Introduction to Theoretical Particle Physics

Lecture: Prof. Dr. K. Melnikov
Exercises: Dr. C. Brønnum-Hansen, Dr. M. Jaquier

Exercise Sheet 12

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Exercise 26: Singlet Higgs Model — 11 points

A common way to extend the Standard Model (SM) of particle physics consists in adding more Higgs bosons. The simplest way to do so is to add an additional scalar boson to the SM $SU(2)$ Higgs doublet. In this exercise, the interactions of the SM Higgs boson are investigated. Then the effect of an additional scalar boson will be studied. Since we are adding a single scalar (a singlet), this extension of the SM is called the singlet Higgs model.

Consider the part of the SM Lagrangian responsible for symmetry breaking,

$$\mathcal{L}_{\text{EWSB}} = -\frac{\lambda}{4}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 , \tag{26.1}$$

where the Higgs field $\phi(x)$ is an $SU(2)$ doublet. After spontaneous symmetry breaking took place, the Higgs field is expanded around its vacuum expectation value (vev) v:

$$\phi(x) = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} , \tag{26.2}$$

where $h(x)$ is a real scalar.

(a) Use Eq. (26.2) to expand the potential in Eq. (26.1) in terms of the Higgs boson h. Determine the mass of the Higgs boson in terms of λ and v.

(b) In the previous sheet, you used the measured values for the masses of the W and Z bosons and the fine structure constant to evaluate $v = 250$ GeV. Knowing that the Higgs boson mass is measured to be $M_H = 125$ GeV, find the value of the coupling λ. Note that this fixes all free parameters of the SM as it has been introduced in the lecture.

(c) After symmetry breaking the terms in Eq. (26.1) with cubic or higher powers of h describe self-interactions of the Higgs boson. List the different interactions given by Eq. (26.1) after expanding in h and determine the respective coupling in terms of λ and v (similarly to the previous exercise sheet).

We now extend the SM by adding a real scalar S. The new scalar couples to the SM Higgs boson through a modified potential $V(\phi, S)$. The remaining SM Lagrangian stays unchanged. In particular, the new scalar S does not couple to the fermions and gauge bosons of the SM. The new Lagrangian term is given by

$$\mathcal{L}_{\text{extended}} = -V(\phi, S) = -\frac{\lambda}{4}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 - \frac{a_1}{2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right) S - \frac{a_2}{2} S^2 \ , \quad (26.3)$$

where ϕ is the SM $SU(2)$ Higgs doublet, S is a scalar boson and a_1 and a_2 are coupling constants.

(d) The vev's of ϕ and S are required to be extrema of the potential $V(\phi, S)$. Writing $\phi^\dagger\phi = r_\phi^2$, this yields the conditions

$$\left.\frac{\partial V(r_\phi, S)}{\partial r_\phi}\right|_{\substack{r_\phi = r_{\phi,\text{vac}} \\ S = S_{\text{vac}}}} \stackrel{!}{=} 0 \ , \quad (26.4)$$

$$\left.\frac{\partial V(r_\phi, S)}{\partial S}\right|_{\substack{r_\phi = r_{\phi,\text{vac}} \\ S = S_{\text{vac}}}} \stackrel{!}{=} 0 \ . \quad (26.5)$$

Show that

$$r_{\phi,\text{vac}}^2 = \frac{v^2}{2} \ , \qquad S_{\text{vac}} = 0 \ , \quad (26.6)$$

is a solution to these conditions.

(e) After expanding the potential Eq. (26.3) around the vev's of ϕ and S as

$$\phi(x) = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} \ , \qquad S(x) = S_{\text{vac}} + s(x) = s(x) \ , \quad (26.7)$$

the mass terms for the field excitations $h(x)$ and $s(x)$ can be written as

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}\left(M_H^2 h^2 + M_S^2 s^2 + M_{HS}^2 hs\right) \ . \quad (26.8)$$

Calculate the masses M_H^2, M_S^2 and M_{HS}^2 using Eq. (26.3).

(f) In order to get rid of the mixed term $\propto hs$ in $\mathcal{L}_{\text{mass}}$, we introduce linear combinations of $h(x)$ and $s(x)$ which depend on a mixing angle θ:

$$h_1(x) = \cos(\theta)h(x) + \sin(\theta)s(x) \ , \quad (26.9)$$

$$h_2(x) = -\sin(\theta)h(x) + \cos(\theta)s(x) \ . \quad (26.10)$$

Determine the angle θ_0 as a function of λ, v, a_1 and a_2 such that the mixed term in $\mathcal{L}_{\text{mass}}$ vanishes.

We can now choose the parameters in Eq. (26.3) such that θ_0 is small (note that measuring the masses of the two bosons does not specify all free parameters of the theory anymore). Then $h_1(x)$ is almost the SM Higgs boson $h(x)$ with a small contribution from $s(x)$. We can then study how the presence of the additional scalar boson modifies the couplings of the SM Higgs boson to other SM particles and to itself.

(g) As stated previously, the remainder of the Lagrangian describing the interaction of the fermions and gauge bosons with the Higgs boson and with themselves is the same as in the SM. In particular, the interaction between the Higgs boson and leptons is given by

$$\mathcal{L}_{\text{lepton}} = y_f(\overline{\psi}_L^f \phi \psi_R^f + \overline{\psi}_R^f \phi^\dagger \psi_L^f) \,, \tag{26.11}$$

where y_f is the Yukawa coupling for the respective lepton family. The left- and right-handed electron fields are

$$\psi_L^e = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \qquad \text{and} \qquad \psi_R^e = e_R \,, \tag{26.12}$$

and similar for the muon and tau fields. Express the coupling of h_1 to a lepton-antilepton pair and of h_2 to a lepton-antilepton pair in terms of the coupling of the SM Higgs boson to a lepton-antilepton pair and the angle θ_0.

(h) After expanding Eq. (26.3) around the vev's of the two bosons and reexpressing them in terms of h_1 and h_2, it will contain cubic and quartic terms in h_1 and h_2 which describe interactions of h_1 and h_2 with themselves as well as with each other. Determine the coupling for the interaction between three h_1 bosons and compare it to the coupling between three Higgs bosons in the SM that you determined in subquestion c).