INSTITUT FÜR THEORETISCHE TEILCHENPHYSIK

Einführung in die Flavourphysik
WiSe 2017/18

PROF. U. NIERSTE
DR. I. NISANDZIC

Exercise Sheet 4

Problem 6: Parametrization of the phase space

The partial decay width for a decay of a particle with mass M and 4-momentum P ($P^2 = M^2$) into n particles with masses m_i ($i = 1, \dots, n$) is given by

$$\Gamma = \frac{1}{2M} \int d^4p_1 \cdots d^4p_n \Phi_n(P, m_1, \dots, m_n; p_1, \dots, p_n) |\mathcal{M}(p_1, \dots, p_n)|^2.$$

where $\mathcal{M}(p_1, \dots, p_n)$ is matrix element of the decay process and

$$\Phi_n(P, m_1, \dots, m_n; p_1, \dots, p_n) = (2\pi)^{4-3n} \delta^{(4)}(P - \textstyle\sum_{i=1}^n p_i) \displaystyle\prod_{i=1}^n \delta(p_i^2 - m_i^2)\theta(p_i^0)$$

is Lorentz-invariant phase-space integration measure.

a) Show that for $r \in \{1, \dots, n-2\}$ the following holds:

$$\begin{aligned}\Phi_n&(P, m_1, \dots, m_n; p_1, \dots, p_n) \\ &= \int_0^\infty \frac{ds}{2\pi} \int d^4q \, \Phi_{r+1}(P, \sqrt{s}, m_1, \dots, m_r; q, p_1, \dots, p_r) \\ &\qquad \times \Phi_{n-r}(q, m_{r+1}, \dots, m_n; p_{r+1}, \dots, p_n).\end{aligned}$$

b) Parametrize the 2-body phase space by integrating all δ-functions in

$$\int d^4p_1 d^4p_2 \, \Phi_2(P, m_1, m_2; p_1, p_2) |\mathcal{M}(p_1, p_2)|^2.$$

Treat the $\mathcal{M}(p_1, p_2)$ as the function of momenta p_1 and p_2. Use the Lorentz invariance of $|\mathcal{M}(p_1, p_2)|^2$ that dictates the dependence of $|\mathcal{M}(p_1, p_2)|^2$ only on p_1^2, p_2^2 and $(p_1 + p_2)^2$.

c) Parametrize the 3-body phase space in

$$\int d^4p_1 d^4p_2 d^4p_3 \, \Phi_3(P, m_1, m_2, m_3; p_1, p_2, p_3) |\mathcal{M}(p_1, p_2, p_3)|^2$$

by integrating out all δ-functions.

Problem 7: Semileptonic $\boldsymbol{B}$-decays

Using the results from problem 5, calculate the inclusive decay width $\Gamma(b \to c\ell\nu_\ell)$ for $m_b = 4.7\,\mathrm{GeV}$ and $m_c = 1.5\,\mathrm{GeV}$. Consider the semileptonic branching ratio $B_{\mathrm{sl}} = \Gamma(\bar{B}^0 \to X_c\ell\nu_\ell)\tau_{\bar{B}^0}$, where $\tau_{\bar{B}^0} = 1.527\,\mathrm{ps}$ is the lifetime of $\bar{B}^0$. Set $\Gamma(\bar{B}^0 \to X_c\ell\nu_\ell)$ equal to $\Gamma(b \to c\ell\nu_\ell)$ and determine the $|V_{cb}|$ from the experimental value $B_{\mathrm{sl}} = 0.105 \pm 0.001$.