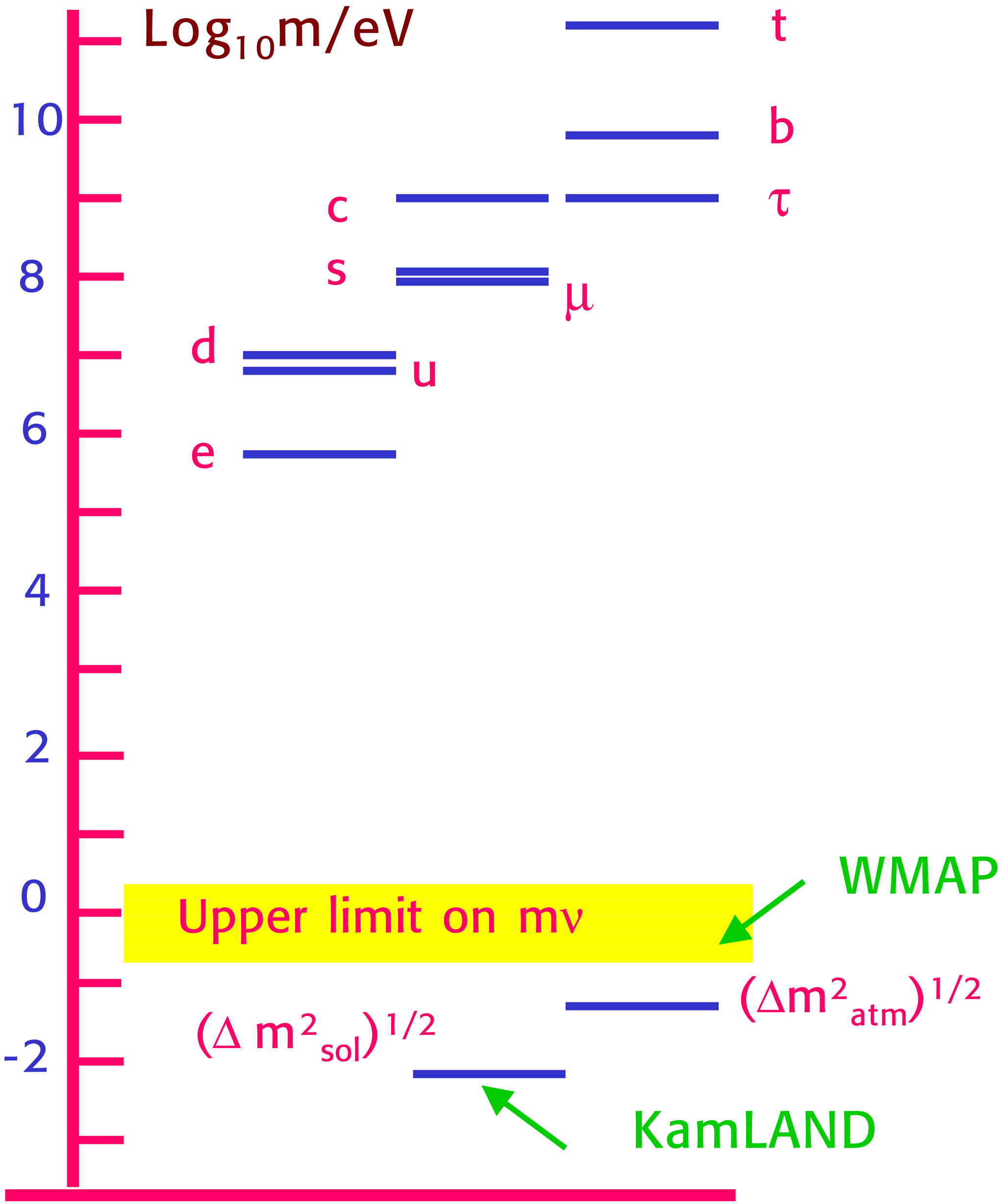
Dark Matter

WMAP, BAO….

Most of the Universe is not made up of atoms: $\Omega_{tot}\sim 1$, $\Omega_b \sim 0.045$, $\Omega_m \sim 0.27$
Most is Dark Matter and Dark Energy

Most Dark Matter is Cold (non relativistic at freeze out)
Significant Hot Dark matter is disfavoured
Hot Dark Matter does not "stick" enough at short distances (Galaxy haloes...)

Neutrinos are not much cosmo-relevant: $\Omega_\nu < 0.015$



Neutrino masses are really special!

$m_t/(\Delta m^2_{atm})^{1/2} \sim 10^{12}$

Massless ν's?

- no ν_R
- L conserved

Small ν masses?

- ν_R very heavy
- L not conserved

Very likely:
ν's are special as they are Majorana fermions

Are neutrinos Dirac or Majorana fermions?

Under charge conjugation C: particle <--> antiparticle

For bosons there are many cases of particles that coincide (up to a phase) with their antiparticle:

$$\pi^0, \rho^0, \omega, \gamma, Z^0.....$$

A fermion that coincides with its antiparticle is called a Majorana fermion

Are there Majorana fermions?
Neutrinos are probably Majorana fermions

The fundamental fermions of the Standard Model:

$$\begin{bmatrix} uuu\nu_e \\ ddde \end{bmatrix} \quad \begin{bmatrix} ccc\nu_\mu \\ sss\mu \end{bmatrix} \quad \begin{bmatrix} ttt\nu_\tau \\ bbb\tau \end{bmatrix}$$

- Of all fundamental fermions only ν's are neutral

  If lepton number L conservation is violated then no conserved charge distinguishes neutrinos from antineutrinos ⟶

  Majorana ν's : neutrinos and antineutrinos coincide

  neutrinos are their own antiparticles

- ν's have very small masses

  The two facts are probably related

The field of an electron (massive, charged) has 4 components

In fact there are 4 dof: $e^-, e^+, h = +, -$
(h is the helicity: component of spin along momentum)

$$
\begin{tikzcd}
|e^-, h = +> \arrow[r, "\text{Lorentz boost}"] \arrow[d, "\text{TCP}"'] & |e^-, h = -> \arrow[d, "\text{TCP}"'] \\
|e^+, h = -> \arrow[r, "\text{Lorentz boost}"] & |e^+, h = +>
\end{tikzcd}
$$

A 2-component description is possible in two cases:

- for a massless neutrino $|\nu_L\rangle = |\nu, h = -1\rangle$ and $|\overline{\nu}_R\rangle = |\overline{\nu}, h = +1\rangle$ can be enough because massless particles go at the speed of light (no boost can flip h)

But now we know that (at least two) neutrinos have non vanishing masses, although very small

- for a completely neutral neutrino there is the possibility that neutrino and antineutrino coincide (Majorana neutrino)

Each neutrino mass eigenstate of definite helicity coincides with its own antiparticle

ν's have no electric charge.
Their only charge is lepton number L.

IF L is not conserved (not a good quantum number)
ν and $\bar{\nu}$ are not really different

TCP, "Lorentz"

$$|\nu, h = -1/2> \longrightarrow |\bar{\nu}, h = +1/2>$$

For a massive Majorana neutrino only two states are enough

A Majorana neutrino is identical with its charge conjugated

$$C|\nu> = |\bar{\nu}> = |\nu>$$

Each neutrino mass eigenstate of definite helicity coincides with its own antiparticle

ν masses:

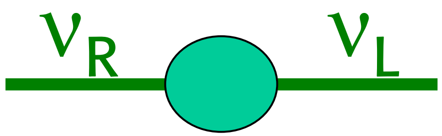
recall: ν_R : ann $|\nu_R>$ creates $|\overline{\nu}_L>$

$\overline{\nu}_L$: ann $|\overline{\nu}_R>$ creates $|\nu_L>$

For massive fermions L,R
refer to chirality, not helicity

Dirac mass: $\overline{\nu}_L \nu_R + \overline{\nu}_R \nu_L$

(needs ν_R)



Lepton number (L)-conserving

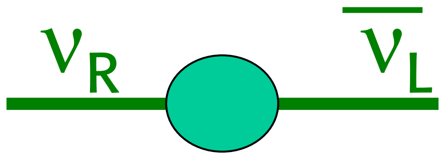
Don't confuse left-chirality and lepton n.

Majorana mass: $\overline{\nu^c}\nu \to \nu^T{}_R C \nu_R$ or $\nu^T{}_L C \nu_L$

$$\boxed{\psi^c = C\overline{\psi}^T}$$

$C = i\gamma^2\gamma^0$

Violates L, B-L by $|\Delta L| = 2$



short-hand: $\longrightarrow$ $\nu^T{}_R\, \nu_R$ or $\nu^T{}_L\, \nu_L$

Weak isospin I

$\nu_L \Rightarrow I = 1/2,\ I_3 = 1/2$

$\nu_R \Rightarrow I = 0,\ I_3 = 0$

Dirac Mass:

$\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L$ $|\Delta I| = 1/2$

For Dirac ν's no explanation of small masses

Can be obtained from Higgs doublets: $\nu_L \nu_R H$

Majorana Mass:

- $\nu^T{}_L \nu_L$ $|\Delta I| = 1$

Non ren., dim. 5 operator: $\nu^T{}_L\ \nu_L HH$

- $\nu^T{}_R \nu_R$ $|\Delta I| = 0$ $\longrightarrow$ Directly compatible with SU(2)xU(1)!

See-Saw Mechanism

Minkowski; Glashow; Yanagida;
Gell-Mann, Ramond , Slansky;
Mohapatra, Senjanovic…..

$M\nu^T{}_R \nu_R$ allowed by SU(2)xU(1)

Large Majorana mass M (as large as the cut-off)

$m_D \overline{\nu}_L \nu_R$

Dirac mass m_D from Higgs doublet(s)

$$\begin{array}{c c} & \begin{array}{cc} \nu_L & \nu_R \end{array} \\ \begin{array}{c} \nu_L \\ \nu_R \end{array} & \begin{bmatrix} 0 & m_D \\ m_D & M \end{bmatrix} \end{array}$$

$M >> m_D$

Eigenvalues

$$|\nu_{light}| = \frac{m_D{}^2}{M} \quad , \quad \nu_{heavy} = M$$

A very natural and appealing explanation:

ν's are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale $M \sim M_{GUT}$

$$m_\nu \sim \frac{m^2}{M}$$

$m: \leq m_t \sim v \sim 200$ GeV

M: scale of L non cons.

Note:

$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.05$ eV

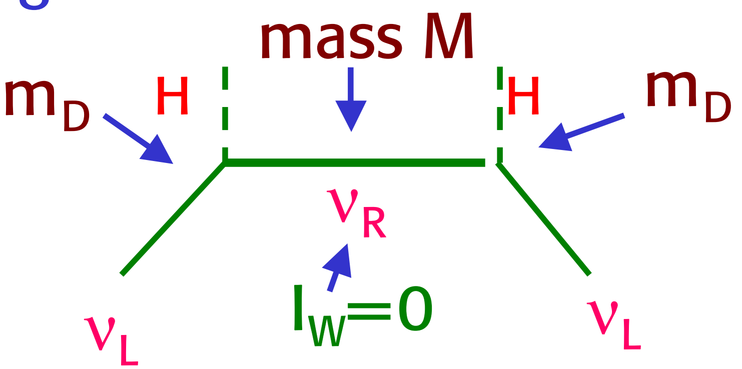
$m \sim v \sim 200$ GeV

$M \sim 10^{14} - 10^{15}$ GeV

Neutrino masses are a probe of physics at M_{GUT} !

See-saw diagrams

Type 1

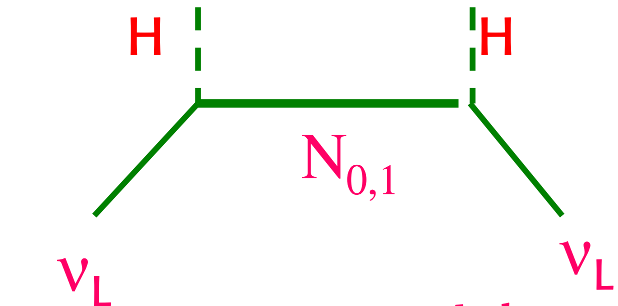


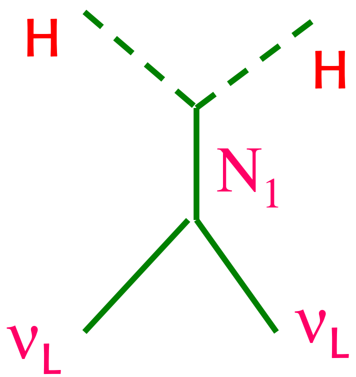
$\nu_L^T m_\nu \nu_L$

$m_\nu = m_D^T M^{-1} m_D$

More in general: non ren. O_5 operator

$$O_5 = \ell^T \frac{\lambda^2}{M} \ell H H$$





e.g from
I_W=1Boson:Type 2

$N_{0,1}$: new particle I_w=0,1

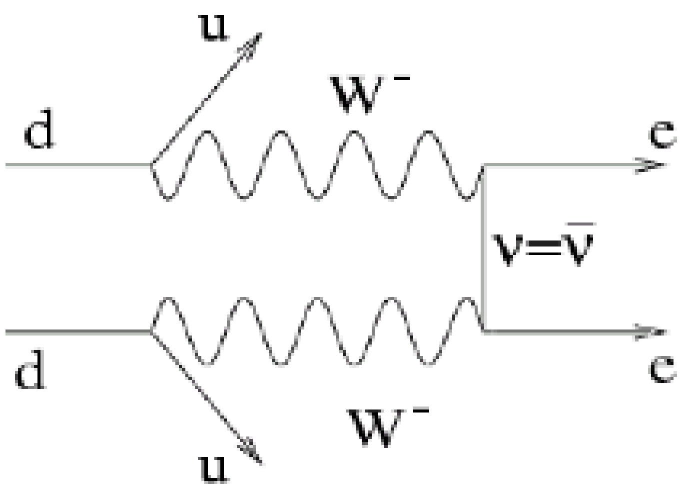
I_W=1Fermion:Type 3

Whatever the underlying dynamics O_5 is a general effective description of light Majorana neutrino masses

ν oscillations point to very large values of M ~ M_{GUT}

How to prove that ν's are Majorana fermions?

All we know from experiment on ν masses strongly indicates that ν's are Majorana particles and that L is not conserved (but a direct proof still does not exist).



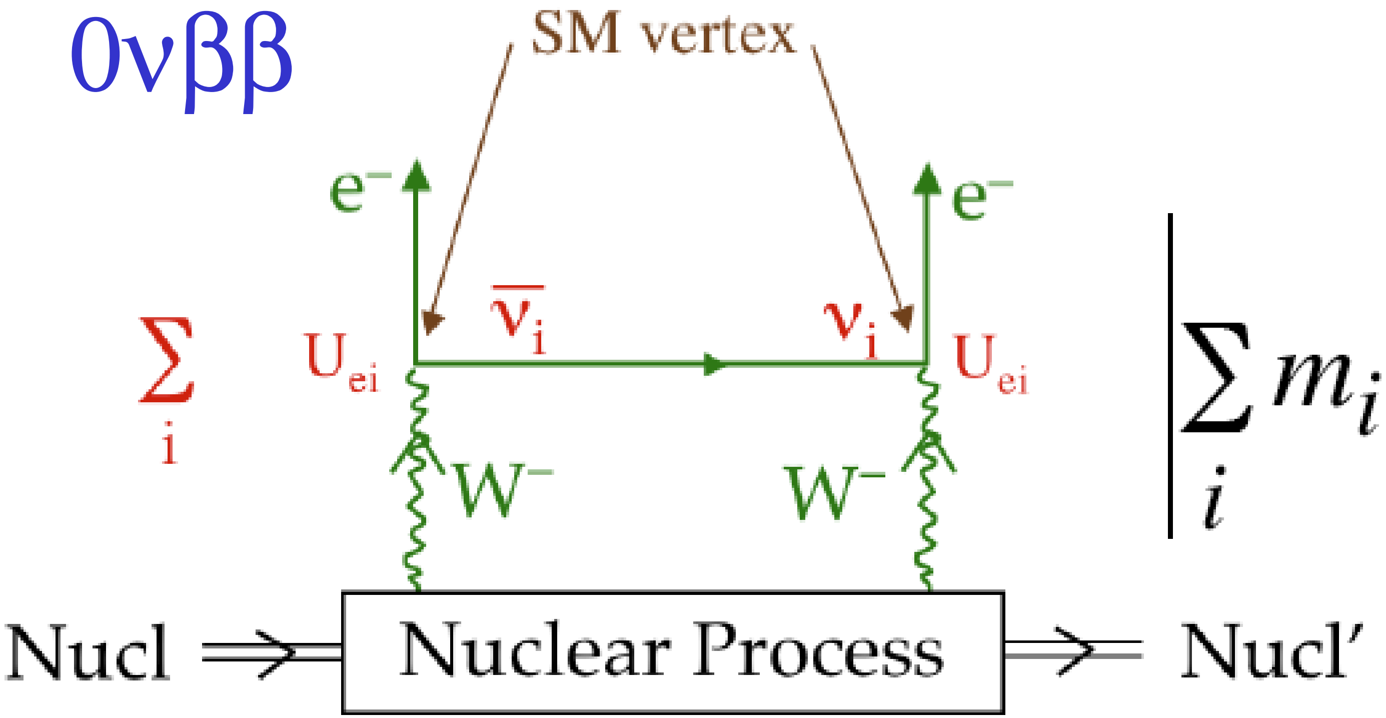
$0\nu\beta\beta$ = dd -> uue⁻e⁻

$$\left|\sum_i m_i U_{ei}^2\right| \equiv m_{\beta\beta}$$

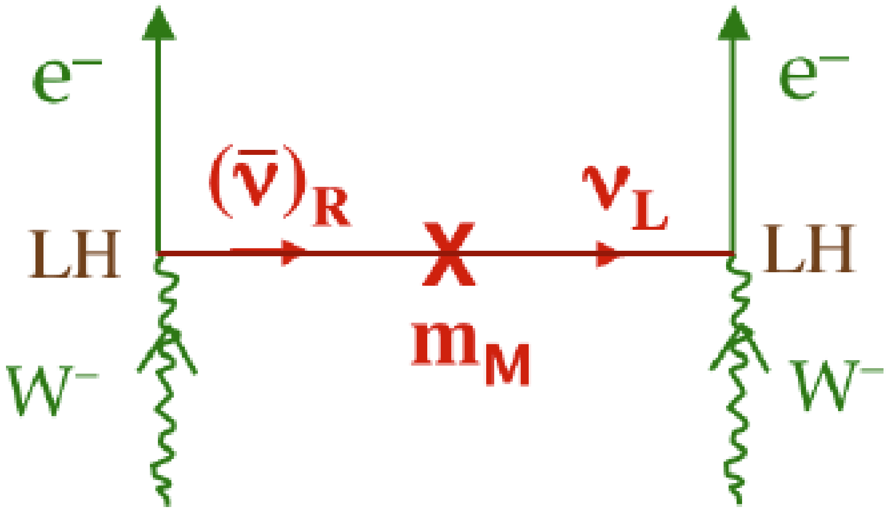
Detection of $0\nu\beta\beta$ (neutrinoless double beta decay) would be a proof of L non conservation ($\Delta L=2$).
Thus a big effort is devoted to improving present limits and possibly to find a signal.

- Heidelberg-Moscow, Cuoricino-Cuore, GERDA, ...

0νββ



$$\left|\sum_i m_i U_{ei}^2\right| \equiv m_{\beta\beta}$$



0νββ signal
would establish
Majorana ν's

$0\nu\beta\beta$ would prove that L is not conserved and ν's are Majorana
Also can tell degenerate, inverted or normal hierarchy

$|m_{ee}|=c_{13}^2\ [m_1 c_{12}^2+e^{i\alpha}m_2 s_{12}^2]+m_3 e^{i\beta}s_{13}^2$

Degenerate: $\sim|m||c_{12}^2+e^{i\alpha}s_{12}^2|\sim|m|(0.3\text{-}1)$

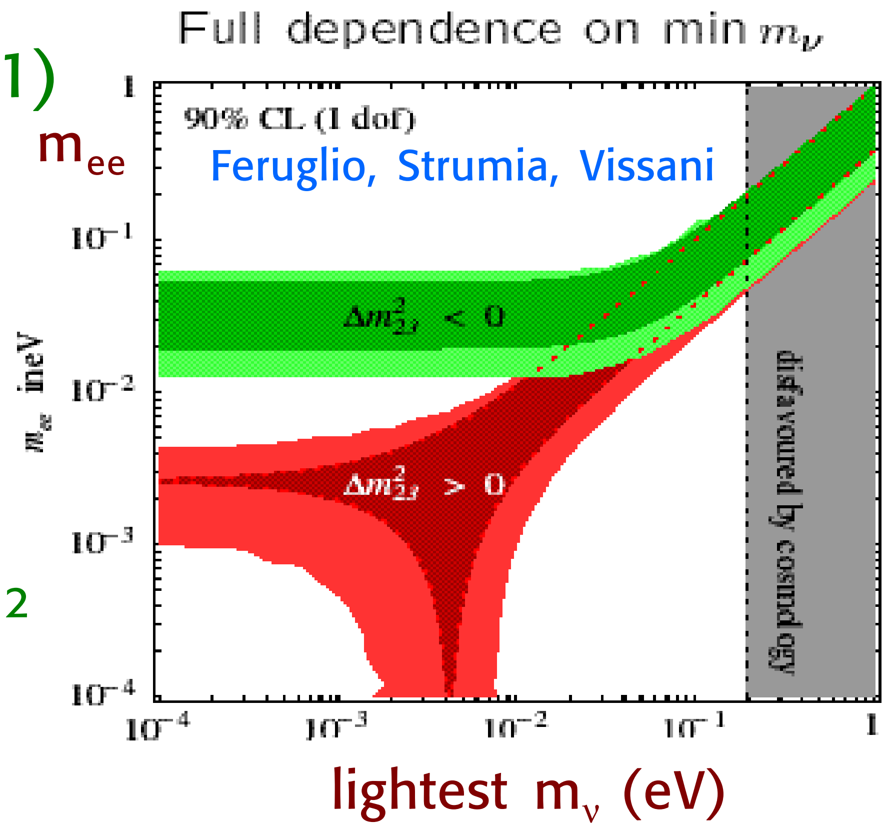
$|m_{ee}|\sim|m|\ (0.3\ \text{-}1)\leq 0.23\text{-}1\ \text{eV}$

IH: $\sim(\Delta m^2_{atm})^{1/2}|c_{12}^2+e^{i\alpha}s_{12}^2|$

$|m_{ee}|\sim(1.6\text{-}5)\ 10^{-2}\ \text{eV}$

NH: $\sim(\Delta m^2_{sol})^{1/2}s_{12}^2+(\Delta m^2_{atm})^{1/2}e^{i\beta}s_{13}^2$

$|m_{ee}|\sim(\text{few})\ 10^{-3}\ \text{eV}$



Present exp. limit: $m_{ee}<0.3\text{-}0.5$ eV

Baryogenesis

$n_B/n_\gamma \sim 10^{-10}$, $n_B >> n_{Bbar}$

Conditions for baryogenesis: (Sacharov '67)

- B (and L) non conservation (obvious)
- C, CP non conserv'n (B-B^{bar} odd under C, CP)
- No thermal equilib'm ($n = \exp[\mu - E/kT]$; $\mu_B = \mu_{Bbar}$, $m_B = m_{Bbar}$ by CPT

If several phases of BG exist at different scales the asymm. created by one out-of-equilib'm phase could be erased in later equilib'm phases: BG at lowest scale best

Possible epochs and mechanisms for BG:

- At the weak scale in the SM Excluded
- At the weak scale in the MSSM Disfavoured
- Near the GUT scale via Leptogenesis Very attractive

Baryogenesis by decay of heavy Majorana ν's

BG via Leptogenesis near the GUT scale

$T \sim 10^{12\pm3}$ GeV (after inflation)

Buchmuller,Yanagida, Plumacher, Ellis, Lola, Giudice et al, Fujii et al ….

Only survives if Δ(B-L) is not zero
(otherwise is washed out at T_{ew} by instantons)

Main candidate: decay of lightest ν_R (M~10^{12} GeV)

L non conserv. in ν_R out-of-equilibrium decay:
B-L excess survives at T_{ew} and gives the obs. B asymmetry.

Quantitative studies confirm that the range of m_i from ν oscill's is compatible with BG via (thermal) LG

In particular the bound was derived for hierarchy

$m_i < 10^{-1}$ eV

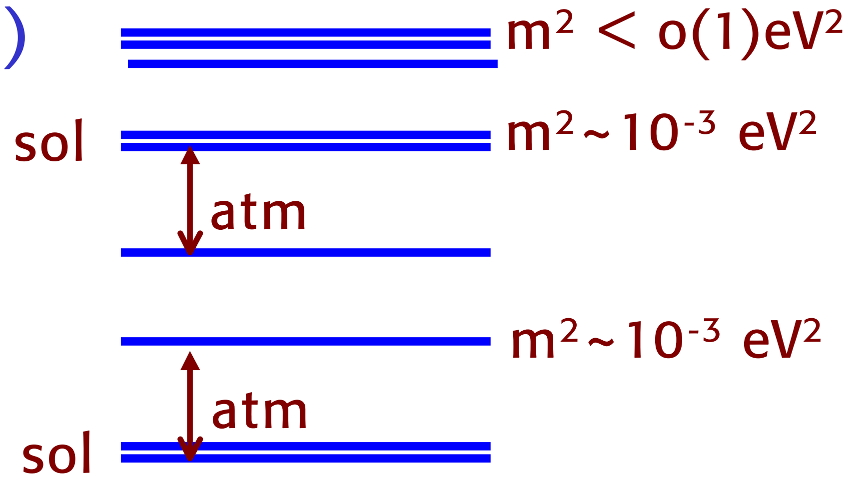
Buchmuller, Di Bari, Plumacher; Giudice et al; Pilaftsis et al; Hambye et al

Can be relaxed for degenerate neutrinos
So fully compatible with oscill'n data!!

The current experimental situation on ν masses and mixings has much improved but is still incomplete

- what is the absolute scale of ν masses?
- precise value of θ_{13}, shift of θ_{23} from maximal, CP viol. phase....
- pattern of spectrum (sign of Δm^2_{atm})

- Degenerate ($m^2 >> \Delta m^2$)
- Inverse hierarchy
- Normal hierarchy



- no detection of $0\nu\beta\beta$ (i.e. no proof that ν's are Majorana)
  see-saw?
- are 3 light ν's OK? (are there sterile neutrinos?)

→ Different classes of models are still possible

8

KIT, 6-10 February '12

Beyond the Standard Model

Neutrino Masses & Mixings 2012

Guido Altarelli

Universita' di Roma Tre
CERN

Models of ν masses and mixings

$$m_\ell \to \bar{R} m_\ell L$$

$$m_\ell' = V_\ell^\dagger m_\ell U_\ell$$

$$m_\ell^{\dagger\prime} m_\ell' = U_\ell^\dagger m_\ell^\dagger m_\ell U_\ell$$

An interplay of different matrices:

$$U_{PMNS} = U_\ell^\dagger U_\nu$$

neutrino diagonalisat'n

charged lepton diagonalisat'n

$$O_5 = \ell^T \frac{\lambda^2}{M} \ell HH \to \nu_L^T m_\nu \nu_L$$

See-saw

$$m_\nu = m_D^T M^{-1} m_D$$

neutrino Majorana mass

neutrino Dirac mass

The large ν mixing versus the small q mixing can be due to the Majorana nature of ν's

$$m_\nu' = U_\nu^T m_\nu U_\nu$$

General remarks

- Finally not too much hierarchy is found in ν masses:

$$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/30$$

Only a few years ago could be as small as 10^{-8}!

Precisely at 3σ: $0.025 < r < 0.039$

Schwetz et al '10

or

$m_{heaviest} < 0.2 - 0.7$ eV
$m_{next} > \sim 8\ 10^{-3}$ eV

For a hierarchical spectrum: $$\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$$

Comparable to $\lambda_C = \sin\theta_C$: $$\lambda_C \approx 0.22 \text{ or } \sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$$

Suggests the same "hierarchy" parameters for q, l, ν
(small powers of λ_C) $\longrightarrow$ e.g. θ_{13} not too small!

I now discuss some current ideas on model building

We go from less to more structure

Models with little symmetry are more qualitative.
Some examples:

Anarchy
Semianarchy
Lopsided models
$U(1)_{FN}$
••••••

With better data the range for each mixing angle has narrowed and precise special patterns are suggested that can be reproduced by specified symmetries :

TriBimaximal (TB), BiMaximal (BM),.......
Discrete non abelian flavour groups A4, S4,.....

An extreme point of view

No order for leptons -> Anarchy

In the lepton sector no symmetry, no dynamics is assumed; only chance

Hall, Murayama, Weiner'00

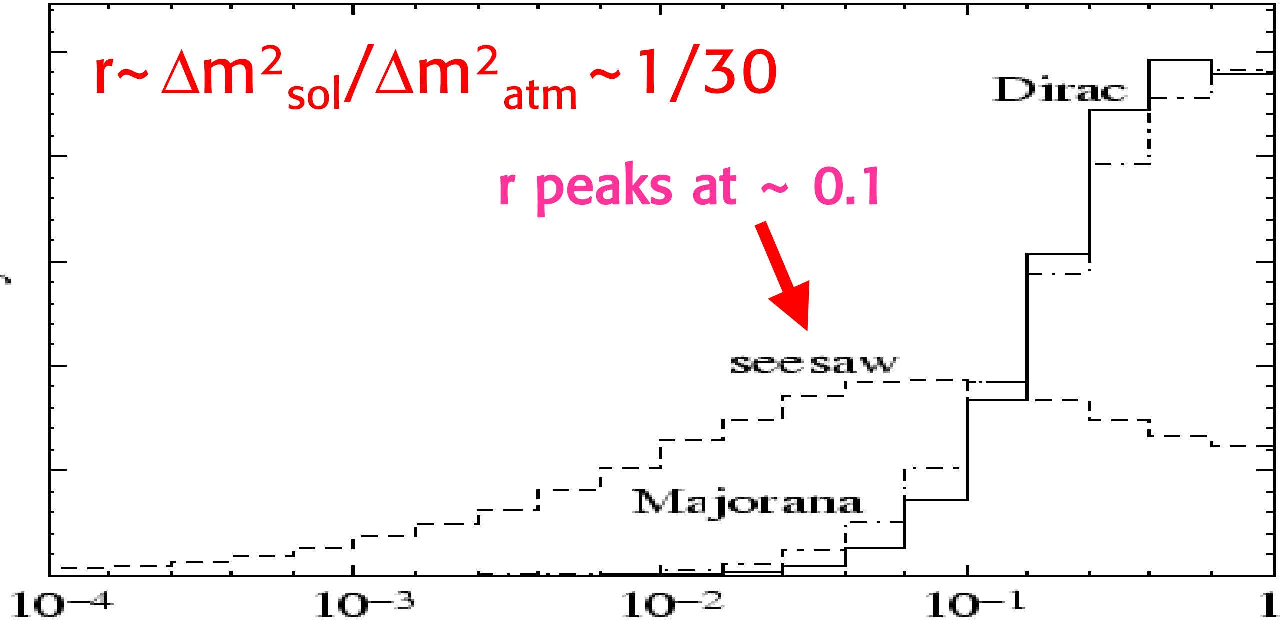
Boosted recently by θ_{13} near the previous bound

Anarchy (or accidental hierarchy): No structure in the neutrino sector

Hall, Murayama, Weiner

See-Saw:
$m_\nu \sim m^T M^{-1} m$
produces hierarchy from random m, M

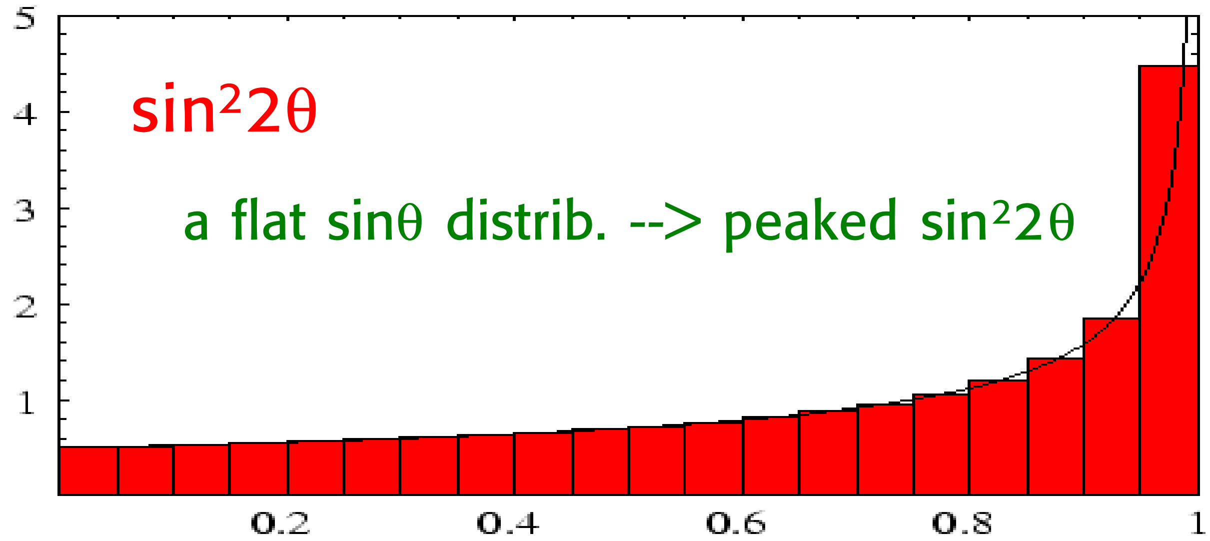
could fit the data on r



But: all mixing angles should be not too large, not too small

Predicts θ_{13} near bound
θ_{23} sizably non maximal

θ_{13} largish is great news for anarchy!



Anarchy and its variants can be embedded in a simple GUT context based on

$SU(5)xU(1)_{flavour}$

Froggatt Nielsen '79

Offers a simple description of hierarchies for quarks and leptons, but only orders of magnitude are predicted (large number of undetermined o(1) parameters)

Hierarchy for masses and mixings via horizontal $U(1)_{FN}$ charges.

Froggatt, Nielsen '79

The simplest flavour symmetry

Principle:

A generic mass term

$$\bar{R}_1 m_{12} L_2 H$$

is forbidden by U(1)
if $q_1+q_2+q_H$ not 0

q_1, q_2, q_H:
U(1) charges of
$\bar{R}_1, L_2, H$

U(1) broken by vev of "flavon" field θ with U(1) charge q_θ= -1.
If vev $\theta = w$, and $w/M=\lambda$ we get for a generic interaction:

Δ_{charge}

$$\bar{R}_1 m_{12} L_2 H \, (\theta/M)^{\,q1+q2+qH} \qquad m_{12} \to m_{12}\, \varepsilon^{q1+q2+qH}$$

Hierarchy: More Δ_{charge} -> more suppression ($\varepsilon=\theta/M$ small)

One can have more flavons ($\varepsilon, \varepsilon'$, ...)
with different charges (>0 or <0) etc -> many versions

Anarchy can be realised in SU(5) by putting all the flavour structure in T ~ 10 and not in F^{bar} ~ 5^{bar}

$m_u \sim 10\,.10$ strong hierarchy $m_u : m_c : m_t$

$m_d \sim 5^{bar}\,.10 \sim m_e^T$ milder hierarchy $m_d : m_s : m_b$

or $m_e : m_\mu : m_\tau$

Experiment supports that d, e hierarchy is roughly the square root of u hierarchy

$m_\nu \sim \nu_L^T m_\nu \nu_L \sim 5^T\,.5$ or for see saw $(5.1)^T\,(1.1)\,(1.5)$

For example, for the simplest flavour group, $U(1)_F$

1st fam. 2nd 3rd

anarchy
$$\left\{\begin{array}{ll} T: & (3, 2, 0) \\ F^{bar}: & (0, 0, 0) \\ 1: & (0, 0, 0) \end{array}\right.$$

A milder ansatz - Semianarchy: no structure only in 23

Consider a matrix like $m_\nu \sim L^T L \sim \begin{pmatrix} \varepsilon^4 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & 1 & 1 \\ \varepsilon^2 & 1 & 1 \end{pmatrix}$ Note: $\theta_{13} \sim \varepsilon^2$

$\theta_{23} \sim 1$

$q(5^{bar}) \sim (2, 0, 0)$

with coeff.s of o(1) and det23~o(1)

["semianarchy", while $\varepsilon \sim 1$ corresponds to anarchy]

After 23 and 13 rotations $m_\nu \sim \begin{pmatrix} \varepsilon^4 & \varepsilon^2 & 0 \\ \varepsilon^2 & \eta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Normally two masses are of o(1) or r ~1 and $\theta_{12} \sim \varepsilon^2$

But if, accidentally, $\eta \sim \varepsilon^2$, then r is small and θ_{12} is large.

The advantage over anarchy is that θ_{13} is naturally small and a single accident is needed to get both θ_{12} large and r small

Ramond et al,
Buchmuller et al, '11

SU(5)xU(1)

1st fam. 2nd 3rd

Ψ_{10}: (5, 3, 0)
Ψ_5: (2, 0, 0)
Ψ_1: (1,-1, 0)

Equal 2,3 ch. for lopsided

Recall: $m_u \sim 10\ 10$
$m_d = m_e^T \sim \bar{5}\ 10$
$m_{\nu D} \sim \bar{5}\ 1$; $M_{RR} \sim 1\ 1$

No structure for leptons

No automatic det23 = 0

Automatic det23 = 0

Model	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	(H_u, H_d)
Anarchical (A)	(3,2,0)	(0,0,0)	(0,0,0)	(0,0)
Semi-Anarchical (SA)	(2,1,0)	(1,0,0)	(2,1,0)	(0,0)
Hierarchical (H_I)	(6,4,0)	(2,0,0)	(1,-1,0)	(0,0)
Hierarchical (H_{II})	(5,3,0)	(2,0,0)	(1,-1,0)	(0,0)
Inversely Hierarchical (IH_I)	(3,2,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)
Inversely Hierarchical (IH_{II})	(6,4,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)

all charges positive

not all charges positive

With suitable charge assignments all relevant patterns can be obtained

Example: Normal Hierarchy

G.A., Feruglio, Masina'02

1st fam. 2nd 3rd

$q(10)$: $(5, 3, 0)$
$q(\bar{5})$: $(2, 0, 0)$
$q(1)$: $(1, -1, 0)$

Note: not all charges positive --> det23 suppression

$q(H) = 0$, $q(\bar{H}) = 0$
$q(\theta) = -1$, $q(\theta') = +1$

In first approx., with $<\theta>/M \sim \lambda \sim \lambda' \sim 0.35 \sim o(\lambda_C)$

$$\underbrace{m_u}_{10_i 10_j} \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix}, \qquad \underbrace{m_d}_{10_i \bar{5}_j} = m_e^T \sim v_d \begin{bmatrix} \lambda^7 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

"lopsided"

$$\underbrace{m_{\nu D}}_{\bar{5}_i 1_j} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda' & 1 \\ \lambda & \lambda' & 1 \end{bmatrix}, \qquad \underbrace{M_{RR}}_{1_i 1_j} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}$$

Note: coeffs. O(1) omitted, only orders of magnitude predicted with $\lambda \sim \lambda'$

$$\underset{\bar{5}_i 1_j}{m_{\nu D}} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda & 1 \\ \lambda & \lambda & 1 \end{bmatrix}, \qquad \underset{1_i 1_j}{M_{RR}} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{bmatrix}$$

see-saw $\quad m_\nu \sim m_{\nu D}{}^T M_{RR}{}^{-1} m_{\nu D}$

$$m_\nu \sim v_u{}^2/M \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & \boxed{\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}} \\ \lambda^2 & & \end{bmatrix},$$

$\det_{23} \sim \lambda^2$

The 23 subdeterminant is automatically suppressed,
$\theta_{13} \sim \lambda^2$, θ_{12}, $\theta_{23} \sim 1$

This model works, in the sense that all small parameters are naturally due to various degrees of suppression.
But too many free parameters!!

Examples of mechanisms for Det[23]~0

based on see-saw: $m_\nu \sim m^T_D M^{-1} m_D$

1) A ν_R is lightest and coupled to μ and τ

King; Allanach; Barbieri et al......

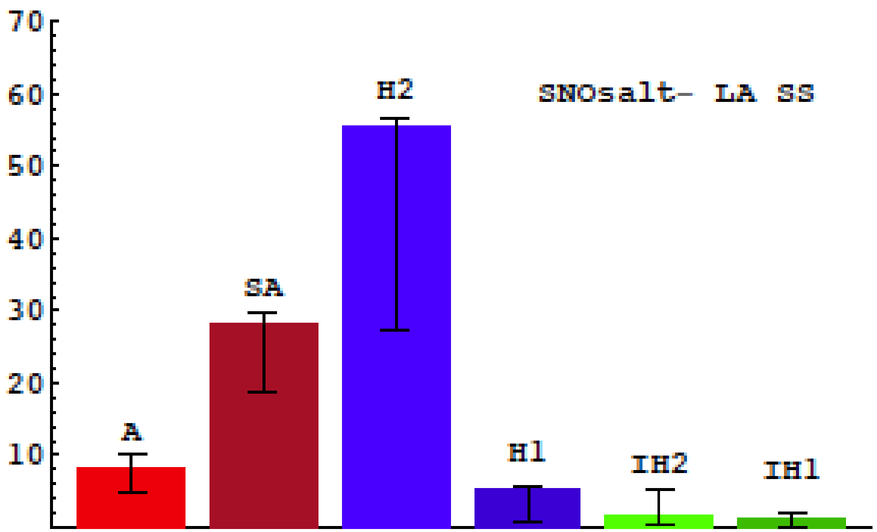
$$M \sim \begin{bmatrix} \varepsilon & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow M^{-1} \sim \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix}$$

$$m_\nu \sim \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} \approx 1/\varepsilon \begin{bmatrix} a^2 & ac \\ ac & c^2 \end{bmatrix}$$

2) M generic but m_D "lopsided" $\qquad m_D \sim \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix}$

Albright, Barr; GA, Feruglio, ...

$$m_\nu \sim \begin{bmatrix} 0 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix} = c \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$$

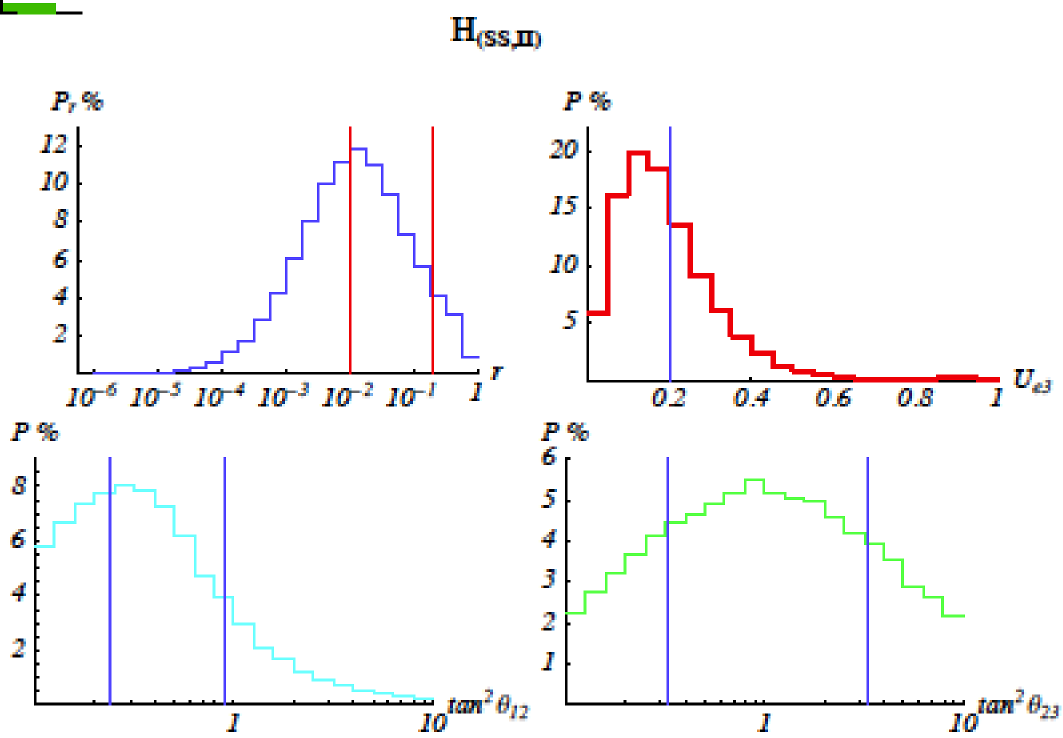


Anarchy: both r and θ_{13}
small by accident

Semianarchy: only r
small by accident

H2: no accidents

GA, Feruglio, Masina'02



We now consider models with a maximum of order:
based on non abelian discrete flavour groups

(a review G.A., Feruglio, Rev.Mod.Phys. 82 (2010) 2701
[ArXiv:1002.0211])

A number of "coincidences" could be hints
pointing to the underlying dynamics

TB Mixing

TB mixing is close to the data:
θ_{12}, θ_{23} agree within ~ 1σ

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

At 1σ: Schwetz et al '11

$\sin^2\theta_{12} = 1/3$: 0.297- 0.329
$\sin^2\theta_{23} = 1/2$: 0.45-0.58
$\sin^2\theta_{13} = 0$: 0.008 - 0.020

A coincidence or a hint?

Called:
Tri-Bimaximal mixing

Harrison, Perkins, Scott '02

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

LQC: Lepton Quark Complementarity

Raidal'04.........

$$\theta_{12} + \theta_C = (47.0 \pm 1.2)^o \sim \pi/4$$

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Suggests Bimaximal mixing corrected by diagonalisation of charged leptons

A coincidence or a hint?

Cannot be all true hints, perhaps none

Golden Ratio

Feruglio, Paris'11

$$\sin^2\theta_{12} = \frac{1}{\sqrt{5}\phi} = \frac{2}{5+\sqrt{5}} \approx 0.276$$

$$U_{GR} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ \frac{\sin\theta_{12}}{\sqrt{2}} & -\frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta_{12}}{\sqrt{2}} & -\frac{\cos\theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

A coincidence or a hint?

Neutrino mixing

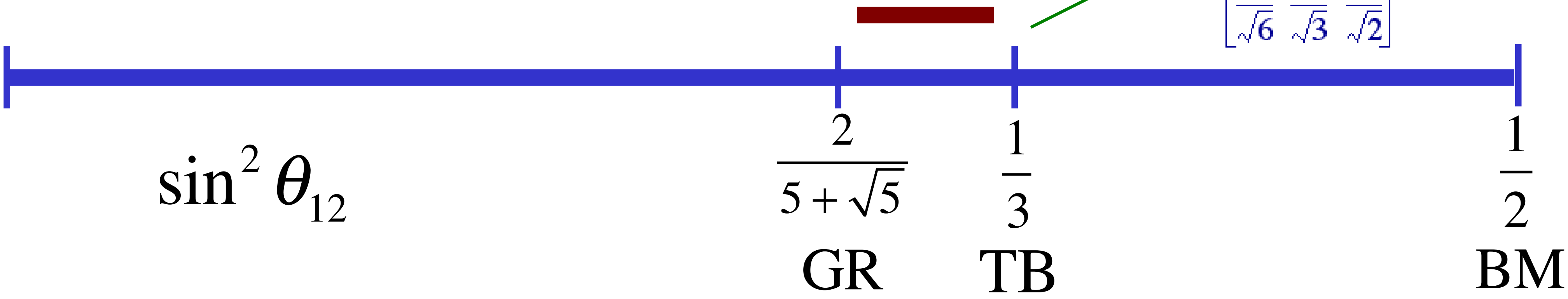
$\sin^2\theta_{23} \sim 1/2$

$\sin^2\theta_{13} \sim 0$

Very different from quarks!

Exp

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$



TB: Group A4, S4..... A vast literature

GR: Golden Ratio - Group A5 Feruglio, Paris '11

$\theta_{13} \sim o(\theta_C^2)$

BM: Group S4 GA, Feruglio, Merlo '09 $\theta_{13} \sim o(\theta_C)$

A recent review of discrete flavour groups:

GA, F. Feruglio, ArXiv:1002.0211 (Review of Modern Physics)

I concentrate now on TB mixing (the most studied)

TB Mixing naturally leads to discrete flavour groups

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

This is a particular rotation matrix with specified fixed angles

TB mixing

Harrison, Perkins, Scott

A simple mixing matrix compatible with all present data

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U diag(m_1, m_2, m_3) U^T$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad m_2 \rightarrow \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad m_1 \rightarrow \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

Compare with quark mixings $\lambda_C \sim (m_d/m_s)^{1/2}$

Why and how discrete groups, in particular A4, work?

TB mixing corresponds to m
in the basis where
charged leptons are diagonal

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

Crucial point 1:
m is the most general matrix invariant under
$SmS = m$ and $A_{23} m A_{23} = m$ with:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2-3 symmetry

$S^2 = A_{23}^2 = 1$

Crucial point 2:

Charged lepton masses:
a generic diagonal matrix
is defined by invariance under T
(or ηT with η a phase):

$$m_l = v_T \frac{v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$m_l^+ m_l = T^+ m_l^+ m_l T$$

a possible T is

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

$\omega^3=1$ --> $T^3=1$

An essential observation is that

S, T and A_{23} are all contained in S4

$S^4=T^3=(ST^2)^2=1$ define S4

Thus S4 is the reference group for TB mixing

Lam

A4: a vast literature

A4 is the discrete group of even perm's of 4 objects.
(the inv. group of a tetrahedron). It has $4!/2 = 12$ elements.

A4 is a subgroup of S4

$S^2 = T^3 = (ST)^3 = 1$ define A4

A4 has 4 inequivalent irreducible representations:
a triplet and 3 different singlets

$3,\ 1,\ 1',\ 1''$ (promising for 3 generations!)

Ch. leptons $l \sim 3$ $e^c, \mu^c, \tau^c \sim 1, 1'', 1'$

Invariance under S and T is automatic in A4 while
A_{23} is not contained in A4 (2<->3 exchange is an odd perm.)
But 2-3 symmetry happens in A4 if $1'$ and $1''$ symm. breaking
flavons are absent or have equal VEV's [2 of S4 = $1' + 1''$ of A4].

Crucial point 3: A4 must be broken: the alignment

Before SSB the model is invariant under the flavour group A4

There are flavons ϕ_T, ϕ_S, ξ... with VEV's that break A4:

ϕ_T breaks A4 down to G_T, the subgroup generated by
1, T, T^2, in the charged lepton sector

ϕ_S, ξ break A4 down to G_S, the subgroup generated by
1, S, in the neutrino sector

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u \quad , \quad \langle \tilde{\xi} \rangle = 0$$

ϕ_T, ϕ_S ~ 3

ξ ~ 1

The 2-3 symmetry occurs in A4 if 1′ and 1″ flavons are absent

This aligment along subgroups of A4 must naturally occur in a good model

At LO TB mixing is exact

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm}$

The only fine-tuning needed is to account for $r^{1/2} \sim 0.2$
[In most A4 models $r^{1/2} \sim 1$ would be expected as l, $\nu^c \sim 3$]

When NLO corrections are included from operators of higher dimension in the superpotential each mixing angle receives generically corrections of the same order $\delta\theta_{ij} \sim o(VEV/\Lambda)$

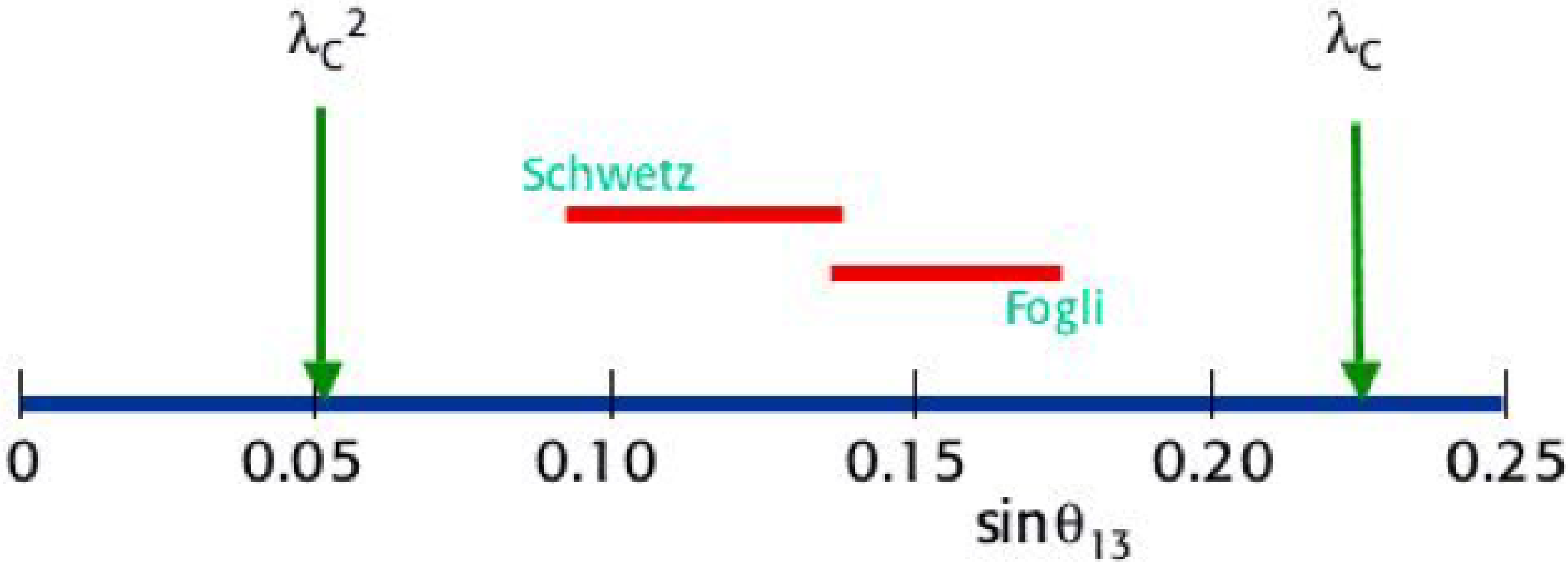
As the maximum allowed corrections to θ_{12} (and also to θ_{23}) are numerically $o(\lambda_C^2)$, we need $VEV/\Lambda \sim o(\lambda_C^2)$ and we typically expect:

$\theta_{13} \sim o(\lambda_C^2)$ data are somewhat undecided

Exp: $\theta_{13} \sim (2.2 - 3.1)\ \theta_C^2$ but also $(0.5 - 0.7)\ \theta_C$

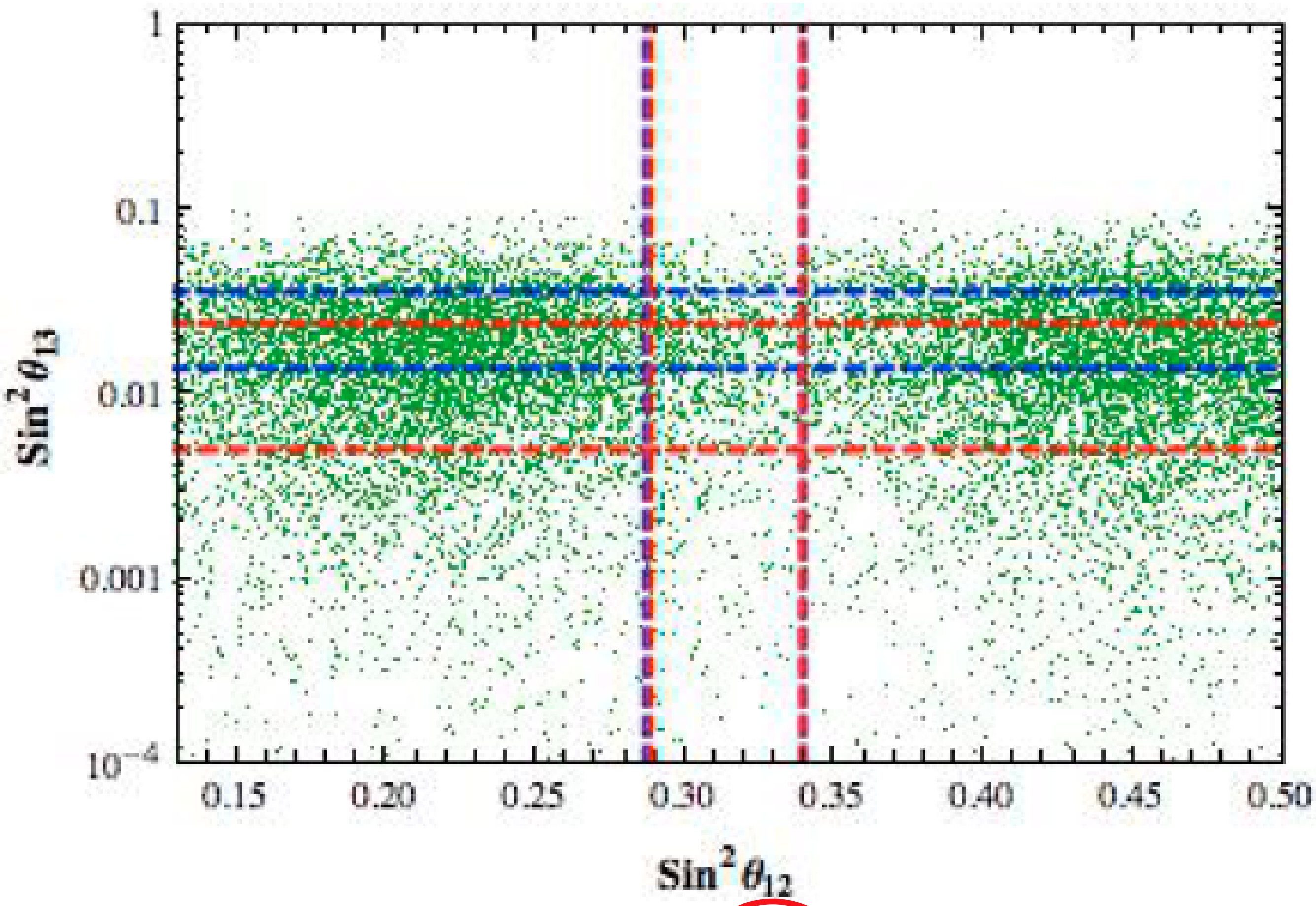
Of course the generic prediction can be altered in ad hoc versions
e.g. Lin '09 has a A4 model where $\theta_{13} \sim o(\lambda_C)$ or by allowing fine tuning

Data are not really clearcut on $q_{13} \sim o(\lambda_C^2)$ or $o(\lambda_C)$



In a typical A4 model the expansion parameter must be relatively large and some fine tuning is needed

GA, Feruglio, Merlo ‘12



(b) $\xi = 0.12$

$$\sin^2\theta_{23} = \frac{1}{2} + \mathcal{O}(\xi)$$

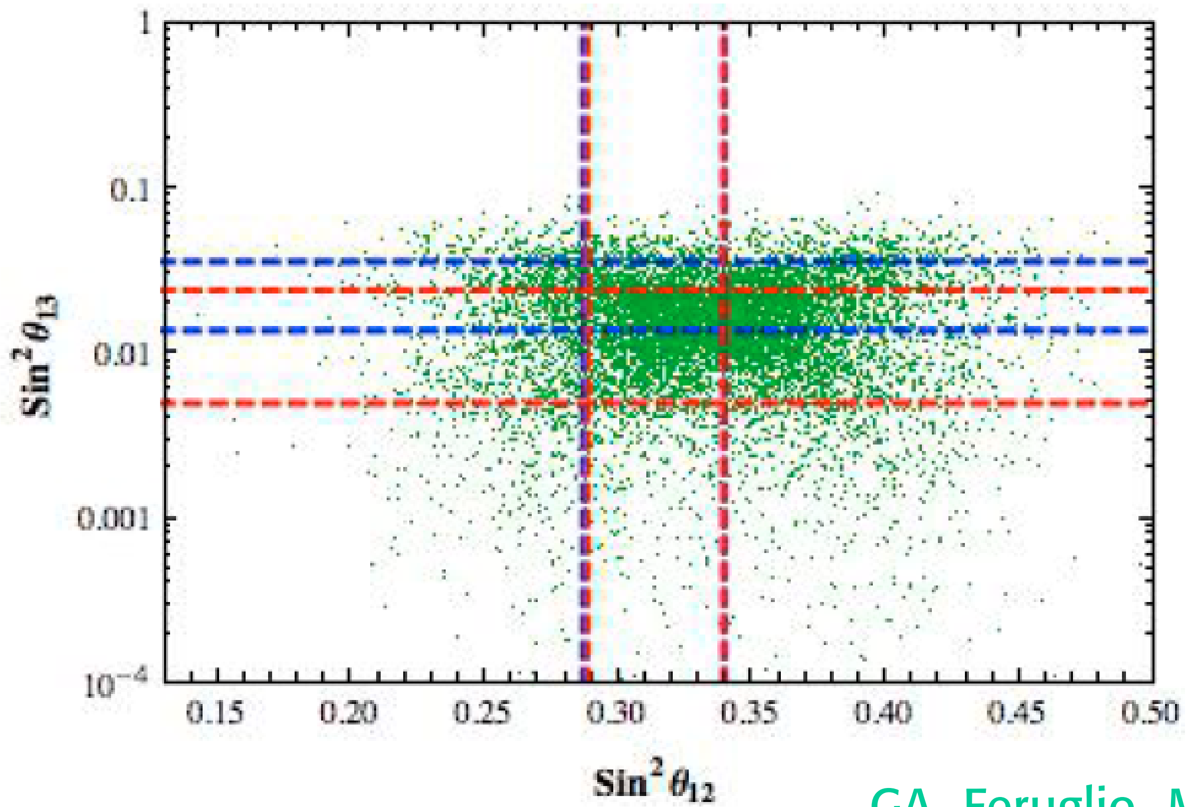
$$\sin^2\theta_{12} = \frac{1}{3} + \mathcal{O}(\xi)$$

$$\theta_{13} = \mathcal{O}(\xi)$$

In the Lin version of A4
ch. leptons and ν's
kept separate also at NLO

[ArXiv:0905.3534]

$$\delta = \arg\beta$$
$$\theta_{13} = \sqrt{\frac{2}{3}}|\beta| + c_{13}\,\xi$$
$$\sin^2\theta_{12} = \frac{1}{3} + c_{12}\,\xi$$
$$\sin^2\theta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}}\cos\delta\sin\theta_{13} + c_{23}\,\xi$$



β~0.15 and ξ~0.005-0.06

Less fine tuning
More natural

GA, Feruglio, Merlo '12

Bimaximal Mixing

Now particularly interesting since θ_{13} largish

Taking the "complementarity" relation seriously:

$$\theta_{12} + \theta_C = (47.0 \pm 1.7)^o \sim \pi/4$$

Raidal'04

leads to consider models that give $\theta_{12} = \pi/4$ but for corrections from the diag'tion of charged leptons

$$U_{PMNS} = U_\ell^\dagger U_\nu$$

Recall:

$$\lambda_C \approx 0.22 \text{ or } \sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$$

Normally one obtains $\theta_{12} + o(\theta_C) \sim \pi/4$ "weak compl." rather than $\theta_{12} + \theta_C \sim \pi/4$

The large deviations from BM mixing could arise from charged lepton diagonalisation

For the corrections from the charged lepton sector, typically $|\sin\theta_{13}| \sim (1 - \tan^2\theta_{12})/4\cos\delta \sim 0.15$

GA, Feruglio, Masina
Frampton et al
King
Antusch et al........

Needs $|\sin\theta_{13}| \sim o(\lambda_C)$ as data now suggest

$$\bar{U}_{12} = -\frac{e^{-i(\alpha_1+\alpha_2)}}{\sqrt{2}} + \frac{s^e_{12}e^{-i\alpha_2} + s^e_{13}e^{i\delta_e}}{2}$$

$$\bar{U}_{13} = \frac{s^e_{12}e^{-i\alpha_2} - s^e_{13}e^{i\delta_e}}{\sqrt{2}}$$

$$\bar{U}_{23} = -e^{-i\alpha_2}\frac{1 + s^e_{23}e^{i\alpha_2}}{\sqrt{2}}$$

$\theta_{12} + \theta_C \sim \pi/4$

difficult to get. Rather:

$\theta_{12} + o(\theta_C) \sim \pi/4$

"weak" LQC

But beware of $\mu \to e\gamma$!

Corr.'s from s^e_{12}, s^e_{13} to U_{12} and U_{13} are of first order (2nd order to U_{23})

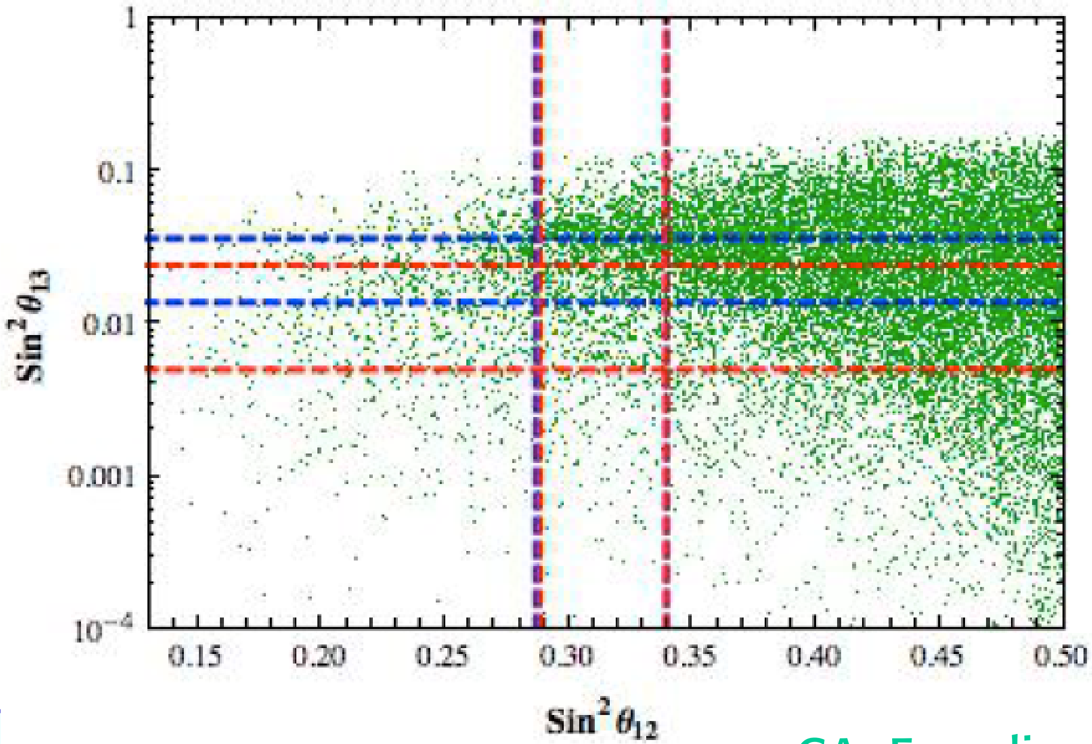
Here is a model based on S4, where BM mixing holds in 1st approximation and is then corrected by terms $o(\lambda_C)$ from the diagonalisation of charged leptons

GA, Feruglio, Merlo '09
D. Meloni '11

LO: $\theta_{12} = \theta_{23} = \pi/4$, $\theta_{13} = 0$ $\longrightarrow$

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\xi = 0.15$



NLO: $$\theta_{13} = \frac{1}{\sqrt{2}}(c_{12} - c_{13})\,\xi$$

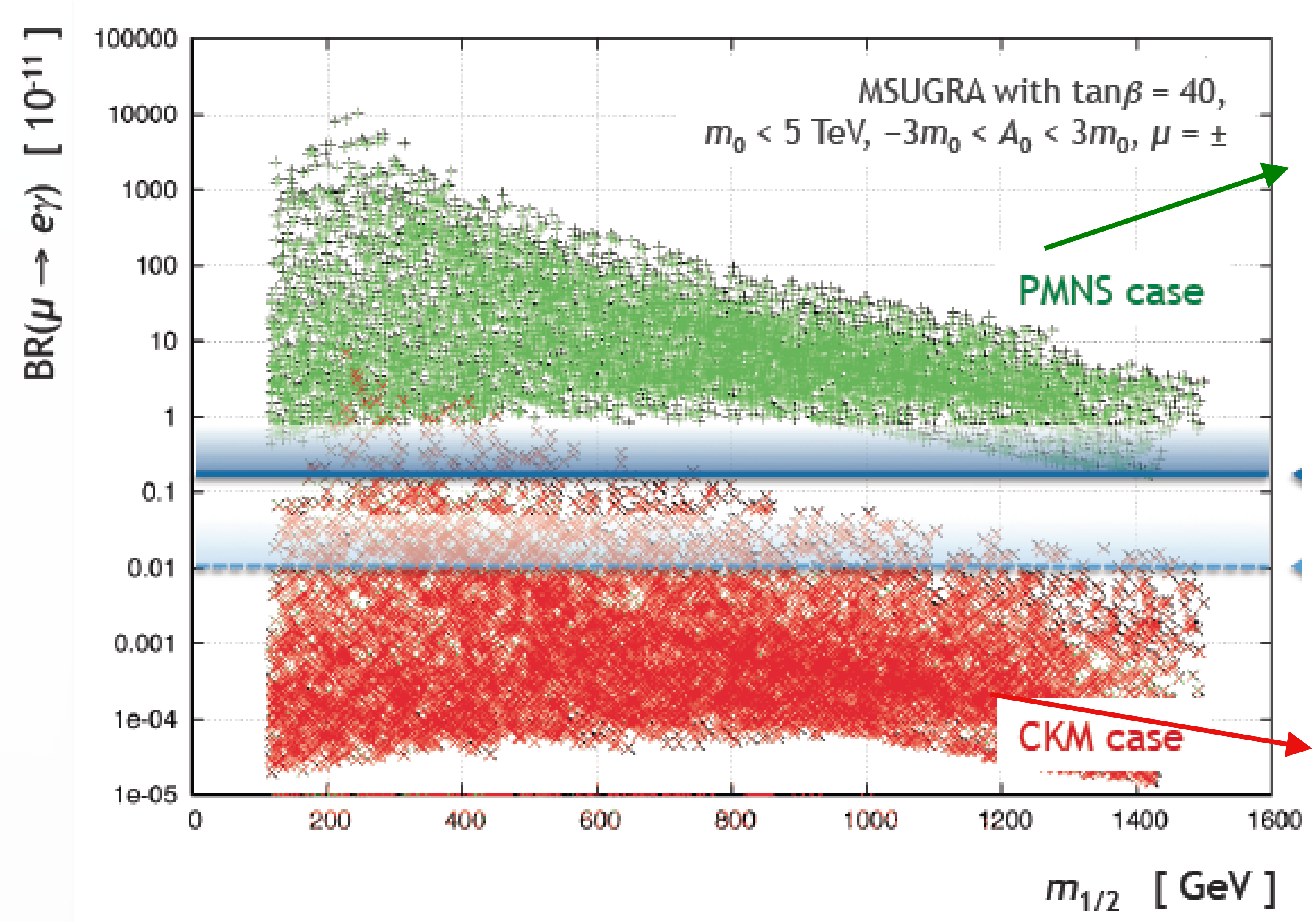
$$\sin^2\theta_{12} = \frac{1}{2} - \frac{1}{\sqrt{2}}(c_{12} + c_{13})\,\xi$$

$$\sin^2\theta_{23} = \frac{1}{2} \quad .$$

GA, Feruglio, Merlo '12

MEG new limit on Br($\mu \to e\,\gamma$) < 2.4 10^{-12}

a serious constraint on SUSY models with non diagonal mass matrices at the GUT scale



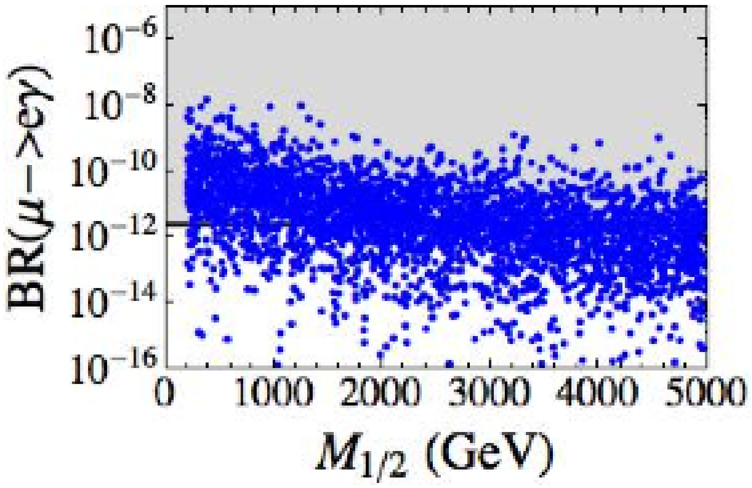
Large mixing in ν Yukawa

MEG now

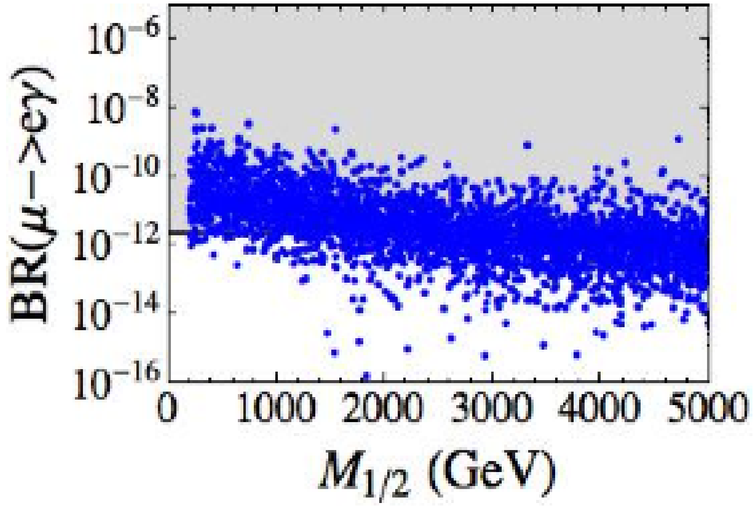
MEG goal

Small mixing in ν Yukawa

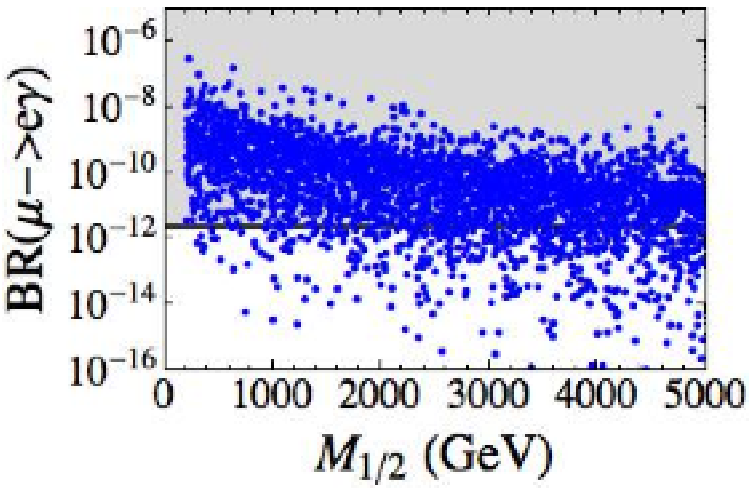
$Br(\mu \to e\gamma) < 2.4\ 10^{-12}$: a serious constraint



Typical A4, $\xi = 0.12$



Lin-type A4, $\beta = 0.15$



S4, $\xi = 0.15$

$m_0 \sim 2$ TeV large!
$\tan\beta \sim 2$-3

Comparable performances when mixing angles are reproduced

GA, Feruglio, Merlo '12

From experiment:

a good first approximation for quarks

$\lambda = \sin\theta_C$

$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & 0 \\ -\lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + o(\lambda^2)$$

$V_{CKM} = U_u^+ U_d$

and for neutrinos

$o(\lambda)$?

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix} + o(\lambda^2) ?$$

In lepton sector TB or GR or BM mixing point to discrete flavor groups

What about quarks?

A problem for GUT models is how to reconcile the quark with the lepton mixings

quarks: small angles, strongly hierarchical masses
abelian flavour symm. [e.g. $U(1)_{FN}$]
neutrinos: large angles, perhaps TB or BM
non abelian discrete symm. [e.g. A4]

Can be accomodated but quarks do not add any indication for discrete flavour groups

Summary on ν mixing

- ν mixing angles are large except for θ_{13} that is small
  but not too small, close to θ_C
- The measured values of ν mixing angles are compatible with TB or GR or BM
- If not a coincidence, this points to discrete flavour groups
  but, on the other extreme, anarchy for leptons is still a possibility
- In principle there is no contradiction between large ν mixings and small q mixings, even in GUT's
- But quarks offer no new supporting evidence for discrete flavour groups
- Natural GUT models describing all fermion masses with TB or GR or BM mixing in the lepton sector are difficult to construct, in particular for SO(10)