Lecture 13

An "alternative" Standard Model

We have seen in the previous lecture that a gauge theory with $SU(2) \otimes U(1)$ gauge group and a Higgs doublet provides a valid theory of Nature *but* predicts the existence of a neutral (i.e. not charged) vector boson Z. When the SM was formulated, the Z-boson and the associated "neutral currents" (in the spirit of Fermi theory) were not yet observed. It was natural to ask then if a formulation of an alternative theory was possible that contained charged currents and electromagnetism but no neutral current.

At first sight such a possibility exists. Indeed, let us consider a pure $SU(2)$ theory and couple it to the Higgs boson in the "adjoint" (vector) representation. This means that we write the Higgs field in a way that is similar to what we do with gauge bosons

$$\hat{\varphi} = \sum_{a=1}^{3} \varphi_a \tau^a, \qquad a = 1, 2, 3. \tag{1}$$

Hence, we describe the scalar field by three *real fields* $\varphi_{1,2,3}$. The Higgs field transforms under gauge transformations as follows

$$\hat{\varphi} \to U \hat{\varphi} U^{-1}. \tag{2}$$

The kinetic term of the Lagrangian is written as

$$\mathcal{L}_{\rm kin} = {\rm Tr}\left(D_\mu \hat{\varphi} D^\mu \hat{\varphi} \right), \tag{3}$$

where

$$D_\mu \hat{\varphi} = \partial_\mu \hat{\varphi} - ig \left[\hat{W}_\mu, \hat{\varphi} \right] = \partial_\mu \hat{\varphi} + g W_\mu^a \varphi^b \epsilon_{abc} \tau^c. \tag{4}$$

It is straightforward to check that the kinetic energy in Eq. (3) is invariant under the gauge transformation of the Higgs field in Eq. (2) and of the gauge bosons

$$\hat{W}_\mu \to U \hat{W}_\mu U^{-1} + \frac{1}{ig} \left(\partial_\mu U \right) U^{-1}. \tag{5}$$

We will break the $SU(2)$ symmetry by introducing the Higgs potential

$$V(\hat{\varphi}) = -\frac{\lambda}{4} \left({\rm Tr}(\hat{\varphi}^\dagger \hat{\varphi}) - \frac{v^2}{2} \right)^2 \tag{6}$$

and choosing the vacuum expectation value for the Higgs field to be

$$\varphi^a_{\rm vac} = v \delta^{a3}. \tag{7}$$

Then, it is easy to see that the two fields $W^{1,2}$ acquire identical masses whereas the field W^3 remains massless. Indeed,

$$\begin{aligned}\mathcal{L}_{\rm kin} \to & {\rm Tr}\left[g\tau^c W^a_\mu \varphi^b_{\rm vac}\epsilon^{abc}\; g\tau^{c_1} W^{a_1,\mu}\varphi^{b_1}_{\rm vac}\epsilon^{a_1b_1c_1}\right] = \frac{\delta^{cc_1}}{2} g^2 W^a_\mu \varphi^b_{\rm vac}\epsilon^{abc}\; W^{a_1,\mu}\varphi^{b_1}_{\rm vac}\epsilon^{a_1b_1c_1} \\ & = \frac{g^2v^2}{2}\left[\vec{W}_\mu \times \frac{\vec{\varphi}_{\rm vac}}{v}\right]\cdot\left[\vec{W}^\mu \times \frac{\vec{\varphi}_{\rm vac}}{v}\right] = \frac{g^2v^2}{2}\left[\left(W^1_\mu\right)^2 + \left(W^2_\mu\right)^2\right].\end{aligned} \tag{8}$$

Similar to the SM, we can associate fields $W^\pm_\mu = W^1_\mu \pm iW^2_\mu$ with charged gauge bosons and the field W^3_μ with a photon.

The "problem" with this model arises if we attempt to couple gauge fields to leptons. Indeed, if we combine the left-handed electron and the neutrino into an $SU(2)$ doublet, the photon W^3 will couple to the neutrino. This, of course, is not compatible with the fact that the neutrino has no electric charge. We can try to get around this problem by considering a larger representation of matter fields, i.e. the adjoint (vector) representation. But this necessarily means that there will be more fermions than just the electron and the neutrino. This is also not a desirable feature of the model but there is nothing one can do about it.

So, we consider a fermion in the adjoint (vector) representation

$$\hat{\psi} = \psi_a \tau^a. \tag{9}$$

Note that at this point ψ is a full Dirac fermion, with its left and right components. This is necessary since photons couple to both left and right fields with equal strength. The interaction of the triplet with the gauge field is written as

$$L_\psi = 2i{\rm Tr}\left[\hat{\bar{\psi}}\left(\hat{\partial}\hat{\psi} - ig[\hat{W},\hat{\psi}]\right)\right] = i\bar{\psi}^a\left(\hat{\partial}\delta^{ac} + g\epsilon^{abc}\hat{W}^b\right)\psi^c. \tag{10}$$

This interaction is invariant under gauge transformations of the field ψ

$$\hat{\psi} \to U\hat{\psi}U^{-1}, \tag{11}$$

and the gauge transformation of the field W, c.f. Eq. (5).

We write explicitely terms in Eq. (10) that describe interactions of gauge bosons and fermions. We find

- W^3_μ interacts with $\bar{\psi}^1\gamma^\mu\psi^2 - \bar{\psi}^2\gamma^\mu\psi^1$,
- W^2_μ interacts with $\bar{\psi}^3\gamma^\mu\psi^1 - \bar{\psi}^1\gamma^\mu\psi^3$,
- W^1_μ interacts with $\bar{\psi}^2\gamma^\mu\psi^3 - \bar{\psi}^3\gamma^\mu\psi^2$.

Since photon interactions do not change the "type" of a lepton, it is useful to make a field redefinition to make this explicit. We write

$$E = \frac{\psi_1 + i\psi_2}{\sqrt{2}}, \quad e = \frac{\psi_1 - i\psi_2}{\sqrt{2}}, \tag{12}$$

so that

$$\psi_1 = \frac{E+e}{\sqrt{2}}, \qquad \psi_2 = \frac{E-e}{\sqrt{2}}. \tag{13}$$

A simple computation gives the interaction term

$$igW_\mu^3 \left[\bar{\psi}_1\gamma^\mu\psi_2 - \bar{\psi}_2\gamma^\mu\psi_1\right] = gW_\mu^3 \left[\bar{E}\gamma^\mu E - \bar{e}\gamma^\mu e\right]. \tag{14}$$

The above equation implies that if we associate the field W_3^μ with a photon, our theory contains two fermions E and e, with electric charges $+g$ and $-g$, respectively. Moreover, since ψ_3 is a full Dirac fermion, we can not identify it with the neutrino since only left-handed neutrinos exist.[1]

To break the symmetry between left and right components of the Dirac field ψ, we introduce yet another fermion – a left-handed *singlet* fermionic field σ_L. We can then write the fermion mass term in the Lagrangian as follows

$$L_{\rm mass} = m_0\bar{\psi}_a\psi_a + f_1\left[\bar{\psi}_a\varphi_a\sigma_L + h.c.\right] + if_2\bar{\psi}_a\epsilon_{abc}\psi_b\varphi_c. \tag{15}$$

We now substitute the vacuum expectation value of the Higgs field $\varphi^a \to \varphi^a_{\rm vac} = v\delta^{a3}$ into $L_{\rm mass}$ and find

$$\begin{aligned} L_{\rm mass} \to & (m_0+f_2v)\,\bar{E}E + (m_0-f_2v)\,\bar{e}e \\ & + \bar{\psi}_{3,R}\left[m_0\psi_{3,L} + f_1v\sigma_L\right] + \left[m_0\bar{\psi}_{3,L} + f_1v\bar{\sigma}_L\right]\psi_{3,R}. \end{aligned} \tag{16}$$

We observe that the masses of the two charged leptons are split and that there is a combination of $\psi_{3,L}$ and σ_L that acquires a mass. We interpret this combination as a massive neutral fermion X. We write its left and right components as

$$X_R = \psi_{3,R}, \quad X_L = \cos\beta\psi_{3,L} + \sin\beta\ \sigma_L, \tag{17}$$

where

$$\cos\beta = \frac{m_0}{\sqrt{m_0^2+f_1^2v^2}}, \quad \sin\beta = \frac{f_1v}{\sqrt{m_0^2+(f_1v)^2}}. \tag{18}$$

Hence, the mass term in the Lagrangian reads

$$m_E\bar{E}E + m_e\bar{e}e + m_X\bar{X}X, \tag{19}$$

where

$$m_E = m_0 + f_2v, \quad m_e = m_0 - f_2v, \quad m_X = \sqrt{m_0^2+(f_1v)^2}. \tag{20}$$

The orthogonal combination of left-handed fields that remains massless is then a massless neutrino

$$\nu_L = -\sin\beta\psi_{3,L} + \cos\beta\sigma_L. \tag{21}$$

To understand the couplings of neutrinos to gauge bosons, we need to write ψ_3 through ν and X. The result reads

$$\psi_{3,L} = \cos\beta\ X_L - \sin\beta\ \nu_L, \qquad \psi_{3,R} = X_R. \tag{22}$$

[1] We know now that this statement is not entirely correct.

Re-expressing currents that describe interactions of fermions with gauge bosons $W^{(1,2)}$, we find that there are right-handed charged currents where the electron e becomes a massive neutral fermion X, in addition to standard $e_L \to \nu_L$ transitions. The right-handed currents are not observed experimentally. To explain this, we can take X to be very heavy. This is achieved by considering $f_1 v$ to be much bigger than m_0. In this case, $\sin\beta \approx 1$, so that X_L is nearly σ_L. The coupling of ν_L to e_L in this limit is given by the gauge coupling g.

Hence, the price we pay for avoiding neutral currents is 1) the right-handed charged currents; 2) the existence of a positively-charged fermion E and 3) the existence of a heavy neutral particle X. It is not possible to decide theoretically what is better – a more complex matter sector or a more complex gauge sector; what is right and what is wrong is decided by experiments. Experiments in the 1970's discovered the neutral currents and, eventually, the Z bosons, but did not discover any additional fermions or right-handed charged currents. These results strongly supported the Standard Model based on the $SU(2) \otimes U(1)$ gauge group as the correct theory of weak interactions.